Optimal Capital Management in Banking

T. Bosch, J. Mukuddem-Petersen, M.P. Mulaudzi and M.A. Petersen *

Abstract—**With the drafting of new banking regulation via the Basel II capital accord, bank regulatory capital and its adequacy has become the subject of much debate. In our contribution, we strive to construct a stochastic dynamic model to describe the evolution of bank capital that incorporates capital gains and losses. In our paper, such gains and losses are represented by loan loss reserves and the unexpected loan losses, respectively. In this regard, we recognize that bank capital consists of Tier 1 capital (mainly equity which is modeled via an exponential Lévy process), Tier 2 and Tier 3 capital (collectively known as supplementary capital). The latter two types of capital mainly consist of (short- and long-term) subordinate debt held by debtholders and loan losses reserves. Furthermore, we setup an optimal capital management problem which maximizes the expectation of bank capital under a risk constraint on the Capital-at-Risk (CaR), where CaR is defined in terms of Value-at-Risk (VaR). In particular, we seek an optimal bank capital management strategy in a mean-CaR paradigm. Historical evidence from Organization for Economic Corporation and Development (OCED) countries assists us in establishing the relationship between output gap (as the proxy of business cycle) and capital adequacy ratios (CARs). Finally, we provide a brief analysis of some of the optimal capital management issues and suggestions for topics of possible future direction.**

Keywords: Dynamic Modeling; Unexpected Loan Losses; Portfolio Optimization; Capital-at-Risk (CaR); Value-at-Risk (VaR).

1 Introduction

With the drafting of new banking regulation via the Basel II capital accord, bank regulatory capital and its adequacy has become the subject of much debate. This accord prescribes that financial institutions (banks) should maintain a minimum level of regulatory capital (see, for instance, [7] and [8]). This regulation is normally justified as a response to the negative externalities arising from bank failures and to the risk-shifting incentives created by deposit insurance. The predecessor of this capital accord, viz., Basel I imposed uniform capital requirements based on risk-adjusted assets, defined as the sum of assets positions multiplied by assets-specific risk weights. These risk weights were intended to reflect primarily the asset's credit risk. In 1996 Basel I was amended to include additional minimum capital reserves to cover market risk, defined as the risk arising from movements in the market prices of trading positions (see [4]). The 1996 Amendment's Internal Models Approach (IMA) determines capital requirements on the basis of the output of the financial institutions' internal risk measurement systems and is related to market risk only. Banks are required to report daily their VaR at 99 % confidence level over both a one day and two weeks (10 trading days) horizon. The minimum capital requirement is then the sum of premiums to cover credit, market risk and operational risk (see, for instance, [12]). The credit risk premium is made up of 8 % of risk weighted assets and the market risk premium is equal to a multiple of the average reported two weeks VaRs in the last 60 trading days. In the sequel, we suppose that ρ is the m-dimensional stochastic process that represents the *current value of risky assets*. In this case, the dynamics of the *current value of the bank's entire asset portfolio*, A, over any reporting period may be given by

$$dA_t = A_t\left\{r^{\mathrm{T}} + \rho^T\left(\mu + \mathbf{E}(d)\right)\right\}dt + A_t\sigma dL_t - r^{\mathrm{T}}D_t dt \quad (1)$$

where D, is the face value of the deposits and $r^{\mathrm{T}}D_t dt$ represents the interest paid to depositors. The charge to cover credit risk equals the sum of the bank's long and short trading positions multiplied by asset specific risk weights. As a result, if we let $\omega \in [0,1]^m$ denote the $m \times 1$ vector of asset risk weights, then the *capital charge to cover credit risk* at time t equals

$$\omega^T\left(\rho_t^+ + \rho_t^-\right), \quad (2)$$

where for any ρ we denote by ρ^+ the $m \times 1$ vector and by ρ^- the $m \times 1$ vector with components

$$\rho_i^+ = \max[0,\rho_i] \text{ and } \rho_i^- = \max[0,-\rho_i], \text{ respectively.}$$

