

# Research on Mechanics Symmetry Ontology

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**Abstract**—On the basis of the specific existence of symmetry in mechanical systems, its existence and functions were analyzed. Based on the above, a framework of mechanics symmetry ontology was established. The definitions and descriptions of the concepts in mechanics symmetry were presented and then the descriptions of the relationships between the elements in mechanics symmetry ontology in the definition range were proposed. Finally, its research prospect was put forward.

**Index Terms**—symmetry, ontology, mechanics symmetry, ontology elements.

## I. INTRODUCTION

There are many kinds of specific existences of symmetry in natural phenomena and artificial systems: such as petal, snowflake and crystal in the former, and building, auto and plane in the latter. The research on symmetry was conducted in many fields, such as aesthetics<sup>[1]</sup>, mathematics<sup>[2][3][4][5]</sup>, biology<sup>[6][7][8]</sup>, physics<sup>[9][10]</sup> and so on. Especially in physics, deep and systematic research on symmetry has been carried out, which has achieved a lot<sup>[11][12][13]</sup>. In different fields, symmetry all plays a crucial role. In biology, the development of biologic symmetry, to some extent, embodies the direction and rule of biologic evolution. In physics, symmetry theory guides the advancement of physical research, and it is an important method in the research of macroscopic and microcosmic physics<sup>[11][12][13]</sup>.

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In mechanics, symmetry develops with the development of mechanical system, and symmetry exists in almost all mechanical systems. Symmetry plays an important role in mechanical system, and therefore deep and systemic research on mechanics symmetry (MS) and the establishment of its theory system can guide the design process of mechanical products, benefit innovative design, and improve the efficiency of product design. But currently systematic research on MS is still a suspending problem. Research on symmetry's application in mechanics can only be referred in Barrenscheen's dissertation (German, 1990)<sup>[14]</sup>.

There are many kinds of different idiographic existences of symmetry in mechanical system, among these existences, difference as well as relationship coexists and so do the commonness and isomerism. Ontology will benefit to the exploration of research on commonness of MS. In this paper, based on the former research, referring to the platforms of biology symmetry ontology, physics symmetry ontology and common symmetry ontology<sup>[15]</sup>, combining the examples of MS, a framework of MS ontology was established; this paper provides a basic platform for subsequent research on MS.

In this paper, MS is defined as follow:

**Definition 1:** MS means an ordered repeating of some element in a mechanical system, which describes the similar rules of this mechanical system. MS exists in four levels of a mechanical system, namely function level, principle level, structure level and process level. MS ontology is defined as  $S_m = \langle C_m, CA_m, E_m, EA_m, R_m \rangle$ , where  $C_m$  - MS concept set,  $CA_m$  - MS concepts' attribute set,  $E_m$  - MS example set,  $EA_m$  - MS examples' attribute set,  $R_m$  - MS relationship set.

## II. MS CONCEPT SET $C_m$

MS concept set  $C_m = \langle c_s, c_{po}, c_f, c_{pi} \rangle$ , is a hierarchical concept system, including structure symmetry  $c_s$ , process symmetry  $c_{po}$ , function symmetry  $c_f$  and principle symmetry  $c_{pi}$ , as figure 1.

