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Study on Mixed Mode Crack-tip Plastic Zones in CTS Specimen

C. M. Sharanaprabhu, S. K. Kudari *Member, IAENG*

***Abstract*—The studies on crack-tip plastic zones are of fundamental importance in describing the process of failure and in formulating various fracture criteria. The material fracture by opening mode (mode I) is not lonely responsible for fracture propagation. Many industrial examples show the presence of mode II and mixed mode I + II loading in machine/structural components. In the present study, an emphasis is laid to study the size and shape of the plastic zone at the crack tip under mode I, mode II and mixed mode I + II loading conditions. The shape and size of crack-tip plastic zones have been estimated in a Compact tensile shear (CTS) specimen under mixed mode loading according to von Mises yield criteria. The results obtained are analyzed with reference to loading angle and effective stress intensity factor.**

***Index Terms*—plastic zone size, mixed mode I/II, finite element analysis, CTS specimen**

I. Introduction

In many practical cases cracks are not normal to the maximum principal stress direction, and a mixed-mode (combined modes I and II) condition prevails at the tip of such cracks. Hence, analysis of mixed mode crack problems is important in structural integrity assessments. Prediction of crack initiation and orientation with its propagation path under mixed-mode loading is desirable for life prediction of engineering materials [1]-[3]. The stress amplitude at the crack tip subjected to a loading provokes a plastic deformation in a localized zone at the crack tip, referred as plastic zone. In this zone emanates a damage, which leads according to the properties of material, to either a total or progressive fracture. The growth of the crack is linked to the existence of this plastic zone at the crack tip, whose formation and intensification are accompanied by energy dissipation. The knowledge of the fracture analysis of a material requires a better and clear understanding of the plastic zone morphology, the deformations and stress field ahead of crack-tip.

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C. M. Sharanaprabhu, is with the Determent of Mechanical Engineering, Bapuji Institute of Enng & Tech., Davanagere, India, Perusing Ph. D at B V B College of Engineering and Technology, Hubli-580031, India (e-mail: cmsharanaprabhu@rediffmail.com).

S. K. Kudari is with the Determent of Mechanical Engineering ,B V B College of Engineering and Technology, Hubli-580031, India (corresponding author phone: +91 0836 2331444; fax: +91 0836 2374985; e-mail: s.kudari@rediffmail.com).

In mixed mode fracture, the studies on crack-tip plastic zones are of fundamental importance in describing the process of failure and in formulating various fracture criteria. In mixed mode fracture, it is known that the crack initiation angle depends on the loading angle [4]. It is also known that loading angle alters the shape and size of crack-tip plastic zone. Several investigators [5]-[7] have proposed fracture criteria on prediction crack-initiation angle based on crack-tip plastic zone in mixed mode fracture. Recently, Bian and Kim [4] have proposed a minimum plastic zone radius (MPZR) theory for crack initiation angle in mixed mode and monotonic loading. This kind of study needs detailed information about the crack-tip plastic zone shape and size in a fracture specimen estimated by numerical method such as finite element method. Benrahou *et al*. [8] have estimated the plastic zone under mixed mode loading by finite element method. But the details of mixed mode plastic zone analysis is missing in there investigation. In the present investigation an effort is made to study the size and shape of plastic zone under mixed mode loading in linear elastic fracture mechanics (LEFM) regime.

II. Finite Element Analysis

The general-purpose finite element (FE) code ANSYS is used in this study. A Compact tensile shear (CTS) specimen [9] under mixed mode loading has been considered in the present study. This kind of specimen is also referred as Compact mixed mode (CMM) specimen [4]. The specimen geometry used in the analysis is shown in Fig.1, the dimensions of the specimen considered in the analysis are similar to the one used in the work of Borrego *et al*. [9]. The loading of specimen is done at various angles (β), 0^o (pure Mode-II), 18^o, 36^o, 54^o, 72^o and 90^o (pure Mode –I) to study the plastic deformation ahead of crack-tip. The load is applied at various angles β using a loading jig [9] along the six holes as shown in Fig.2. In the present FE analysis the specimen loading at various angles (β) is carried out in the similar manner as demonstrated by Borrego *et al* [9].