We suppose that the bank reports its current VaR at the beginning of each reporting period as well as its recorded profits and losses from the previous reporting period. The charge to cover market risk equals the VaR reported at the beginning of the current reporting period times a multiple k. As a consequence of the above, if $VaR \geq 0$ denotes the VaR reported to regulators at the beginning of the current reporting period and k is the currently-applicable multiple, the bank's total risk-weighted assets are given by

*Department of Mathematics and Applied Mathematics, North-West University (Potchefstroom), Private Bag X6001 Potchefstroom, 2520, North-West, South Africa; Email: Thahir.Bosch@nwu.ac.za; Email: Janine.MukuddemPetersen@nwu.ac.za; Email: Mmboniseni.Mulaudzi@nwu.ac.za; Email: Mark.Petersen@nwu.ac.za

$$A_t^r = kVaR + \omega^T\left(\rho_t^+ + \rho_t^-\right) + \max\left[\sum_{k=1}^{8}\beta_k g_k,\ 0\right], \quad (3)$$

at all times during the reporting period. The reported VaR can differ from the true VaR since the bank's future trading strategy, and hence the bank's true VaR, are unobservable by regulators. In inequality (3), the term

$$\max\left[\sum_{k=1}^{8}\beta_k g_k,\ 0\right]$$

is the charge to cover operational risk under the standardized approach from Basel II. One of the main objectives of the Basel II is to align economic and regulatory capital (see, for instance, [5] and [6]). Furthermore, the new accord is more attuned to risk sensitivity issues than Basel I with the introduction of, for instance, operational risk. Moreover, the capital adequacy ratio (CARs) from Basel II provide a summary measure of the extent to which weakness in the banking system may have created a drag on the macroeconomy. More specifically, if weakened banks seek to rebuild or even raise their CARs, this will tend to restrict the supply of credit with resultant effects on the macroeconomy. The move to Basel II capital standards may cause CARs to rise. This tendency may occur through and beyond a recession (see, for instance, [8]).

In this paper, we strive to construct a stochastic dynamic model to describe the evolution of bank capital that incorporates capital gains and losses, where such gains and losses are defined as the addition of loan loss reserves and the unexpected loan losses, respectively. In addition, our paper models the value process of equity (risky) and the unexpected loan losses by an exponential Lévy process and a compound Poisson process, respectively (see, for instance, [9] and [10]). Here the subordinate debt is modeled as a riskless bond. These models enable us to set-up an optimal capital management problem, which maximizes the expectation of bank capital subject to a risk constraint on the CaR.

The problems that we address in this paper may be stated as follows.

Problem 1.1 (Dynamic Modeling of Bank Capital): *Can we construct a stochastic dynamic model to describe the evolution of bank capital that incorporates capital gains and losses ?* **(Section 2)**

Problem 1.2 (Optimal Capital Management): *Can we set-up an optimal capital management problem, which maximizes the expectation of bank capital under a risk constraint on the Capital-at-Risk (CaR) ?* **(Section 2)**

2 Banking Model

In this section, a short description of the banking model is given. The bank starts out with initial capital, k and receives capital gains at the rate $r^l > 0$. The capital losses are given by unexpected loan losses, D^u, which are modeled by a compound Poisson process (see Subsection 2.1 below). In order to model the uncertainty associated with these items we consider the filtered probability space $(\Omega_1,\ \mathbf{F},\ (\mathcal{F}_t)_{0\le t\le T},\ \mathbf{P})$. Furthermore, for the planning period $[0,T]$, we consider a characterization of the *value of bank capital* of the form

$$\text{Total Bank Capital } (K_t) = \text{Initial Bank Capital } (k) \quad (4)$$
$$+ \text{ Capital Gains } (r^l t) - \text{Capital Losses } (D_t^u),$$

where $r^l > 0$ is the constant rate of inflow of loan loss reserves. In the sequel, capital gains are represented by loan loss reserves only.