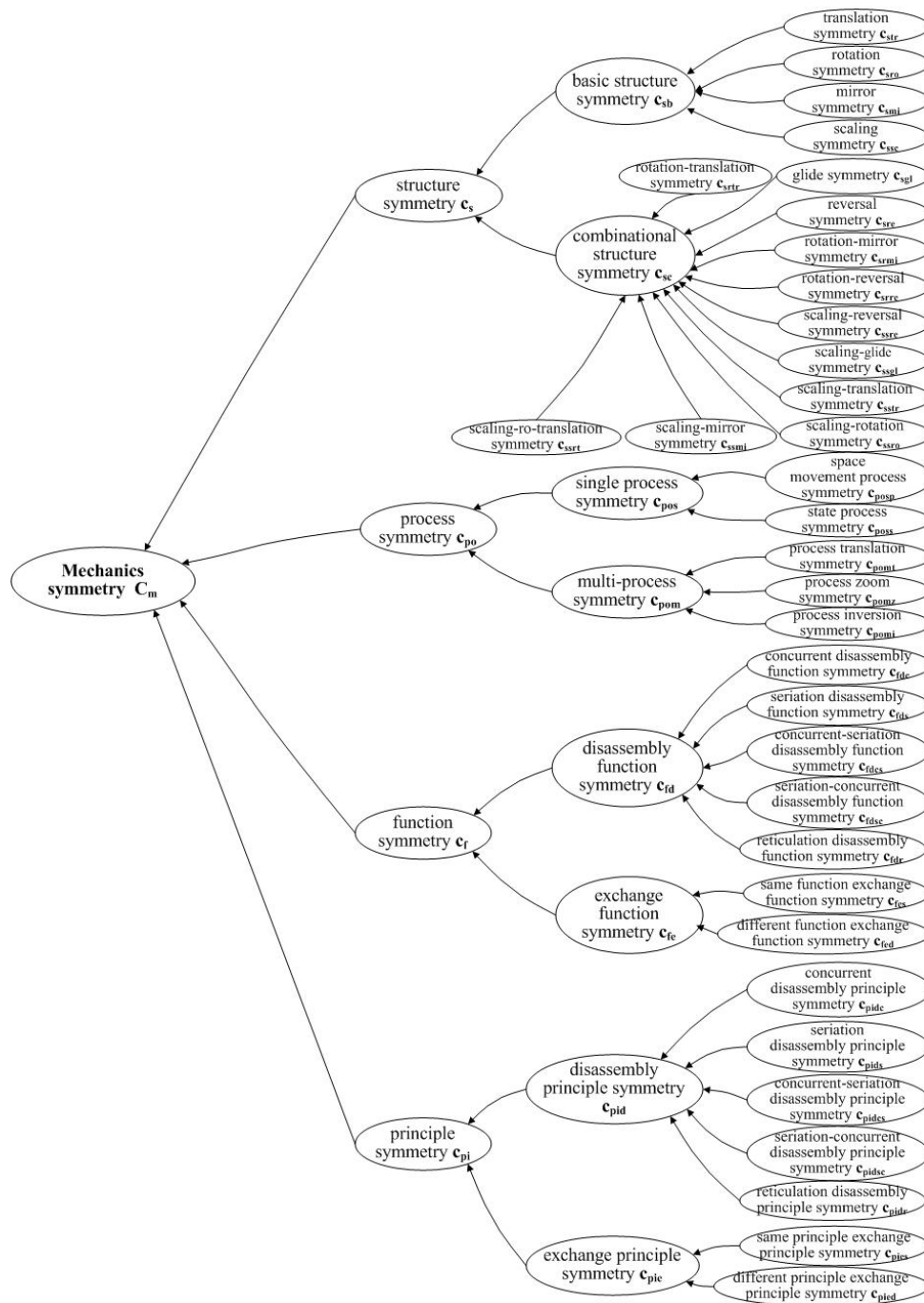


Figure 1: elements and hierarchical levels of mechanics symmetry ontology

**Definition 2:** If a mechanical system has two or more identical parts in geometric structure, and these parts are arranged orderly according to some rule or regulation, then the mechanical system has structure symmetry.

Structure symmetry  $C_s = \langle C_{sb}, C_{sc} \rangle$ , where  $C_s$ - basic structure symmetry  $C_{sb} = \langle C_{str}, C_{sro}, C_{smi}, C_{ssc} \rangle$ , combination structure symmetry  $C_{sc} = \langle C_{strtr}, C_{sgl}, C_{srre}, C_{srmi}, C_{srre}, C_{sstr}, C_{ssro},$

$C_{ssmi}, C_{ssgl}, C_{ssre}, C_{ssrt} \rangle$ . Combination structure symmetry is composed of basic structure symmetries. The sketch map of structure symmetries is showed in table 1.

Figure 2 is an example having mirror symmetry, figure 3 is an example having scaling-translation symmetry.

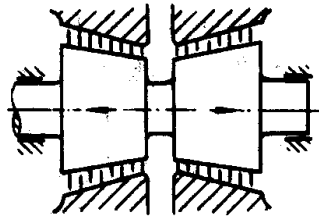


Figure 2: mirror symmetry structure<sup>[16]</sup>

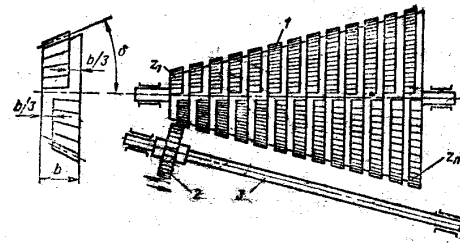
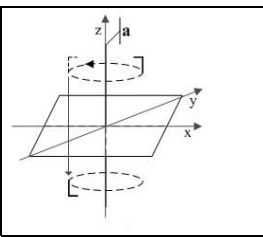
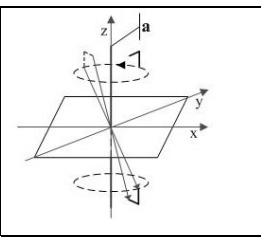


