Penetration Depth Prediction in Mesh-less Metal Targets

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Abstract— **This paper described a general framework of the penetration mechanics for the rigid projectile. The framework is then applied in numerical simulation using mesh-less method for predict the penetration process. The target structures is modelled in a mesh-less way by finite layers of the target materials, which impose penetration resistance on the projectile through resistance function based on dynamic cavity expansion theory. The penetration resistance on the surface of the rigid projectile is a function of the instantaneous velocity of that surface, which can be determined by the rigid body motion of the projectile. Standard finite element method is introduced to model the rigid body motion of the projectile and is coupled with the mesh-less target by exchanging the velocities and stresses through user-interfaces. Predictions of the final penetration depth are compared with corresponding experimental data.**

***Index Terms*—Dynamic cavity expansion, mesh-less, penetration, rigid body.**

I. Introduction

Penetration in metal medium may be described by the travelling distance, i.e. penetration depth of a projectile into a massive metal medium without perforation phenomena. It may occur when there exists relative velocity between projectile and concrete target. Depending on their relative movement, mechanical properties and geometries, different penetration mechanisms may be involved during their collision. For a solid projectile, three penetration regimes were identified in Chen and Li(2004), viz. (i) the rigid projectile penetration regime, where the deformation and damage of the projectile are negligible during penetration, (ii) the semi-hydrodynamic penetration regime, which is represented by the Alekseevskii-Tate model [1]-[2], and (iii) the hydrodynamic penetration regime, which treats the projectile as a fluid. These three regimes appear with increasing impact velocity. [3] recommended a simple method to determine the transition point between the rigid projectile regime and the semi-hydrodynamic regime. In case when the strength and rigidity of the projectile object are greater than those associated with the target, the projectile may be approximately idealised as a rigid projectile without considering its deformation and damage.

Analytical penetration studies were basically based on the relationship between penetration resistance and impact velocity from the motion of the projectile, which can be determined by Newton's second law and the associated initial conditions. The details of penetration mechanics, especially in rigid projectile regime has been greatly promoted by experiments and theoretical models developed by research group in Sandia National Laboratories, who successfully applied dynamic cavity expansion theory in rigid projectile penetration analyses, e.g. in metal application [4]-[6], which were further generalised in [7]. Recent developments about the hard projectile impact are also summarised in [8].

Significant progresses in computer capability and computational mechanics have been made in past two decades. The changes of interest from empirical and analytical models to numerical simulation become inevitable because of its efficiency, low cost and versatility (e.g, it is difficult to use analytical models to predict the trajectory of a projectile and its penetrability in a practical problem). Traditionally, numerical simulation of penetration usually employs a fully coupled analysis, i.e, both projectile and target are discreted in computational code. However, there are weaknesses in the fully coupled numerical simulation, where new difficulties are encountered, i.e., failure criterion, contact problem [9], mesh distortion for the large deformation [10]-[13] and reliable material model [9]. These weaknesses in numerical simulation have challenged researchers to introduce new techniques to deal with these problems. Nevertheless, among various methods, the method called differential area force law (DAFL) appears to be most realistic approach especially in predicting deep penetration and trajectory of a projectile. It provides explicit formulations for normal and tangential stresses on the projectile surface [14], which, together with momentum equations for rigid body and initial conditions, controls the dynamics of the projectile during penetration. The DAFL approach was then adopted and modified by the US Army Waterways Experiment Station (WES) and the Sandia National Laboratories (SNL) to provide 2D and 3D codes for projectile trajectory analyses [14]-[16] and produced good agreement with experimental results. However, the documents containing details of these studies are categorized to be either classified or restricted and are prohibited from public access.

In this paper, a framework of the penetration mechanics for a rigid projectile is described based on rigid body motion. The implementations of a proposed framework is developed using finite element software package ABAQUS 6.4.5 and Compaq Visual FORTRAN 6 and are conducted in selected

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mesh-less metal targets to predict the final penetration depth of the projectiles.

II. THE FRAMEWORK OF THE PENETRATION MECHANICS OF A RIGID PROJECTILE

The hard projectile under consideration is simplified as a rigid body, which has a revolutionary body with a concave-outward surface. The theoretical framework may be extended to irregular rigid projectiles when more advanced contact conditions are introduced. The limitation of the rigid projectile assumption may be determined by the method proposed in [3].