2.1 Unexpected Loan Losses

In the sequel, we assume that unexpected loan losses are recorded at the times

$$0 = T_0 < T_1 < T_2 < \ldots,$$

where the corresponding unexpected loan losses amounts are describe by the non-negative random variables

$$l_1 < l_2 < \ldots,$$

called the *unexpected loan losses sizes*. Let

$$N_t = \sup\{n \ge 1 :\ T_n < t\}, \quad \sup(\emptyset) \equiv 0,$$

be the number of loan losses recorded during the interval $[0,t]$. We start with some initial bank capital, $k > 0$, which may, if necessary, be augmented by a constant interest rate of loan loss reserves $r^l > 0$. The *unexpected loan losses* is modeled by a compound Poisson process

$$D_t^u = \begin{cases} \sum_{j=1}^{N_t} l_j, & N_t > 0; \\ 0, & N_t = 0. \end{cases} \quad (5)$$

Here $(l_n)_{n\in\mathbb{N}}$ is a sequence, independent of N, of positive i.i.d. random variables with a distribution function F and a mean $\mu = \mathbf{E}[l] < \infty$, modeling the values of the unexpected loan losses. Throughout our contribution, we will denote the value of the generic unexpected loan losses by l. The times between loan losses being recorded

$$T_n - T_{n-1}, \;\; n \geq 1$$

are i.i.d. exponentially distributed random variables with parameter $\iota > 0$. The processes $\{T_n\}_{n\geq 1}$ and $\{l_n\}_{n\geq 1}$ are independent. It follows that the *unexpected loan loss number process*, $N = (N_t)_{t\geq 0}$, is a homogeneous Poisson process with an intensity $\iota > 0$, i.e.,

$$\mathbf{P}(N_t = i) = \exp\{-\iota t\}\frac{(\iota t)^i}{i!}, \;\; i = 0,\; 1,\; 2,\; \ldots$$

so that $\{D_t^u\}_{t\geq 0}$ is a compound Poisson process, which is a Lévy process.

2.2 Bank Capital

In this subsection, we discuss equity and subordinate debt, a model for bank capital and the discounted net capital loss process.

2.2.1 Equity and Subordinate Debt

The total value of the bank capital, $K = (K_t)_{t\geq 0}$, can be expressed as

$$K_t = K_t^{T1} + K_t^{T2} + K_t^{T3}, \tag{6}$$

where K^{T1}, K^{T2} and K^{T3} are Tier 1, Tier 2 and Tier 3 capital, respectively. *Tier 1 (T1) capital* is the book value of the bank's equity, $E = (E_t)_{t\geq 0}$, plus retained earnings, $E^R = (E_t^R)_{t\geq 0}$. Tier 2 (T2) and Tier 3 (T3) capital (collectively known as *supplementary capital*) is the sum of subordinate debt, O, and loan-loss reserves, R^l. However, for sake of argument, we suppose that

$$K_t = E_t + R^l + O(t). \tag{7}$$

The value processes for subordinate debt, O, and equity, E_t, are given by

$$O(t) = e^{\delta t} \;\text{ and }\; E_t = e^{L_t}, \;\; t \geq 0,$$

where $\delta > 0$ is the riskless interest rate and L is the Lévy process defined below. The corresponding stochastic differential equations (SDEs) for the above value processes are obtained via Itô's formula for Lévy process (see [10]) as

$$dO(t) = \delta O(t)dt, \;\; t > 0, \;\; O(0) = 1 \tag{8}$$

and

$$\begin{aligned} dE_t &= E_{t^-}d\widetilde{L}_t \\ &= E_{t^-}\left\{dL_t + \frac{\sigma^2}{2}dt + e^{\Delta L_t} - 1 - \Delta L_t\right\}, \\ & \quad t > 0, \;\; E_0 = 1. \end{aligned} \tag{9}$$