Figure 3: scaling-translation symmetry structure<sup>[18]</sup>

Table 1: sketch map of structure symmetries

| symmetry                                    | sketch map | description   | symmetry  | sketch map | description   |
|---|------------|---|---|------------|---|
| translation symmetry<br>$C_{str}$           |            | Translate with equal interval $d$ along the direction of vector $b$   | scaling-translation symmetry<br>$C_{sstr}$          |            | the locations of symmetry elements are arranged as translation symmetry, the sizes change regularly     |
| rotation symmetry<br>$C_{sro}$              |            | rotate along the axis vertical to rotating plane with angle $a$   | scaling-rotation symmetry<br>$C_{ssro}$             |            | the locations of symmetry elements are arranged as rotation symmetry, the sizes change regularly        |
| mirror symmetry<br>$C_{smi}$                |            | mirror operation can be processed in two-dimensional plane and also in three-dimensional space                          | scaling-mirror symmetry<br>$C_{ssmi}$               |            | the locations of symmetry elements are arranged as mirror symmetry, the sizes change regularly          |
| rotation-translation symmetry<br>$C_{srtr}$ |            | Rotate along axis $a$ which is vertical to rotating plane and then translate along the direction parallel to axis $a$ . | scaling-rotation-translation symmetry<br>$C_{ssrt}$ |            | the locations of symmetry elements arranged as rotation-translation symmetry, the sizes changed orderly |
| glide symmetry<br>$C_{sgl}$                 |            | the direction of mirror and the direction of translation are vertical mutually  | scaling-glide symmetry<br>$C_{ssgl}$                |            | the locations of symmetry elements are arranged as glide symmetry, the sizes change regularly           |
| reversal symmetry<br>$C_{sre}$              |            | reversal symmetry can be regarded as combination of mirror symmetry and rotation symmetry                               | scaling-reversal symmetry<br>$C_{ssre}$             |            | the locations of symmetry elements are arranged as reversal symmetry, the sizes change regularly        |

|  |   |   |  |  |   |
|--|---|---|--|--|---|
| rotation-mirror symmetry<br>$C_{srmi}$ |  | Rotate along axis a and then carry out mirror operation by taking xy plane as datum plane | rotation-reversal symmetry<br>$C_{srre}$ |  | Rotate along axis a and then carry out reversal operation by taking xy plane as datum plane |
|--|---|---|--|--|---|

**Definition 3:** If a mechanical system has one or more parts that have orderly time-changing process, then this mechanical system has process symmetry.

Process symmetry  $C_{po} = \langle C_{pos}, C_{pom} \rangle$ , where:  $C_{pos}$  - single process symmetry,  $C_{pos} = \langle C_{posp}, C_{poss} \rangle$ ,  $C_{posp}$  - space movement process symmetry, the symmetry of space movement,  $C_{poss}$  - state process symmetry, the symmetry of transformation of materiel, energy or information;  $C_{pom}$  - multi-process symmetry,  $C_{pom} = \langle C_{pomt}, C_{pomz}, C_{pomi} \rangle$ , there exists three relationship, namely translation, zoom and inversion, between multiple symmetry process.

For the crank-slider linkage in Figure 4, when crank AB moves at uniform speed, both the moving tracks of crank and slider are regular time-changing functions. The moving process of crank AB is a rotating symmetric circle track while the one of slider C is a reciprocating linear moving process. Therefore this mechanism has single moving process symmetry.

For the double-crank-slider in figure 5, the moving processes of crank AB and crank CD have identical time-changing law, so this mechanism has multi-process symmetry.

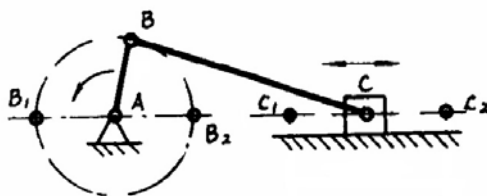


Figure 4: crank-slider linkage

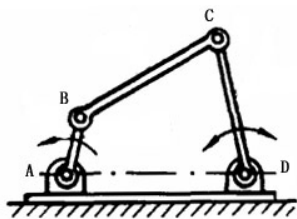


Figure 5: double-crank-slider

**Definition 4:** If a mechanical system has two or more functions and these functions can exchange, and the exchanges do not influence the realization of the function

requirement of this system, then this mechanical system has function symmetry.

Function symmetry  $C_f = \langle C_{fd}, C_{fe} \rangle$ , where  $C_{fd}$  - disassembly function symmetry,  $C_{fd} = \langle C_{fdc}, C_{fds}, C_{fdes}, C_{fdsc}, C_{fdr} \rangle$ , the disassembly types of function disassembly are showed in table 2;  $C_{fe}$  - exchange function symmetry,  $C_{fe} = \langle C_{fes}, C_{fed} \rangle$ .