The orientation and position of the revolutionary rigid body at time t is shown in Fig.1. The fixed global reference frame is represented by [$\vec{i}$, $\vec{j}$, $\vec{k}$] and the rigid body reference frame is represented by [$\vec{e}_1^{'}$, $\vec{e}_2^{'}$, $\vec{e}_3^{'}$]. An intermediate reference frame, [$\vec{e}_1$, $\vec{e}_2$, $\vec{e}_3$], is introduced, which has the same origin as the rigid body reference frame, but does not rotate. C is the centroid of the rigid body and I_1, I_2 and I_3 are three principal moments of inertia about axes of [$\vec{e}_1^{'}, \vec{e}_2^{'}, \vec{e}_3^{'}$]. It is obvious that I_2=I_3 in the studied problem.

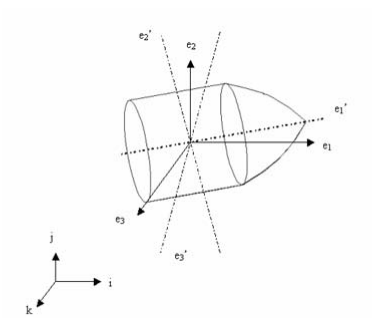

Fig. 1 Reference frame for projectile motion

The motion of the rigid projectile is controlled by the rigid body mechanics, i.e.

$$M\frac{d\vec{V}_c}{dt} = \vec{F}_R \ , \tag{1}$$

$$\left(\frac{d\vec{L}_c}{dt}\right)_{\vec{e}'} + \vec{\omega} \times \vec{L}_c = \vec{N}_c \tag{2}$$

and the initial conditions of the translation and angular velocities and the positions of the projectile. In Eqs.(1)-(2), M is the total mass of the projectile, $\vec{V}_c$ is the velocity of the centroid of the projectile, $\vec{\omega} = \omega_1\vec{e}_1^{'} + \omega_2\vec{e}_2^{'} + \omega_3\vec{e}_3^{'}$ is the angular velocity of the rigid projectile, $\left(\frac{d}{dt}\right)_{\vec{e}'}$ represents the time derivative with respect to the rigid body reference frame, $\vec{L}_c$ is the angular momentum about the centroid of the projectile, determined by

$$\vec{L}_c = I_1\omega_1\vec{e}_1^{'} + I_2\omega_2\vec{e}_2^{'} + I_3\omega_3\vec{e}_3^{'} \ . \tag{3}$$

$\vec{F}_R$ and $\vec{N}$ are the resultant resistant force and the resultant moment of the contact forces about the centroid of projectile, which are applied on the surface of the projectile by surrounding target media and depend on the contact resistance of the target media.

III. IMPLEMENTATION OF THE THEORITICAL FRAMEWORK

In order to implement the proposed model into a numerical simulation, the target media is assume that are not disturbed by the motion of the projectile. The original target space is filled with various target media, M_k, and the territory of the target T_k is defined by

$$G^k(\vec{X}) \le 0 \tag{4}$$

where G^k is a continuous function of global position vector $\vec{X}$ and $G^k\left(\vec{X}\right) = 0$ defines the surface closure of the target T_k. The expression of the normal resistance stress in target T_k is $\sigma_n^k = \sigma_n^k\left(x, V_n\right)$, where the penetration depth x is the minimum distance between the position vector $\vec{X}$ and the surface $G^k\left(\vec{X}\right) = 0$.

Meanwhile, the projectile surface is divided into finite number of discrete meshes. The position vector of mesh M_j in the fixed reference frame is $\vec{X}_j$. The normal resistance stress $\sigma_{nj}^k = \sigma_{nj}^k\left(x^j, V_n^j\right)$ is employed when the condition $G^k(\vec{X}_j) \le 0$ is satisfied, where x^j is the minimum distance between the position vector $\vec{X}_j$ and the surface $G^k\left(\vec{X}_j\right) = 0$ and V_n^j is the normal component of $\vec{V}^j = \frac{d\vec{X}_j}{dt}$.

The general implementation of the proposed framework is shown in Fig.2. The details of its parallel processing and explicit dynamic finite element algorithm are further described in sections IV and V.