Note that $\Delta L(t,\omega) = L(t,\omega) - L(t^-,\omega)$ for all $\omega \in \Omega$, denotes the jump of L at time $t > 0$, and $\widetilde{L}_t$ is a Lévy process such that

$$e^L = \xi(\widetilde{L}_t). \tag{10}$$

In the sequel ξ represents the stochastic exponential of a Lévy process (see, for instance, [10], Section 2.8, and [9, Section 8.4.2]). The Lévy process L has a characteristic exponent ψ, so that

$$\mathbf{E}[e^{isL_t}] = e^{t\psi_s}, s \in \Re, \; t \geq 0,$$

where ψ has a Lévy-Khintchine representation

$$\begin{aligned} \psi_s &= is\gamma - \frac{\sigma^2}{2}s^2 \\ &+ \int_{-\infty}^{+\infty}(e^{isx} - 1 - isx1_{\{|x|\leq 1\}})\nu(dx), s \in \Re, \end{aligned}$$

with $\gamma \in \Re$, $\;\sigma \geq 0$ and Lévy measure ν with

$$\int_{\Re}(x^2 \wedge 1)\nu(dx) < \infty, \;\; \nu(\{0\}) = 0.$$

In this case, the characteristic triplet (γ, σ^2, ν) completely determines the distribution of L. From (8), we see that the SDE of subordinate debt, O, becomes an ordinary differential equation (ODE) which is an indication that the value process for the subordinate debt is riskless.

2.2.2 Dynamic Modeling of Bank Capital

In the sequel, the total bank capital is made for the unexpected loan losses. In particular, the bank capital portfolio consists of the subordinate debt, O_t, equity, E_t, and the loan loss reserves, R^l. Furthermore, at each point in the planning period $[0, T]$ a fixed fraction, denoted by $\pi \in [0, 1]$, is assigned to the equity while the rest, $1 - \pi$, constitute the subordinate debt (sometimes called a constant mix strategy). We call the fraction, π, the bank capital management strategy, K_π the total bank capital under strategy π. For $t > 0$ and D^u, E_t, $\widetilde{L}_t$ and

ISBN:978-988-17012-3-7

O_t given by (5), (9), (10) and (8), respectively. Following (4) the stochastic dynamic model of bank capital is given by

$$dK_t^\pi = K_{t-}^\pi \left\{ (1-\pi)\delta dt + \pi d\widetilde{L}_t \right\} + r^l dt \quad (11)$$
$$-dD_t^u, \;\; t > 0, \;\; K_0^\pi = k$$

where $k > 0$ is the initial capital, $r^l > 0$ is the constant interest rate from loan loss reserves, D_t^u is the unexpected loan losses, δ is the riskless interest rate and $\widetilde{L}_t$ is a Lévy process such that $e^L = \xi(\widetilde{L}_t)$.

If the unexpected loan losses, D^u, and the bank capital process, K, are independent, for $t \geq 0$, then by [1, Lemma 1.4] the solution for (11) is given by

$$K_t^\pi = e^{L_t^\pi}\left(k + \int_0^t e^{-L_s^\pi}(r^l ds - dD_s^u)\right), \quad (12)$$

where the Lévy process, L_π, has a characteristic triplet $(\gamma_\pi, \sigma_\pi^2, \nu_\pi)$ specified with respect to L introduced above.

2.2.3 Discounted Net Capital Loss Process

We deal with discounted losses by transforming the bank capital process in order to obtain the *discounted net capital loss process* given by

$$\begin{aligned} w_t^\pi &= K_t - e^{-L_t^\pi} K_t^\pi \quad (13) \\ &= \int_0^t e^{-L_s^\pi}(dL_s - kds), \;\; t \geq 0. \end{aligned}$$

This process characterizes the total discounted (to time 0) net loss of the bank from its capital allocation and loan losses. An important relation between the bank capital process and the discounted net loss process is

$$\mathbf{P}(K_t^\pi < 0 | K_0^\pi = k) = \mathbf{P}(w_t^\pi > k), \; t \geq 0. \quad (14)$$

In addition to the characteristic exponents, ψ and ψ_π, of L and L_π, respectively, we consider the corresponding Laplace exponents, φ and φ_π, (assuming their existence) defined by

$$\begin{aligned} \varphi(z) = \psi(iz) = \log \mathbf{E}[e^{-zL_1}] \quad (15) \\ \varphi_\pi(z) = \psi_\pi(iz) = \log \mathbf{E}[e^{-zL_1^\pi}]. \end{aligned}$$