A series of finite element calculations have been made on the specimen (Fig.1) considering full specimen geometry due to lack of loading symmetry. A typical 2-dimensional FE mesh used in the analysis is shown in Fig.3. The loading and displacement boundary conditions used in this analysis are similar to the one used in the work of Borrego *et al*. [9]. Two-dimensional elastic FE calculations were performed using eight noded isoparametric quadrilateral elements considering plane stress condition. The number of elements used in the FEA was 2844. In these calculations, the material behaviour has been considered to be linear elastic type pertaining to interstitial free steel (IF) possessing yield strength (σ_y) of 155 MPa, Poisson's ratio (υ) 0.3 and elastic modulus (E) of 197 GPa.

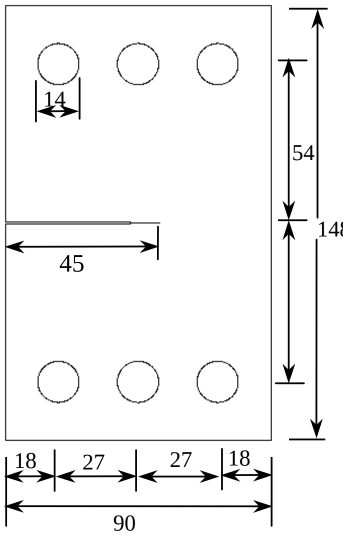


Fig.1 Specimen configuration used in the analysis (all dimensions in mm), thickness = 3 mm.

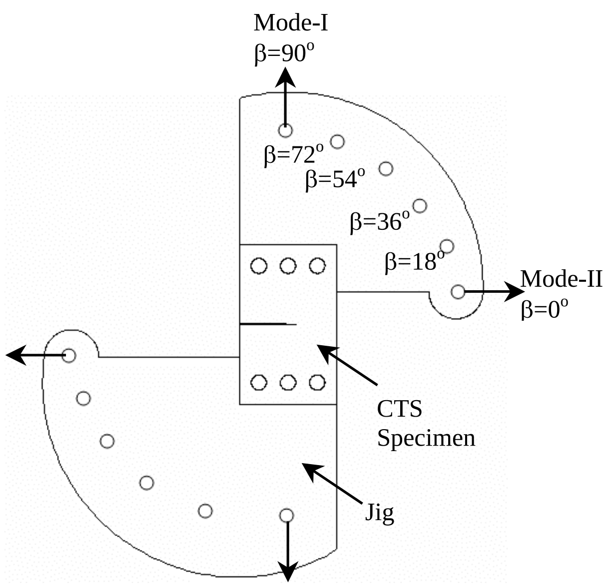


Fig.2 Loading Jig.

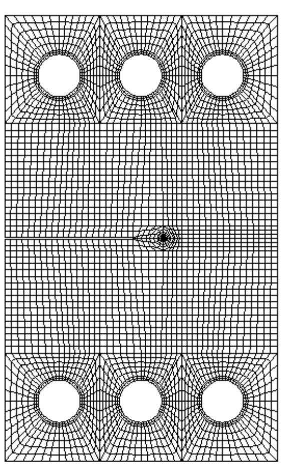


Fig.3 FE mesh used in the analysis.

III. Results and discussion

Different load steps were applied on the specimen to estimate the stress intensity factor and to study the plastic zone shape and size ahead of a crack-tip. The stress intensity factors in mixed mode loading (K_I and K_{II}) have been computed for various load steps and loading angle (β) using ANSYS post processor. The failure locus under mixed mode loading *i.e* variation of K_{II} *vs*. K_I for various loading is depicted in Fig.4. This figure indicates that for the similar applied load, the stress intensity factor in mode-I is more than that of mode-II. This nature of variation of K_{II} vs. K_I is in good agreement with the results shown by Benrahou *et al*. [8] and Kudari and Sharanaprabhu [10] on a SEN specimen under mixed mode loading. The magnitudes of K_I and K_{II} have also been computed by analytical formulations cited in the earlier investigation [4]. The estimated theoretical values of stress intensity factors have been superimposed in Fig.4 typically for load 4kN and 10 kN. This plot indicates that there exists some discrepancy in estimation of stress intensity factors by analytical formulation [4] and present FE results. It is found that there is 10.5% and 2.4% error in estimation of K_I and K_{II} respectively. This discrepancy in estimated magnitudes of stress intensity factor attributed to varied loading condition in FE analysis through loading Jig. The effective stress intensity factors (K_{eff}) in mixed mode loading have been computed using the relation [8]:

