Proceedings of the World Congress on Engineering 2008 Vol II
WCE 2008, July 2 - 4, 2008, London, U.K.

An Insight of Torque and Stress Balance Behaviour of A Contrahelically Armored Cable

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Abstract— **This paper deals with the necessary conditions to achieve a torque balance construction and requirements for equal load sharing among the armoring wires. Among the several mathematical model for predicting the mechanical response of a helical wire strand to axisymmetric loading derived in the literature over five decades, the linear elastic symmetrical model which considered correct generalized strains and depicted the origin of deviation in the earlier works was extensively applied to accurately describe the physical behavior of the cable**.

***Index Terms*— Cable mechanics, helical strand mechanics, wire rope mechanics.**

I. Introduction

In the world today helical wires strands, cables and ropes are in abundant use as structural members, as electrical or optical communications links and for transmitting power. Yet, it is only within the last decade that a series of attempt has been made to develop engineering models to accurately predict the mechanical behavior of such cables.

In many applications where such cables are axially loaded in tension, there will be a torsional coupling that causes twisting of the cable. To prevent any cable twist, when found undesirable, external torque needs to be applied. However if the layer are designed adjusting suitably the lay angles, number, diameter and material of the wires in a layer, the axial twist could be avoided without any external torque.

The present effort introduces a simple technique to determine the torque characteristics as well as stress distribution among the wires. A procedure is suggested to achieve torque balance and stress balance. An example of two opposing layers of helical wires wound over an elastic core of different diameters and materials is elaborately dealt. In light of availability of the computers and of the more accurate symmetric stiffness matrix, an analysis for the two balances is attempted in this paper.

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Manuscript received February 14, 2008.
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II. Brief review of the previous literature

In the recent years, various mathematical models are available for predicting mechanical behavior of helical wire strands. They are classified into four categories:

a) Models based on purely tensile wires
b) Models additionally with wire bending and torsional stiffness.
c) Models additionally with core radius variation
d) Models additionally with Poisson's effect in core and layer wires.
e) Orthotropic sheet model considering the Hertzian contact stress between the layer wires and associated elastic wire flattening and elastic slip.
f) Model considers the layers as an orthotropic tube classified the models(a) to (d) as 'packing geometry models', the orthotropic model with line contacts as the 'wire rope geometry model' and that the Hertzian contact as the 'wedge geometry model'.

In a paper Raoof and Kraincanic [1] have critically examined some typical thin rod models with the orthotropic sheet model. Of the six models only model (a) and the orthotropic models are found to yield strict symmetry. In all the other models , there is a asymmetry, though small. It should be noted, however, that all of them assumed linearity to be valid, on the basis of small deformation, friction less contacts, no slip when friction exists etc. It is well established that the response of a linear elastic strand system has the following form of stiffness equation for axisymmetric loading

$$\begin{bmatrix} Fs \\ Ms \end{bmatrix} = \begin{bmatrix} F\varepsilon & F\phi \\ M\varepsilon & M\phi \end{bmatrix} \begin{bmatrix} \varepsilon \\ \gamma \end{bmatrix} \tag{1}$$

An accurate formulation of a linear elastic model should possess the symmetry of stiffness matrix so that

$$F\phi = M\varepsilon \tag{2}$$

It is necessary at this stage to recall what has been stated in earlier works on symmetry or lack of it. Typical linear mathematical models of cables subjected to axisymmetrical loading have been compared. The equations are rewritten with same notations, similarities and differences in the various models have been highlighted [10].

In the model developed by Hurska [2,3], the bending and torsional constituent wires are neglected. Existing models are linear, variation of the lay angle neglected and the stiffness matrix is a constant. In a more rigorous way Machida and

Durelli [4] studied the effects of bending and torsion stiffness of individual wires on the cable rigidity matrix, but failed to consider the effect of binormal forces N'. Paper by Costello and Phillips [5] presented a general non linear theory for a layer of helically wound wires, without a core, which included the effects of radius and helix angle variations , as well as wire bending and torsion moments. The linear analysis presented by Sathikh et al[6] which includes wire shear forces and couples with their correct relationship and ensures symmetry of stiffness matrix. The validation of these stiffness constants were compared with those of experimental findings of Utting [7,8] and FEM analysis performed by Ghoreishi et al [9] and used if for prediction of response in this paper.