Proposition 2.1 (Almost Surely Limit of w^π): *Let us assume that*

$$\mathbf{E}[l] = \mu < \infty, 0 < \mathbf{E}[L_1] < \infty, r < \varphi(-1).$$

As a consequence, we have that if $\varphi_1 < \iota$, then, for all $\pi \in [0,1]$, we have that

$$w_t^\pi \to w_\infty^\pi \; a.s., \;\; t \to \infty, \quad (16)$$

for the random variable $w_\infty^\pi < \infty$. Moreover, if $\varphi_1 \geq \iota$, then (16) *holds for all $\pi \in [0, \pi_k)$, where $\pi_k \in (0,1]$ is the unique strictly positive solution to $\varphi_1^\pi = \iota$.*

2.3 Value-at-Risk

In this subsection, we discuss Value-at-Risk and its definition and measurement.

2.3.1 Definition of Value-at-Risk

In this subsection, we provide a definition of the VaR for our model. In the banking industry, VaR is standard risk measure. Generally, VaR is specified as some high quantile of the corresponding loss distribution. The Basel II capital accord (see, for instance, [2] and [3]) permits an approach involving internal VaR models in order to determine the bank's capital requirements. This approach is not only allowed but in fact encouraged under Basel II.

2.3.2 Measure of Value-at-Risk

The distribution of the a.s. limit w_π^∞ in Proposition 2.1 allows us to obtain a measure of the risk in a stationary sense. This fact is borne out by the following definition.

Definition 2.2 (Value at Risk with Probability α):

Suppose that Proposition 2.1 *holds. Let $\pi \in \Pi \subseteq [0,1]$ be the non-empty interval for which* (16) *is satisfied. In this case, for some probability $\alpha \in (0,1)$, we define*

$$\mathbf{VaR}_\alpha(w_\pi^\infty) = \inf\{y \in \Re : \mathbf{P}(w_\pi^\infty > y) \leq \alpha\}.$$

2.3.3 Definition of Capital-at-Risk

In this section, we define the Capital-at-Risk that is appropriate to our model. Suppose that k is the initial bank capital and T a given planning horizon. In addition, let y_α be the α-quantile of the distribution of $\xi(\pi\widetilde{L}_t)$ for some bank capital management strategy $\pi \in [0,1]$, $\pi \leq 1$, and K_T^π the corresponding terminal bank capital. Furthermore, we consider $\mathbf{VaR}_\alpha(w_\pi^\infty)$ as given by definition 2.2, then

$$\mathbf{CaR}_\alpha(w_\pi^\infty) = ke^{\delta T} - \mathbf{VaR}_\alpha(w_\pi^\infty)$$

represents the Capital-at-Risk of the capital portfolio strategy π.

3 Optimal Capital Management

The statement of the optimal capital management problem based on issues related to bank capital and regulatory constraints from the Basel II capital accord is given as follows.

Problem 3.1 (Statement of the Optimal Capital Management Problem): *Suppose that K_t^π is defined by* (11) *and* (12). *Then, for $t > 0$, we formulate the optimal capital management problem for bank as*

$$\max_{\pi\in\Pi} \mathbf{E}[K_t^\pi] \text{ subject to } \mathbf{CaR}_\alpha(w_\infty^\pi) \leq C \qquad (17)$$

for given constraint, $C > 0$, and probability, α, and some fixed time $t > 0$.

Unfortunately, there is no analytic method that can be used to determine the solution of the optimal capital management problem above. However, we can be able to determine the behavior of the solution to this problem as follows. In the sequel, the result below suggests that the solution to the optimal capital management problem does not depend on initial bank capital, $k > 0$, and time, $t > 0$.