$$K_{eff} = \sqrt{K_I^2 + K_{II}^2} \quad (1)$$

The computed magnitudes of K_{eff} are plotted against loading angle (β) in Fig.5. This figure indicates that for a particular load the magnitude of K_{eff} increases as β increases. It is also clear from Fig.5 that for a particular applied load K_{eff} in mode –I (β=90°) is more than that of mode-II (β=0°). The nature of variation of K_{eff} vs. β is in good agreement with similar earlier results [8], [10].

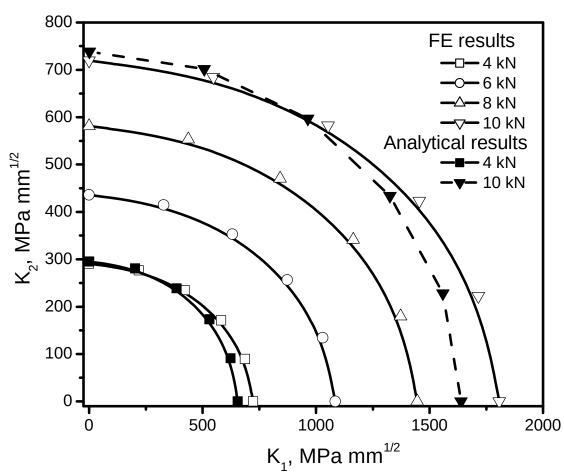


Fig.4 Variation of K_{II} vs. K_I for different applied loads.

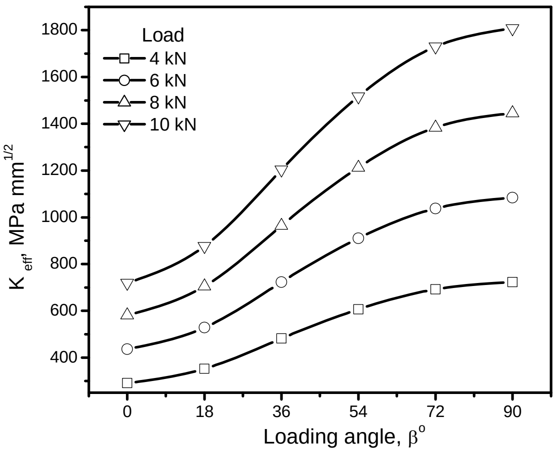
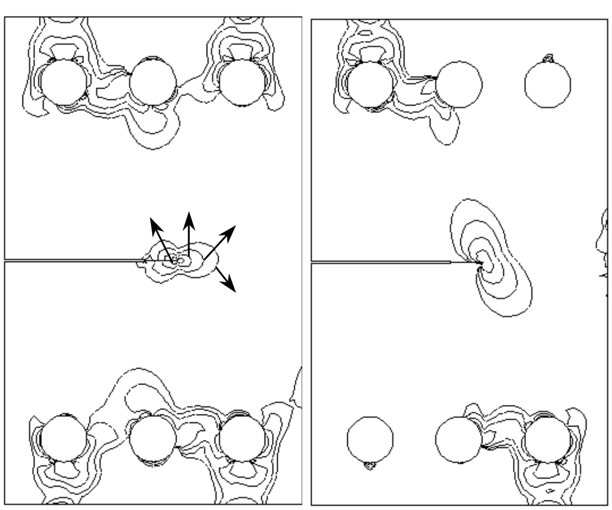


Fig.5 Variation of K_{eff} vs. β for different applied loads.