Sathikh et al have derived the symmetric matrix for the case of radially core in terms of helix angle as

$$F\varepsilon = E_C A_C + \sum_{i=1}^{n} m_i \begin{bmatrix} E_i A_i \sin^3\alpha_i + [G_i J_i \cos^2\alpha_i \\ + E_i I_i \sin^2\alpha_i] \dfrac{\cos^4\alpha_i \sin\alpha_i}{r_i^2} \end{bmatrix} \qquad (3)$$

$$F_\phi = M_\varepsilon = \sum_{i=1}^{n} m_i \begin{bmatrix} E_i A_i r_i \sin^2\alpha_i \cos\alpha_i + [G_i J_i \sin^2\alpha_i \\ - E_i I_i (1+\sin^2\alpha_i)) \dfrac{\cos^3\alpha_i \sin^2\alpha_i}{r_i} \end{bmatrix} \qquad (4)$$

$$M_\phi = G_C J_C + \sum_{i=1}^{n} m_i \begin{bmatrix} E_i A_i r^2{}_i \sin^2\alpha_i \cos\alpha_i + \\ G_i J_i \sin^7\alpha_i + \\ E_i I_i (1+\sin^2\alpha_i)^2 \end{bmatrix} * \left(\sin\alpha_i \cos^2\alpha_i\right) \qquad (5)$$

III. Present work

These stiffness matrices are used to describe the torsion response of the cables and the internal stress distribution is derived. The following assumptions have been made in deriving the torque and stress balance equations.

The geometry of deformation of a helical wire can be described by its centroid axis; i.e the diameter of the wire is small in comparison with the pitch length of the helix.

Plane section of the cable cross section remain plane before and after deformation

The helically wound wires in any layer are equally spaced around the circumference of the cable.

The helical wires are homogenous, isotropic and linearly elastic.

The core element is a linearly elastic element and can be represented as a structure which is allowed to elongate with the cable while being either rigid or incompressible in the radial direction.

Reduction of wire diameter due to inter-wire contact is neglected.

Cable elongation and rotation strain parameters are considerably less than unity ($\varepsilon i, \gamma i << 1$)

The stiffness equation were been used to evaluate the design of a two layer, contra helically armored KEVLAR EM cable used as a segment link between a surface support ship and a deep sea unmanned work system[5]. A representative model is shown in Fig.1. This cable has selected for this comparative study since it was rigorously designed, manufactured and tested. As built- geometrical and material properties have been accurately determined.

The properties of Table 1 were used in stiffness equation to produce the curves shown in Fig. 2. These curves represent the relationship between the inner and outer helix angles to achieve torque and stress balance.

IV. Torque Balance

The helical armoring wires, which render the cable flexible, induce a torque as the helical wires try to "unwind" during axial loading. Induced torque can be undesirable from several points of view. Cable rotation may loosen some wires and tighten others depending on the direction of lay. This, of course, means that some wire layers will be stressed at higher levels than others. Thus, the efficiency of the cable is reduced and the breaking strength may be appreciably lowered. Long cables which are restrained from rotating may develop a sufficiently large induced torque that slight relaxations of cable tension (momentary slack cable) can result in hockling (looping) due to instability. Upon reapplication of the cable load, the hockle radius may be decreased sufficiently to fail the armoring wires due to the large bending stresses. Furthermore, there are numerous cable applications which require torque-free performance, such as in long oceanographic cables used for towed bodies.

The condition of torque balance requires that no external torque be developed for a cable pulled in tension and restrained from rotating at both ends. For torque balance $\dfrac{d\phi}{h} = 0$ and $M_s = 0$ this requires also $M_\varepsilon = 0$.

The parameters are given in Table I. For different helix angles α_1 of the inner layer, the suitable helix angle α_2 is computed to satisfy $M_\varepsilon = 0$. The relationship between α_1 and α_2 is so obtained is shown in Fig.1.