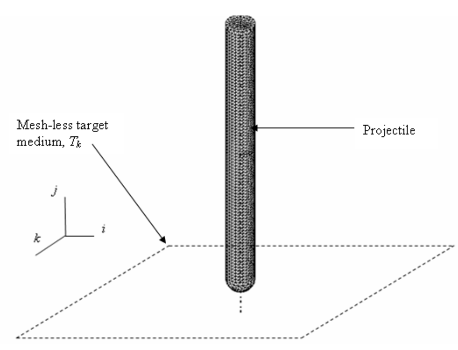

Fig.2 Framework description

IV. PARALLEL PROCESSING IN SIMULATION MODEL

Principally, ABAQUS 6.5.4 and Compaq Visual FORTRAN 6 are the main software that used to implement the proposed framework model. The core program is developed in ABAQUS 6.5.4 software, which is involved with projectile modelling, i.e. defined the boundary condition and elements meshed. Whilst the sub-program is written in Compaq Visual FORTRAN 6 to create the mesh-less of thickness layer of target medium by imposing the target's pressure resistance on the projectile surface through resistance function. Basically, the operation of combination of both programs is simple. At a given time, *t* and an initial velocity, V, the projectile will travel directly to the mesh-less target surface. When the projectile entered the target, the global load vector on the front nose is created from elemental contributions to σ_n. Next, the velocity will reduce to certain value and then this value will supply to user sub routine program for analyzing the load value that is use to apply to the element. This process will repeat until its conditions are satisfied, i.e. value of velocity normal, $V_n = 0$ or $\vec{X}_j > T_k$ (it is noted that when the condition are satisfied, the communication of both core and sub program will be stop). The detail of these descriptions is shown in Fig.3.

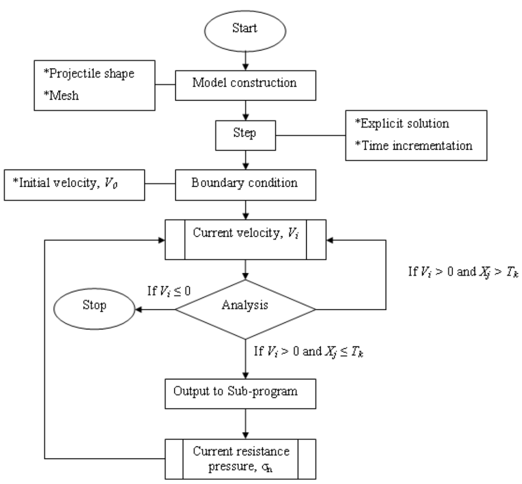

Fig.3 Flow chart of communication between core and sub programs

V. EXPLICIT DYNAMIC FINITE ELEMENT ALGORITHMS IN SIMULATION MODEL

In this simulation model, the explicit dynamical analysis procedure is applied. The equation of motion for the rigid body of projectile at the time of i is given by

$$Ma^{(i+1)} = \vec{F}_R^{\ i} \tag{5}$$

where the superscript (*i*) refers to the increment number, *M* is the mass of the projectile, *a* is the acceleration vector of the centroid of the projectile and $\vec{F}_R$ is resultant force vector applied on the surface of the projectile due to the resistance of the foamed concrete target. The applied resultant force vector is determined using the finite element interpolation functions, i.e.,

$$\vec{F}_R^{\ i} = \oint_s \sigma_n \vec{n} ds \tag{6}$$

where σ_n is determined as resultant pressure at projectile surface (as elaborated in section 3.4) and $\vec{n}$ is the normal direction of the infinitesimal area ds of the projectile surface.

The relationships between velocity and acceleration and between velocity and displacement are evaluated using explicit central difference method [17], i.e.,

$$V^{(i+\frac{1}{2})} = V^{(i-\frac{1}{2})} + \frac{\Delta t^{(i+1)} + \Delta t^{(i)}}{2} a^{(i)} \tag{7}$$

$$u^{(i+1)} = u^{(i)} + \Delta t^{(i+1)} V^{(i+\frac{1}{2})} \tag{8}$$

where u is displacement, V is velocity and $(i - \frac{1}{2})$ and $(i + \frac{1}{2})$ refer to the averages of the incremental neighbouring values. The central difference integration operator is explicit, which means that the velocity state at step $i + \frac{1}{2}$ can be obtained using values of $V^{(i-\frac{1}{2})}$ and $a^{(i)}$ from previous states and the displacement state at $i+1$ can be obtained from $u^{(i)}$ and $V^{(i+\frac{1}{2})}$.

VI. RESULTS AND DISCUSSIONS

In this section, the proposed model is applied to the normal penetration situations. The penetration process is modelled using ABAQUS finite element software package. The projectile is modelled in standard finite element and treated as a discrete rigid body. The target structure is treated as a mesh-less by finite layers of material, which impose penetration resistance on the projectile through resistance function based on dynamic cavity expansion theory as shown in Fig.3. The dynamic cavity expansion theory in [18]-[21], which have been successfully applied to define the normal penetration resistance stress in [4]-[7], i.e.

$$\sigma_n = AY + B\rho V_n^2 \tag{9}$$

is employed, where Y and ρ are yielding stress and density of target material, respectively. A and B are non-dimensional material constants.