The shape of the plastic zone ahead of a crack-tip has been ascertained by plotting iso-contours of the effective stress, which causes yielding according to von Mises criterion [11]. The sequential development of crack-tip plastic zone for various applied loads and for loading angles $\beta=0^o$ (Mode-II), 54^o (Mixed mode-I and II) and 90^o (Mode-I) are shown in Fig. 6. The contours in Fig.6 are obtained by superimposing the plastic zone contour obtained in each load step. For simplicity the displacement scaling of the specimens shown in Fig.6 are set to zero. Figure 6 demonstrates that the plastic zone grows in horizontal direction for loading angle $\beta=0^o$, as loading angle is changed (Mixed mode) the angle of stretch (θ to crack plane) of plastic zone also changes, and for $\beta=90^o$ the plastic zone grows vertically. The natures of plastic zones obtained in this analysis are in good agreement with the theoretical plastic zone shapes presented in [4], [10]. It is also seen from Fig.6 that for similar applied load the stretch of plastic zone ahead of crack-tip for $\beta=0^o$ is higher compared to one for $\beta=54^o$ and 90^o. It is well known that the fracture toughness of the material is governed by the size of plastic zone stretch ahead of crack-tip [12]. The plastic stretch for $\beta=90^o$ (Mode-I) is minimum as compared to $\beta < 90^o$. Therefore these results clearly demonstrate why Mode-I loading is more dangerous than mixed mode or Mode-II loading.



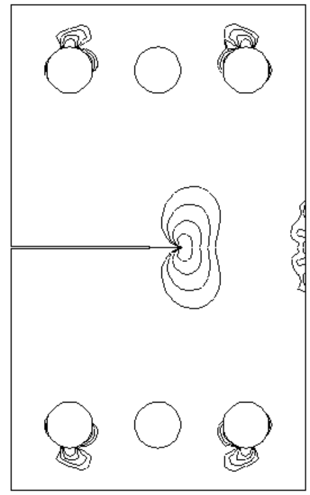


Fig.6 Sequential development of plastic zone for various applied loads. Number 1,2,3,4 indicates the plastic zone for applied load 4, 6, 8 and 10 kN.

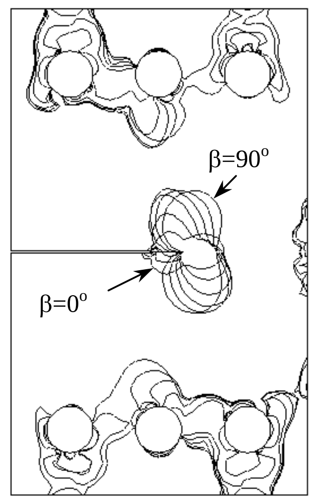


Fig.7 Typical shapes of crack-tip plastic zones for $\beta=0^o$ - 90^o under 10kN load.

Typically, development of crack-tip plastic zones for various loading angles, $\beta=0^o$ to 90^o and applied load 10kN are superimposed and shown in Fig. 7. From this figure one can find that for similar applied load at various loading angles (β), tilts the direction of growth plastic zone. It is also interesting to know that the shape of plastic zone at various β remains almost similar with some change in the size and orientation. From these results of plastic enclaves shown in Fig. 6 and Fig.7, several plastic-zone characterizing parameters such as: (i) plastic zone size along the crack plane, r_p, (ii) maximum plastic zone size, $(r_p)_{max}$, (iii) angle at which the maximum extent of plastic zone occurs, θ, measured from crack plane, (iv) minimum plastic zone radius (MPZR) and (v) angle at which MPZR occurs, θ_o, have been estimated at various load steps and loading angle (β). These parameters are schematically illustrated in Fig.8.