Table I. As-Built Properties of a KEVLAR-Armored Cable

Wire	Inner Layer	Outer Layer
$A_i\ [mm^2]$	4.976	2.965
$r_i [mm]$	12.51	14.78
$\alpha_i [\deg]$	+69.0 RHL	76.1 LHL
$E_i [GPa]$	75.36	83.74
$Sy_i [MPa]$	1,309	1.509
$m_i [Nbr.Wires]$	28	44

Core

$Rc[mm]$	11.31
v_c	0 (assumed for rigid core)

Where m is number of wires in layer i , α_1 the helix angle, r_i the helix radius, m_i the number of wires, $A_i = \pi R_i^2$, R_i is the wire radius, E_i and G_i the elastic and shear modulii

ISBN:978-988-17012-3-7

respectively of the wire of layer *i*. E_c and G_c are the elastic and shear modulii of core wire and $A_c = \pi R_c^2$, R_c = core wire radius. For computational purpose $Gi = E/2(1+\nu)$ is used where ν is Poisson's ratio = 0.3

V. STRESS BALANCE

For stress balance, the wire axial stress, wire bending stress and wire twisting stress are considered together to determine the maximum stress. In a wire in the layer i, assuming that the torque is not balanced the stresses are:

Axial $fi = E_i \ \varepsilon_{wi}$ (6)

Bending $f_{bi} = E_i \ R_i \ \omega_{2i}$ (7)

Torsional $f_s = G_i \ R_i \ \omega_{3i}$ (8)

Where

$$\varepsilon_{wi} = \varepsilon \sin^2 \alpha_i + \gamma \cos^2 \alpha_i \tag{9}$$

$$\omega_{2i} = \frac{-\varepsilon \cos^2 \alpha_i \sin^2 \alpha_i}{r_i} + \frac{\gamma \cos^2 \alpha_i (1 + \sin^2 \alpha_i)}{r_i} \tag{10}$$

$$\omega_{3i} = \frac{\varepsilon \sin \alpha_i \cos^3 \alpha_i}{r_i} + \frac{\gamma \sin^3 \alpha_i \cos \alpha_i}{r_i} \tag{11}$$

$$\gamma = r_i \frac{d\phi}{h} \tan \alpha \tag{12}$$

Maximum stress is given by

$$f_{\max i} = \frac{(f_i + f_{bi})}{2} + \left[\left(\frac{f_i + f_{bi}}{2} \right)^2 + f_s^2 \right]^{0.5} \tag{13}$$

For equal strength, it is not the wire stress in every layer that needs to be equal. What is required is, for all layers

$$f_{\max i} = S_{yi} \text{ or } f_{\max i} / S_{yi} = 1 \tag{14}$$

where S_{yi} is the allowable design stress of the wire material of layer i. The relationship between α_1 and α_2 that yields the strength balance for $\gamma = 0$, not necessarily $M_\varepsilon = 0$, is given in Fig.2.

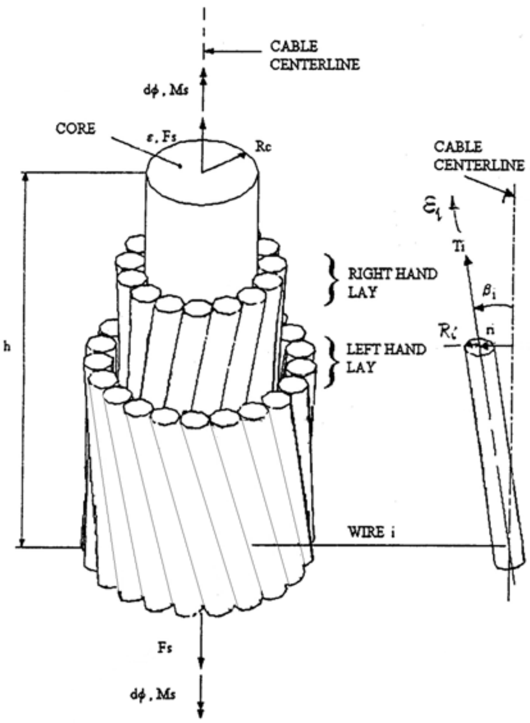

Fig. 1 Strand geometry and wire forces and couples.