Lemma 3.2 (Time and Initial Value Independence): *Suppose that K_π have the form (12) with $\mathbf{E}[l] = \mu < \infty$. In addition, assume that*

$$\varphi(-1) = k\mathbf{E}[e^{L_1}] < \infty.$$

Then, for $t \geq 0$, we have that $\mathbf{E}[K_t^\pi]$ exists and

$$\mathbf{E}[K_t^\pi] = k\mathbf{E}[e^{L_t^\pi}] + (r^l - \iota\mu)\int_0^t \mathbf{E}[e^{L_s^\pi}]ds,$$

where

$$\mathbf{E}[e^{L_t^\pi}] = \exp\{t(\delta + \pi(\varphi(-1) - \delta))\}.$$

Proposition 3.3 ($\mathbf{E}[K_\pi(t)]$ is Increasing Function in $0 \leq \pi \leq 1$): *Assume that the safety loading condition $r^l > \iota\mu$ is satisfied and $\delta < \varphi(-1)$. Then, for $t > 0$, we have that $\mathbf{E}[w_t^\pi]$ is increasing in $\pi \in [0,1]$.*

If we consider the above lemma, it follows that $\mathbf{E}[K_t^\pi]$ is an increasing function of π for every fixed time period $t > 0$ and initial bank capital $k > 0$. It then follows that the optimal capital management problem in (17) is equivalent to

$$\max\{\pi \in \Pi : \mathbf{CaR}_\alpha(w_\infty^\pi) \leq C\} \qquad (18)$$

which depends on the risk measure itself.

3.1 Solution of the Optimal Capital Management Problem

As we stated under Problem 3.1 that there is no analytic method to determine the solution of our problem. Then Proposition 3.3 reveals the behavior of solution of Problem 3.1. In particular, since $\mathbf{E}[K_t^\pi]$ is increasing function in $0 \leq \pi \leq 1$. It then follows that the optimal solution of Problem3.1 is the largest bank capital management strategy $\pi \in [0,1]$ that satisfies the CaR constraint. This strategy can be found through numerical iteration methods for optimization problems of this kind.

3.2 Simulations of Output Gap vs CARs for OCED Countries

In this section we discuss the capital-to-total assets ratio and capital-to-risk weighted assets ratio given by

$$\frac{K_t}{A_t} \text{ and } \frac{K_t}{A_t^r}, \qquad (19)$$

respectively. Here A and A^r is given by (1) and (3) , respectively. Subsequently we simulate the output gap versus the CARs mentioned earlier for Australia (period 1990-2000), Norway (period 1992-2000) and United Kingdom (period 1990-2000).

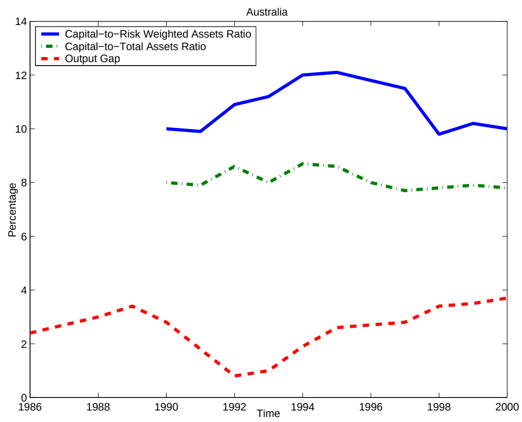

Figure 1: Capital-to-Risk Weighted Assets Ratio vs Capital-to-Total Assets Ratio for Australia