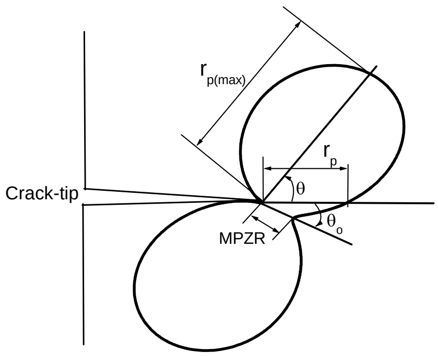


Fig.8 Schematic representation of plastic zone characterizing parameters.

The variation of plastic zone characterizing parameters r_p and $(r_p)_{max}$ vs. K_{eff} and θ vs. β for CTS specimen are shown in Fig.9, Fig.10 and Fig.11 respectively. Fig.9 shows the variation of r_p vs. K_{eff} for various loading angles (β). This figure illustrates that the plastic zone size ahead of crack-tip increases with K_{eff}. It is also clear from Fig.9 that for a particular magnitude of K_{eff} (for example 500 MPa $mm^{1/2}$) the value of r_p is least for $\beta=90^o$ (mode-I) and it is highest for $\beta=0^o$ (mode-II). The difference in magnitudes of r_p for $\beta=0^o$ and 90^o for K_{eff} = 500 is about 5 mm, which is 3.33 times that of Mode-I. These results infer that due to minimum plastic zone radius ahead of crack-tip for a particular value of K_{eff}, mode-I loading can lead to material fracture earlier than any mixed mode or mode-II loading.

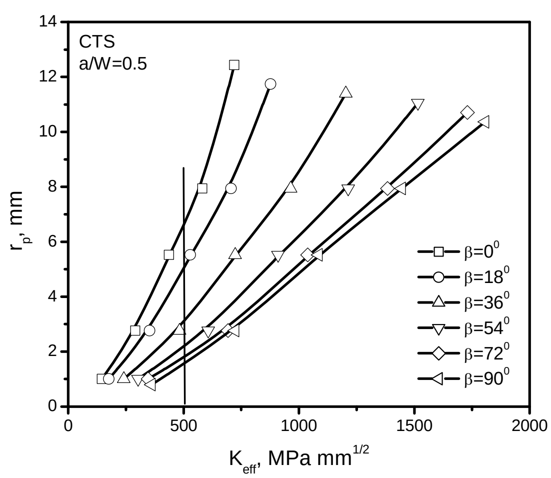


Fig.9 Variation of r_p vs. K_{eff} for various β

The variation of $(r_p)_{max}$ vs. K_{eff} for various loading angles (β) is depicted in Fig.10. This figure also indicates that the magnitude of $(r_p)_{max}$ for a particular value of K_{eff} is least for $\beta=90^o$ (mode-I) and it is highest for $\beta=0^o$ (mode-II). These results (r_p and $(r_p)_{max}$) apparently indicate that the area of plastic zone in mode-I is much lesser than that of mode-II for similar magnitude of K_{eff}. This analysis infers that for the similar magnitude of K_{eff} the energy absorption capacity of the material in Mode-I is much lesser than that of Mode-II loading. One can conclude from this analysis that due to lesser amount plastic area ahead of crack-tip, the mode –I loading leads to early fracture, hence in fracture, Mode-I loading is considered to be more dangerous than mode-II. The plot of angle at which the maximum extent of plastic zone occurs,θ, *vs.* loading angle, β, is shown in Fig.11. This plot indicates that the angle at which the maximum extent of plastic zone size occurs (θ) changes from 0^o to 90^o as β is varied form 0^o to 90^o. It is interesting to find that β=θ for mode-I and mode-II loading conditions only. But, for mixed mode (β= 18^o to 72^o) there is considerable amount of deviation between β and θ is observed. The nature of variation of θ vs. β is almost similar for all various applied loads. From Fig.11 one can understand that the growth of plastic zone takes place almost in a similar angle θ for a constant loading angle, β.