Figure 6 is the schematic diagram of the wheel fixing bolt. This mechanism has parallel disassembly function symmetry  $C_{fdc}$ . The five fixing bolts on wheel has the same function and co-work to implement the function of fixing wheel. If one of them doesn't work, the others can work normally, which is parallel function disassembly design.

Figure 7 is the schematic diagram of a single universal joint. This mechanism has exchange function symmetry. The driving shaft and driven shaft of the joint are two components with identical functions and they can exchange. However, each of them is indispensable, and they must mutually cooperate to implement the universal joint transmission.

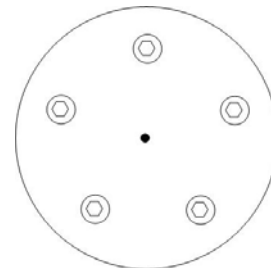


Figure 6: a wheel fixing bolt

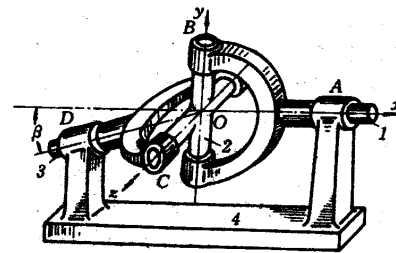


Figure 7: a single universal joint<sup>[17]</sup>

**Definition 5:** If a mechanical system has two or more principles and these principles can exchange, and these exchanges do not influence the realization of the principle requirement of the system, then this mechanical system has principle symmetry.

Principle symmetry  $c_{pi} = \langle c_{pid}, c_{pie} \rangle$ , where  $c_{pid}$  - disassembly principle symmetry,  $c_{pid} = \langle c_{pidc}, c_{pids}, c_{pidsc}, c_{pidr} \rangle$ , the disassembly types of principle disassembly are showed in table 2;  $c_{pe}$ -exchange principle symmetry,  $c_{pie} = \langle c_{pies}, c_{pied} \rangle$ .

For the example in figure 8, to make the jamming plate 3 keep the direction of AB, this mechanism utilizes two symmetric springs to hold the two sides of jamming

plate 3 and these two springs cooperate to realize the function requirement. The structure, function and principle of these two springs are all the same. Therefore, the two springs and jamming plate have mirror symmetry in structure, function symmetry in function and principle symmetry in principle.

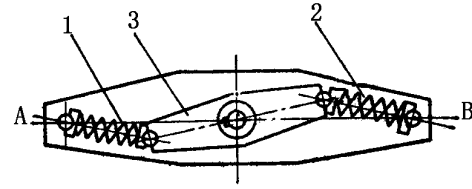


Figure 8: a structure having multi-level symmetry<sup>[19]</sup>

Table 2: sketch map of the style of decomposability

| concurrent disassembly | seriation disassembly | seriation-concurrent disassembly | concurrent-seriation disassembly | reticulation disassembly |
|------------------------|-----------------------|----------------------------------|----------------------------------|--------------------------|
|                        |                       |                                  |                                  |                          |

### III. MS CONCEPTS' ATTRIBUTE SET $CA_m$

MS concepts' attribute set contains all the attributes of all the elements in MS concept set,  $CA_m = \langle ca_{mb}, ca_b, ca_o, ca_{dn}, ca_{ax}, ca_{mt}, ca_n, ca_{df}, ca_{dp}, ca_{ds}, ca_c, ca_{th}, ca_p, ca_f \rangle$ . In this set:  $ca_{mb}$  - main body of MS;  $ca_b$  - level of MS, including four levels: function level, principle level, structure level and process level.  $ca_o$  - symmetric operation of MS, symmetric operation connected with symmetry closely, every symmetry corresponds to a symmetric operation;  $ca_{dn}$  - dimensionality of symmetric structure;  $ca_{ax}$  - symmetric axis or symmetric plane of one symmetric structure;  $ca_{mt}$  - mathematical description of symmetry;  $ca_n$  - sum of elements of MS;  $ca_{df}$  - similarities and differences of elements' functions of MS.  $ca_{dp}$  -

similarities and differences of elements' principle of MS.  $ca_{ds}$  - similarities and differences of elements' structure of MS.  $ca_c$  - combinability of MS, including common combinative symmetries and non-combinative symmetries;  $ca_{th}$  - theory of symmetry design;  $ca_p$  - principle of symmetry design;  $ca_f$  - function of mechanics symmetry, in a high level, the function can be divided into three types: economic function, technical function and social function.

MS concepts' attribute set  $CA_m$  has many elements, the attribute elements of each element of MS concept set  $C_m$  are a subset of  $CA_m$ . Taking mirror symmetry  $c_{smi}$  as an example, the attribute  $CA_{m(csmi)}$  of  $c_{smi}$  is showed in table 3.