ISBN:978-988-17012-3-7

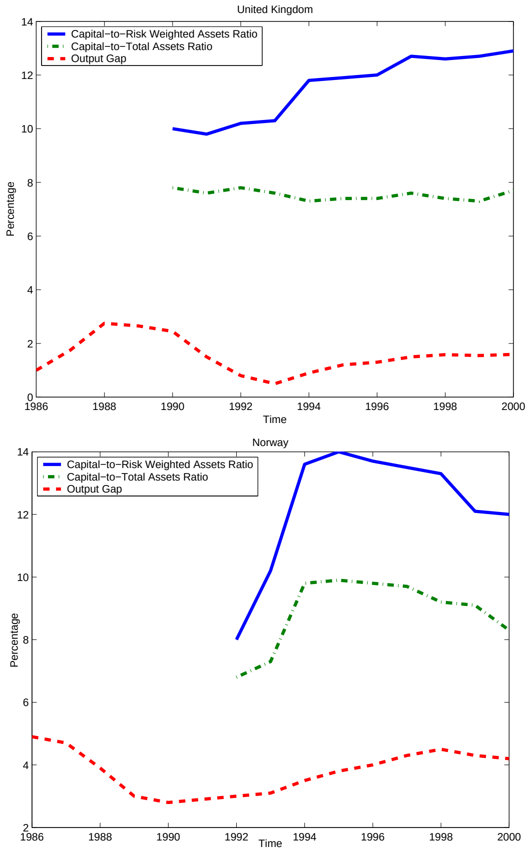

Figure 2: Capital-to-Risk Weighted Assets Ratio vs Capital-to-Total Assets Ratio for United Kingdom and Norway

If we observe the Figures 1 and 2 more closely, we can see that for Australia, Norway and the United Kingdom the capital adequacy ratio is negatively correlated with the output gap. As a result of the above, under Basel II, capital requirements are likely to increase in recessions and decrease in the boom. Yet if capital requirements shows this tendency - when building reserves from decreasing profits is difficult or raising fresh capital is likely to be extremely costly - banks would have to reduce their lending and the subsequent credit squeeze would add to the downturn. This would make the recession deeper, thus setting in motion an undesirable vicious circle that might ultimately have an adverse effect on the stability of the banking system. This is why capital requirements are said to be procyclical despite actually increasing (decreasing) during a downturn (upturn). The implications of this link between financial stability and macroeconomic stability in terms of the soundness of credit institutions merit being taken into account in the final design of Basel II.

4 Conclusions and Future Directions

In this paper, we analyzed the issue of bank capital management for banks that can raise their capital through equity, subordinate debt and loan loss reserves. In Section 2, we constructed a stochastic dynamic models for bank capital that involves the capital gains and losses. Our paper assumed that such gains and losses are coming from loan loss reserves and the unexpected loan losses, respectively. In Section 3, we constructed an optimal capital management problem in the banking industry that cannot be solved by analytical approach. However, through Proposition 3.3, the solution of optimal capital management problem found to be the largest bank capital management strategy $\pi \in [0,1]$ that satisfies the CaR constraint. Future research topics will involve a numerical approach to Problem 3.1. Also, we may wish to solve the mean-variance problem in [11] in a mean-CaR context.

References

[1] Klüppelberg C, Kostadinova R. Integrated insurance risk models with exponential Lévy investment (2005). *http://www-m4.ma.tum.de/paper/.*

[2] Basel Committee on Banking Supervision. *The New Basel Capital Accord*. Bank for International Settlements, 2001, http://www.bis.org/publ/bcbsca.htm.

[3] Basel Committee on Banking Supervision. *International Convergence of Capital Measurement and Capital Standards; A Revised Framework*. Bank for International Settlements, June 2004, http://www.bis.org/publ/bcbs107.pdf.

[4] Basel Committee on Banking Supervision (BCBS). *Amendment to the Capital Accord to incorporate market risk*. Bank for International Settlements Technical Report 1996.

[5] Basel Committee on Banking Supervision (BCBS). *International convergence of capital measurement and capital standards*. Bank for International Settlements Technical Report June 2004.

[6] Basel Committee on Banking Supervision (BCBS). *Implementation of Basel II: Practical Considerations*. Bank for International Settlements Technical Report July 2004.

[7] Gersbach H. *The optimal capital structure of an economy*. March 2002.

[8] Illing M, Paulin G. *Basel II and required bank capital*. Financial System Review 2004.

[9] Cont R, Tankov P. *Financial Modelling with Jump Processes*. Chapman & Hall/CRC, 2004.

[10] Protter P. *Stochastic Integration and Differential Equations* (2nd edn). Springer: Berlin, 2004.

[11] Gideon F, Mukuddem-Petersen J, Mulaudzi MP, Petersen MA. Optimal provisioning for bank loan losses in a robust control framework, *Optimal Control Applications and Methods*, February 2008, DOI:10.1002/OCA.846.

[12] Mukuddem-Petersen J, Petersen MA. *Stochastic behavior of risk-weighted bank assets under Basel II capital accord*. Applied Financial Economics Letters. 2005; **1**:133-138.