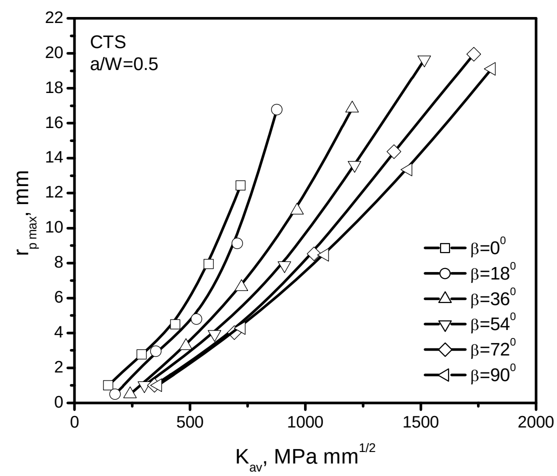


Fig.10 Variation of $(r_p)_{max}$ vs. K_{eff} for various β

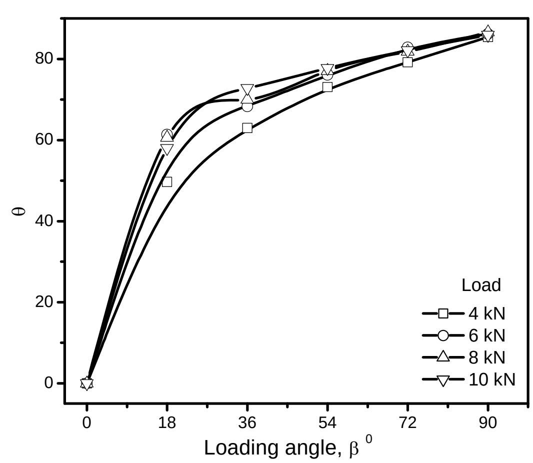


Fig.11 Variation of θ vs. β for various loads

The variation of minimum plastic zone radius (MPZR) vs. loading angle,β, for various applied loads is depicted in Fig.12. This figure indicates that the magnitude of MPZR increases as loading angle changed from 0^o (Mode-II) to 90^o (Mode-I). It is interesting to know that the magnitude of MPZR in Mode-II loading is about 5 times lesser than that of Mode-I for applied load 4kN. It is observed from Fig.12

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that the ratio of MPZR between Mode-I and Mode-II decreases with increase in applied load. In case of applied load of 10 kN the ratio is found to be 2.7. These results clearly demonstrate that the specimen experiences minimum plastic zone radius under Mode-II loading only. The results of MPZR estimated using FEM in this study can be used as inputs for minimum plastic zone radius (MPZR) criterion for crack initiation [4] in mixed mode loading for a CTS specimen.

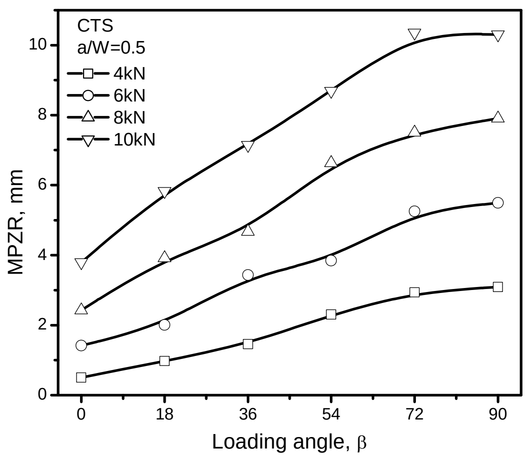

Fig.12 Variation of MPZR vs. β for various loads

The variation of minimum plastic zone radius (MPZR) *vs*. effective stress intensity factor (K_{eff}) is also studied; the plot of MPZR vs. K_{eff} for various loading angles is depicted in Fig.13. It is interesting know from this figure that the variation of MPZR vs. K_{eff} is linear and the slope of this variation for different magnitudes of β appears to be almost equal for various loading angles. This figure infers that for any loading angle growth of MPZR is proportional to the effective stress intensity factor. The proportionality constant can be evaluated by fitting a straight line equation to all the MPZR data. Such a linear fit is shown in Fig.13; the slope of the estimated linear fit line is 0.0685. From these results, the relation between MPZR and K_{eff} independent of loading angle can be expressed as:

$$\frac{MPZR}{K_{eff}} = 0.0685 \qquad (2)$$

The above Equation (2) can be used to estimate MPZR in a CTS specimen independent of loading angle if K_{eff} is known or vice versa. The proposed Equation (2) can be of great importance in MPZR criteria.