Table 3: the attribute  $CA_{m(csmi)}$  of  $c_{smi}$

|                | $ca_{mb}$              | $ca_b$          | $ca_o$           | $ca_{dn}$         | $ca_{ax}$                    | $ca_{mt}$                                  | $ca_n$ | $ca_{ds}$ | $ca_c$  | $ca_{th}, ca_p, ca_f$ |
|----------------|------------------------|-----------------|------------------|-------------------|------------------------------|--|--------|-----------|---|-----------------------|
| $CA_{m(csmi)}$ | structure of mechanism | structure level | mirror operation | two-dimensional   | symmetric axis is y axis     | $X_2 = -X_1$<br>$Y_2 = Y_1$                | 2      | same      | it can be combined with $C_{str}$ , $C_{sro}$ , $C_{ssc}$ and so on |                       |
|                |                        |                 |                  | three-dimensional | symmetric plane is y-z plane | $X_2 = -X_1$<br>$Y_2 = Y_1$<br>$Z_2 = Z_1$ |        |           |   |                       |

IV. MS EXAMPLE SET  $E_m$

MS example set  $E_m$  includes all mechanical systems and elements having symmetry. The elements of  $E_m$  could be some large and complicated mechanical systems, such as plane, auto, and engineering machine etc., and could also be some small and simple standardized parts and components, such as bolt, nut, and spring etc. Corresponding to MS concept set  $C_m$ ,  $E_m$  is also a hierarchical system. Figure 9 is a symmetric example- an oriented mechanism, marked as  $E_{om}$ .

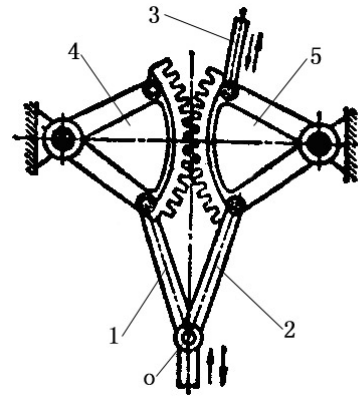


Figure 9: an oriented mechanism<sup>[18]</sup>

Table 4: the attributes matrix of  $E_{om}$

| $ea_t$   | $ea_{mb}$                                       | $ea_b$          | $ea_o$                   | $ea_n$ | $ea_{dr}$ | $ea_{dp}$ | $ea_{ds}$ | $ea_c$  | $ea_{th}$ | $ea_p$ | $ea_r$   |
|--|---|-----------------|--------------------------|--------|-----------|-----------|-----------|---|-----------|--------|--|
| 1. disassembly function symmetry<br>$C_{fd}$   | sector gear 4, link 1 and sector gear 5, link 2 | function level  | reticulation disassembly | 2      | same      | —         | —         | —   |           |        |  |
| 2. disassembly principle symmetry<br>$C_{pid}$ | sector gear 4, link 1 and sector gear 5, link 2 | principle level | reticulation disassembly | 2      | —         | same      | —         | —   |           |        |  |
| 3. rotation symmetry<br>$C_{sro}$              | gear teeth                                      | structure level | rotation operation       | 12     | —         | —         | same      | the four symmetries are hierarchical, No4 include No3, No6 is composed of No3, No4 and No5. |           |        | the rotation symmetric and mirror symmetric structures can ensure the up-down reciprocating linear motion of point "o" |
| 4. mirror symmetry<br>$C_{smi}$                | sector gear 4 and 5                             | structure level | mirror operation         | —      | —         | —         | —         | —   |           |        |  |
| 5. mirror symmetry<br>$C_{smi}$                | link 1 and 2                                    | structure level | mirror operation         | 2      | —         | —         | —         | —   |           |        |  |
| 6. mirror symmetry<br>$C_{smi}$                | sector gear 4, link 1 and sector gear 5, link 2 | structure level | mirror operation         | 2      | —         | —         | same      | the four symmetries are hierarchical, No4 include No3, No6 is composed of No3, No4 and No5. |           |        | the rotation symmetric and mirror symmetric structures can ensure the up-down reciprocating linear motion of point "o" |