The resistance on the surface of rigid projectile is a function of the instantaneous velocity of projectile nose surface, which can be determined by the rigid motion of the projectile. Coupling between motion of rigid projectile and mesh-less target is made by exchanging the velocities and stresses through user-interfaces.

The results from simulations model are compared with experimental data conducted by [4]-[6]. Final penetration depths of projectiles obtained from simulation and experimental data are summaries in Fig.4-6. Clearly, the simulation results give encouraging predictions, which follow the general trend of experimental results and some offer an upper bound of experimental data.

In simulation model, it is assumed that the projectile strikes target with normal incidents (i.e., angle of obliquity is zero and therefore the projectile is constrained in the transverse directions allowing it to move only in the axial direction. However, in the real situation projectiles may have angles of yaw and pitch [4]-[6], which may influence penetration process and these factors should be considered in further study.

In addition, the rigid assumption of projectile model may somehow influence predicted results especially when impact velocity high, it may deform projectile and this violates the rigid projectile assumption

Further more, the approach of neglecting the sliding fictional resistance during penetration process in simulation model should be reconsidered, especially when the tearing strength of the aluminium alloy material is high and the side surface of projectile is rough. In other hand, this factor may increase with the decrease in impact velocity. Nevertheless, this factor does not generally belong to dominant factors in a penetration problem.

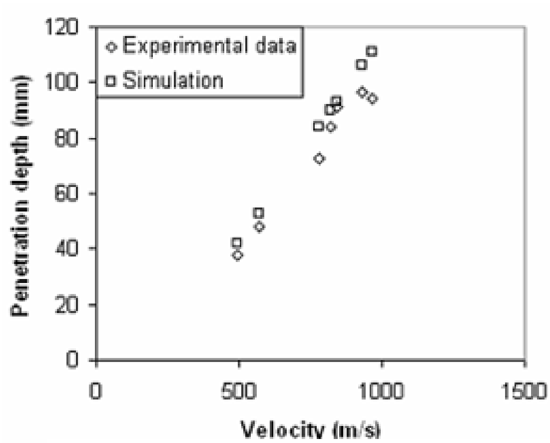

Fig.4 Penetration depth versus velocity for 6061-T6511 aluminium alloy by spherical nose projectile (experimental data are from [4])

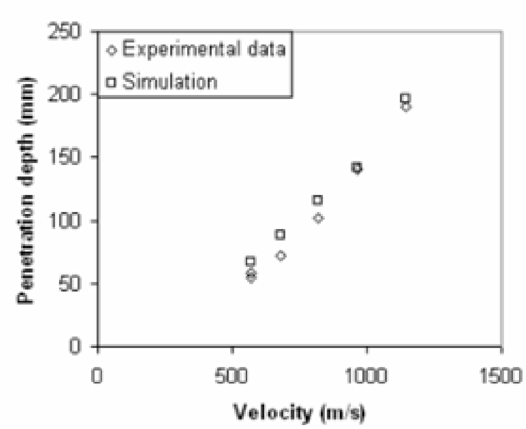

Fig.5 Penetration depth versus velocity for 6061-T6511 aluminium alloy by ogive nose projectile (experimental data are from [5])

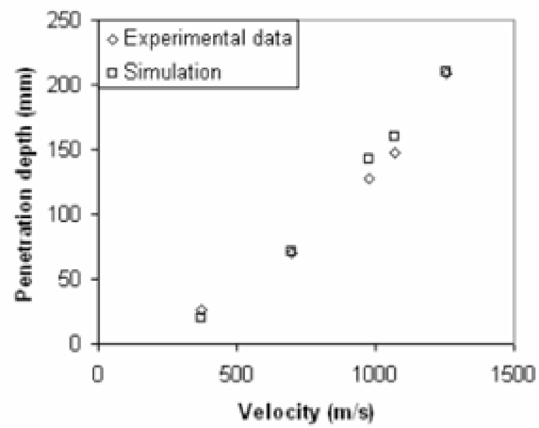

Fig.6 Penetration depth versus velocity for 7075-T651 aluminium alloy by ogive nose projectile (experimental data are from [5])

VII. Conclusion

Investigations on deep penetration of metal targets are conducted based on proposed rigid body motion framework. The detail of framework implementation is described. Selected experimental data obtained by [4]-[6] were used to verify the proposed model. Encouraging predictions were observed when the prediction results are compared to experimental data.

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