In this study the angle at which MPZR occurs, θ_o, is also studied with respect to loading angle. The variation of θ_o vs. loading angle,β, for various applied loads depicted in Fig.14. This figure shows that the magnitude of θ_o decreases from 90^o to 0^o as β changes from 0^o to 90^o. The results shown in Fig.14 indicate that the magnitude of θ_o computed for various applied loads and a particular β is almost similar. A small dissimilarity observed in estimated θ_o for various β can possibly be attributed to measurement difficulties. Bin and Kim [4] have considered that the crack in mixed mode loading initiates at MPZR in a CTS specimen. These investigators have used the magnitude of θ_o for defining the crack initiation angle. The nature of variation of θ_o vs. β shown in Fig.14 is in good agreement with the results presented in the investigation of Bin and Kim [4]. Figure14 shows that for specimen under Mode-I loading crack initiates along the ligament and for specimen under mode-II loading crack initiates almost perpendicular to the ligament. In this investigation an effort is made to study the details of crack-tip plastic zones and its various parameters under mixed mode loading. The detailed study of comparison of MPZR and θ_o estimated by analytical method proposed by Bian and Kim [4], and validity of MPZR theory can be taken up as future work.

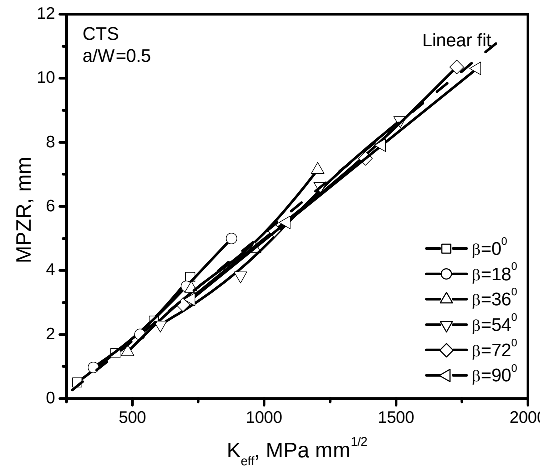

Fig.13 Variation of MPZR vs. K_{eff} for various β

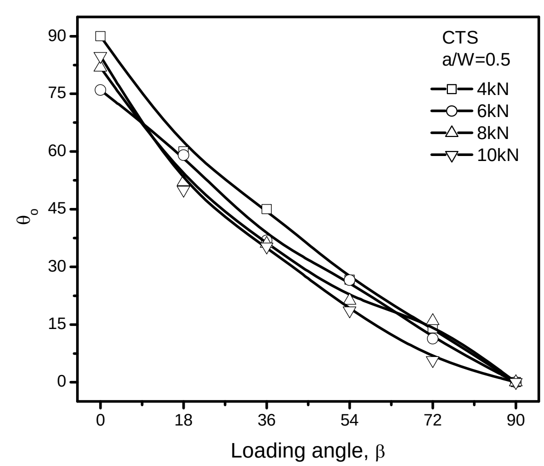

Fig.14 Variation of $\theta_o$vs. β for various loads

IV. Conclusion

Following conclusions are drawn from the present study:

(i) the magnitude of plastic zone size ahead of crack-tip (r_p) for a particular value of K_{eff} is least for β=90^o (mode-I) and it is highest for β=0^o (mode-II)

(ii) the angle at which the maximum extent of plastic zone size occurs (θ) changes from 0^o to 90^o as β is varied form 0^o to 90^o

(iii) minimum plastic zone radius (MPZR) and the angle at which MPZR occurs, θ_o, depends on the loading angle β
(iv) the variation of MPZR with K_{eff} is linear and is independent of loading angle and
(v) a simple relation between MPZR and K_{eff} is proposed, which can be useful in MPZR criterion for mixed mode fracture problems.

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