V. MS EXAMPLES' ATTRIBUTE SET  $EA_m$

MS examples' attribute set  $EA_m$  is the description of MS example set  $E_m$ .  $EA_m$  can be described by a two-dimensional matrix: the row element is the symmetry type of MS example  $ea_i$ ; the column elements include  $\{ea_{mb}, ea_b, ea_o, ea_n, ea_{df}, ea_{dp}, ea_{ds}, ea_c, ea_{th}, ea_p, ea_f\}$ , where:  $ea_{mb}$  — main body of symmetry,  $ea_b$  — symmetry level, including four levels: function level, principle level, structure level and process level,  $ea_o$  — symmetric operation, different for different main body,  $ea_n$  — sum of symmetric elements,  $ea_{df}$  — similarities and differences of elements' functions of symmetry examples,  $ea_{dp}$  — similarities and differences of elements' principle of

symmetry examples,  $ea_{ds}$  — similarities and differences of elements' structure of symmetry examples,  $ea_c$  — combinability of example's symmetry, an complicated example can have many kinds of symmetry and these symmetries may be combined with some hierarchical rules,  $ea_{th}$  — theory of symmetry design,  $ea_p$  — principle of symmetry design,  $ea_f$  — function of mechanics symmetry, the same to  $ca_f$ ,  $ea_f$  and can also be divided into three types of functions: economic function, technical function and social function. Taking  $E_{om}$  as an example, the attribute matrix of  $EA_m(E_{om})$  is showed in table 4.

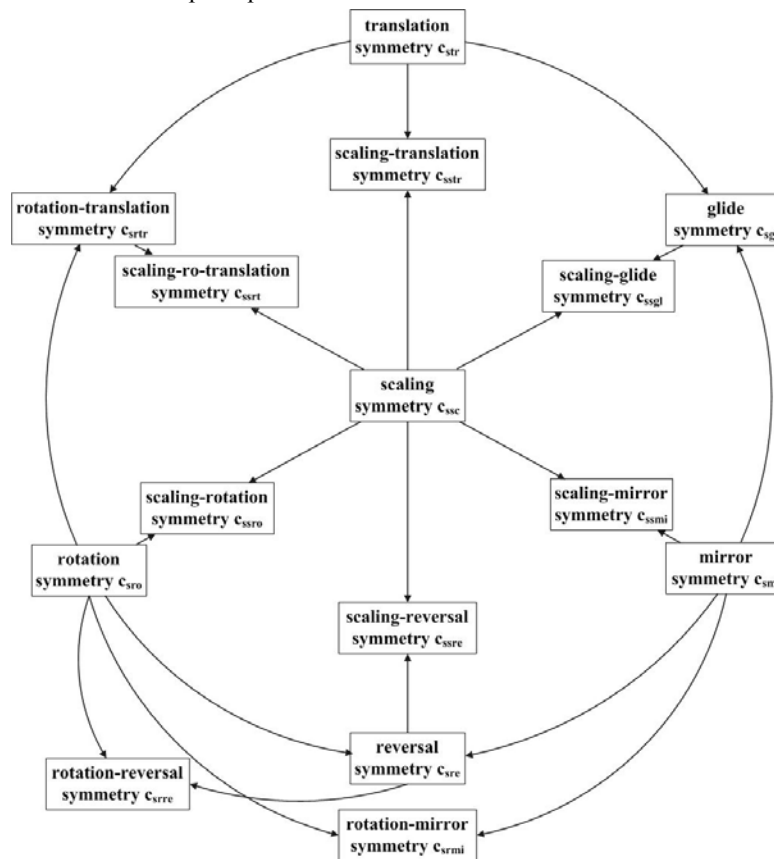


Figure 10: the combinational relationship between structure symmetry

VI. MS RELATIONSHIP SET  $R_m$

MS relationship set  $R_m$  describes the interior relationships and interrelation between the elements of MS ontology, such as  $C_m$ ,  $CA_m$ ,  $E_m$ ,  $EA_m$ .

— In MS concept set  $C_m$ , there are two kinds of affiliating relationship in the elements in the same class: “equal to” and “is a”. If concept  $c_1$  and  $c_2$  are completely the same, then  $c_1$  equal to  $c_2$ , marked as  $c_1 \equiv c_2$ . If concept  $c_1$  is a subclass of  $c_2$ , then  $c_1$  is a  $c_2$ , marked as  $c_1 \subseteq c_2$ , such as  $c_{pie} \subseteq c_{pi}$ ,  $c_{str} \subseteq c_s$ ,  $c_{sre} \subseteq c_s$ .

— The combinational relationship between structure symmetry  $c_s$  in MS concept set  $C_m$ . The elements in combinational structure symmetry  $c_{sc}$  are composed of the elements in basic structure symmetry  $c_{sb}$ . For instance,

rotation-translation symmetry  $c_{srt}$  is composed of rotation symmetry  $c_{sro}$  and translation symmetry  $c_{str}$  and reversal symmetry  $c_{sre}$  is composed by mirror symmetry  $c_{smi}$  and rotation symmetry  $c_{sro}$ . The specific combinational relationship is showed in figure 10.