VI. TORQUE AND STRENGTH BALANCE

In addition to strength balance, if torque balance is also required, then the conditions $M_\varepsilon = 0$ with $\gamma = 0$, and (14) should be satisfied simultaneously. The point of intersection between the two relations shown in Fig.2 yields the desired results.

VII. CONCLUSION

A set of equations to determine the conditions of torque and stress balance of a multilayered, helically armored cable have been derived.

The improved linear equation should provide a useful design tool, particularly in a preliminary design situation, capable of revealing the physical characteristics of both torque balance and load-sharing among armoring wires.

The values of (α_1 and α_2) in degrees for torque and strength balance from the present analysis are (75º) and (-79º) respectively. The torque balance and strength analysis carried out has considered the symmetric stiffness matrix and included bending and twisting of the wire in addition to the wire stretching. It is hoped that this improved torque and strength balance model should be helpful to designers of cable and wire ropes.

ISBN:978-988-17012-3-7

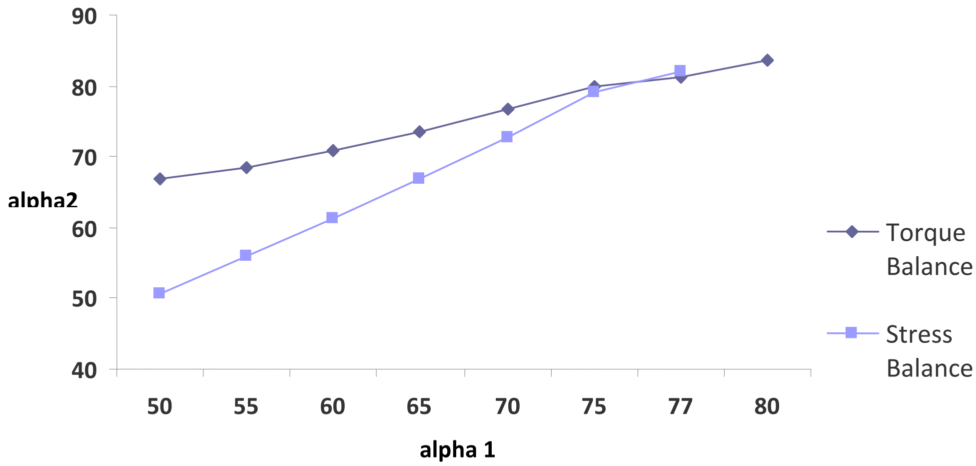

Fig. 2 Torque balance and Stress balance

APPENDIX

A Area of cross-section of helical wire = πR2
E_C Area of cross-section of core wire= πRc2
Ec,E Elastic modulus of core and helical wire
L Length of the helical wire
H Strand length
R Wire helix radius
ϕ Angle of strand rotation
α, α' Initial and Final Helix angle
ε Strand axial strain= δh/h
ε_w Helical wire axial strain
γ ,γ_w Shear strain of strand and helical wire
F Component of axial wire force in the strand axial direction
F_s Axial force of the strand
Gc Shear modulus of core wire
G, G' Wire moment about wire normal and binormal axis
H Wire axial moment
I_c I_i Moment of inertia of core ($\pi Rc^4/4$) and wire ($\pi R^4/4$)
Jc, J Polar moment of inertia of core (πRc4/2) and helical wire(πR4/2)
K, K', Components of external moment
Θ per unit length
m Number of wires in a layer
M Component of wire moment about strand axial direction
Ms Strand axial moment
T, N, N' Wire axial, normal and binormal force
Rc R Core radius and wire radius
X, Y, Distributed wire unit force in wire
Z normal, binormal and axial direction

ACKNOWLEDGMENT

The authors are profoundly indebted to Professor Sathikh, S for his constructive criticisms and extensive discussions during the preparation of the manuscripts of this paper.

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