— In MS concept set  $C_m$ , there is a relationship called “mapped” between the elements in different levels. The elements in function symmetry  $c_f$ , principle symmetry  $c_{pi}$ , structure symmetry  $c_s$  and process symmetry  $c_{po}$  have a multi to multi bidirectional mapped relationship, showed in figure 11.

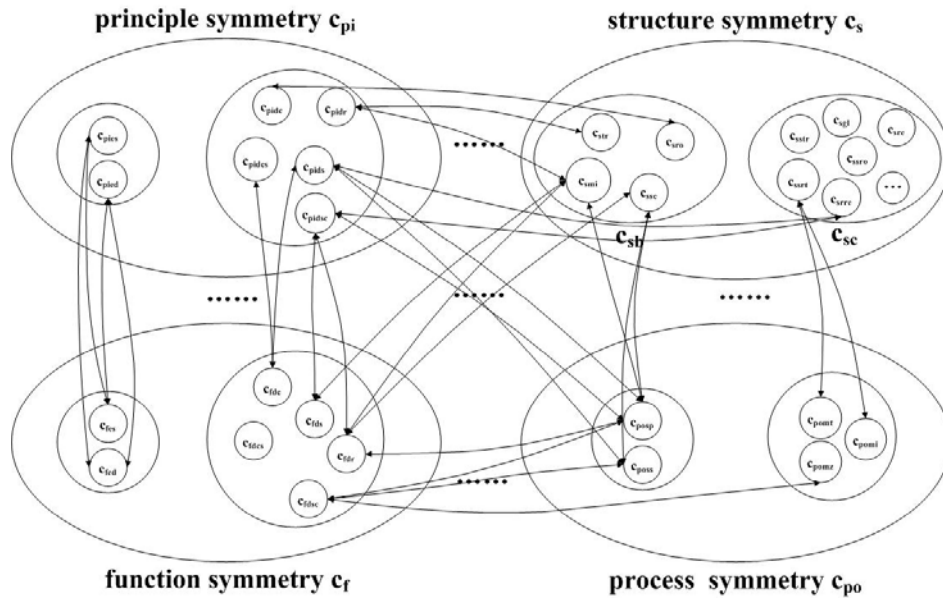


Figure 11: the mapped relationship between the levels in  $C_m$

—The elements in MS concepts' attribute set  $CA_m$  are detailed descriptions of the elements in MS concept set  $C_m$ , and the relationship between  $C_m$  and  $CA_m$  is a multi to multi unidirectional mapped relationship, showed in figure 12. Similarly, the elements in MS examples' attribute set

$EA_m$  are detailed descriptions of the elements in MS example set  $E_m$ , and the relationship between  $E_m$  and  $EA_m$  is still a multi to multi unidirectional mapped relationship, showed in figure 13.

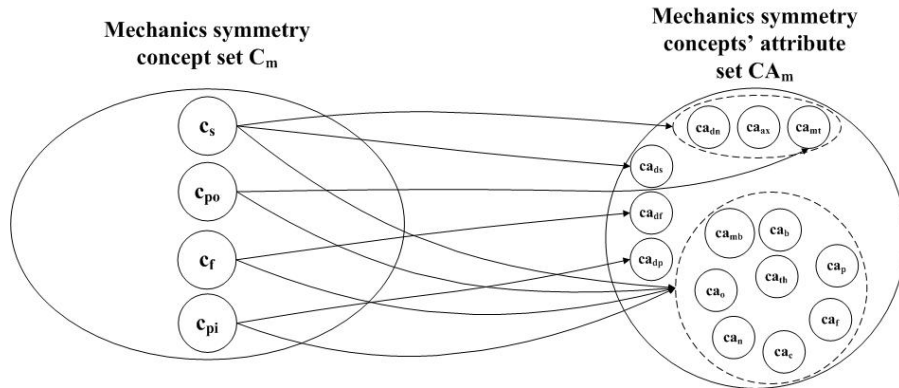


Figure 12: the relationship between  $C_m$  and  $CA_m$

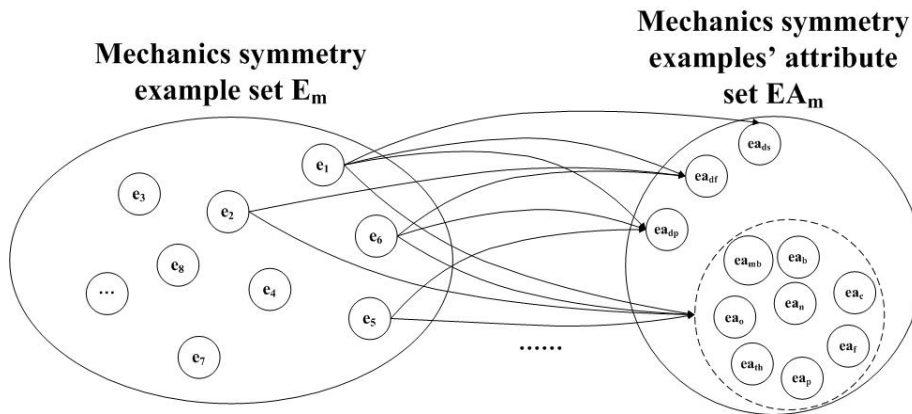


Figure 13: the relationship between  $E_m$  and  $EA_m$

—The elements in MS example set  $E_m$  are the instantiation of the elements in MS concept set  $C_m$ , the relationship between  $C_m$  and  $E_m$  is a multi to multi

unidirectional mapped relationship, showed in figure 14. Similarly,  $CA_m$  and  $EA_m$  is also a multi to multi unidirectional mapped relationship, showed in figure 15.



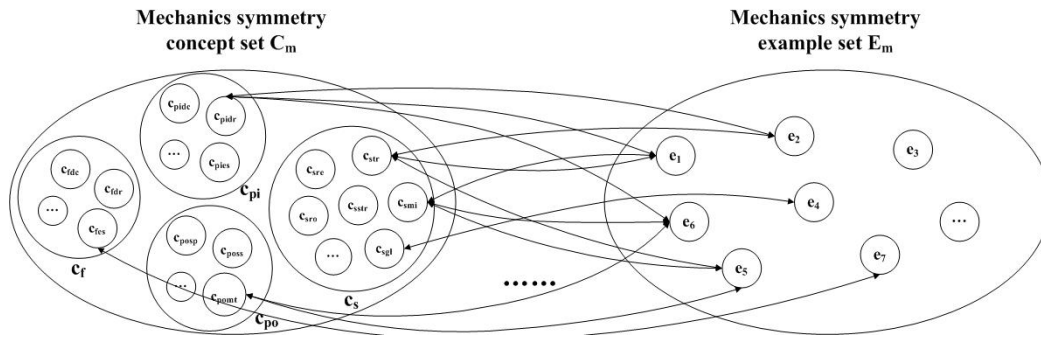


Figure 14: the relationship between  $C_m$  and  $E_m$

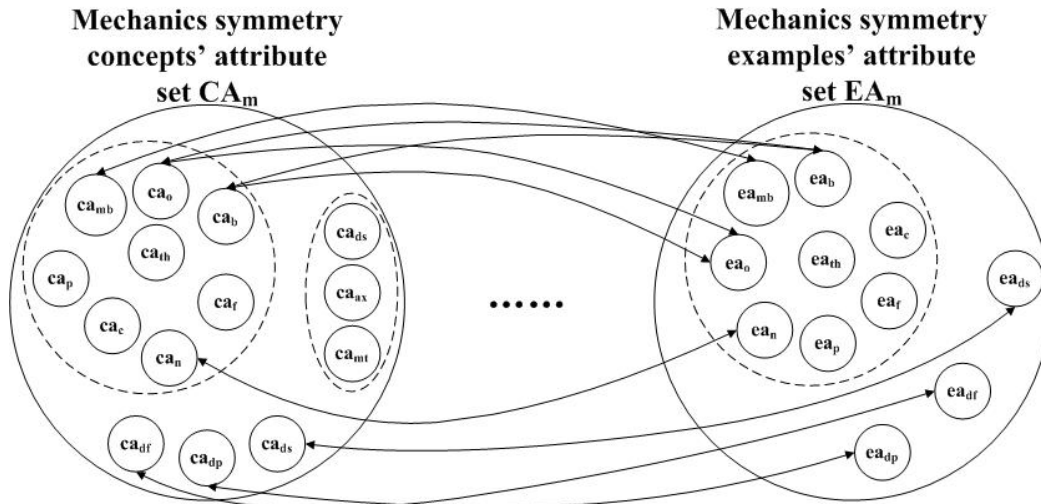


Figure 15: the relationship between  $CA_m$  and  $EA_m$

## VII. CONCLUSION

Symmetry exists anywhere in mechanical system and mechanical engineering, in which it plays an important role. The research on concept, principle, rule of MS is helpful to further understand the objective laws of product design and mechanical systems, benefit innovative design, enhance the efficiency of product design, and improve technology, economy, sociality of product design. In this paper, a framework of mechanics symmetry ontology was established; the definitions and descriptions of the concepts in MS and the relationships and descriptions of the elements in MS ontology in the definition range were proposed. Moreover, this paper established foundation for further research on MS examples and the principles of MS. Further achievement will be posted latterly

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