Numerical Analysis of Two-Phase Turbulent Flow in Horizontal Channel with Experimental Verification

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***Abstract*— in this study, large eddy simulation method (LES) has been used for simulating the particle-laden turbulent flow. We used one-way coupling in our simulations. In one-way coupling model the presence of particles has negligible effect on the carrier flow. We suppose that our particle is spherical and the drag, buoyancy and gravity forces affect the movement of the particles. The numerical solution has been verified with LDA experimental tests. Also we have used Al_2O_3 particles in LDA tests.**

***Index Terms*— Large eddy simulation, particle-laden flow, Horizontal channel, Laser Doppler Anemometry**

I. INTRODUCTION

A. Numerical Solution

Two-phase flows consist of two different phases e.g. gas and liquid, gas and particle or liquid and particle. In general, different phases in two-phase flows interact each other, change the shape of their interface, and transit from one flow pattern to another. When the carrier flows are turbulent in dispersed two-phase flows, those flows are called dispersed two-phase turbulent flows. Examples for such numerous industrial processes are coal combustion, dust deposition and removal in clean rooms, droplets deposition in gas-liquid flows, etc... Accurate prediction of particle-laden turbulence is important in order to gain a better understanding of particle transport by turbulence flow.

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Since the Navier-Stokes equation involves a nonlinear advection term, extra unknowns called Reynolds stresses appear in the averaged equation.
Therefore relevant closure models are needed to close the equation. One of the most popular closure models is the k-ε model (Jones & Launder, 1972), where k is the turbulent kinetic energy and ε is the dissipation rates which are obtained by solving the transport equations. Due to the rapid development of computational ability some common method has been made. Traditional methods are usually based on the Reynolds-Averaged Navier-Stokes (RANS) equations in which the entire spectrum of velocity fluctuations is represented indirectly using various parameters , see for example Chang *et al.* [1] as well as Berlemont *et al.* [2]. A primary shortcoming of RANS methods for the prediction of particle-laden turbulent flows is related to deficiencies associated with the model used to predict properties of the Eulerian turbulence field. This problem can be solved by using a more approach like DNS but the drawback is that it is only applicable to low or moderate Reynolds numbers. In between there is the large-eddy simulation (LES) which is less sensitive to modeling errors than in RANS calculation and less restricted to low Reynolds number than DNS, see for a review Yeung [3], Crowe *et al.* [4], Michaelides [5] and Peng *et al.* [6]. Concerning the effect of the particles on the carrier flow, two different approaches called one-way and two-way coupling can be used. In one-way coupling model the presence of particles has negligible effect on the carrier flow but in a two-way coupling model the effects of particles is taken into account in the carrier flow (Segura [7]). In this paper an overview of the particle motion simulation including the calculation of particle trajectories, particle velocity statistics and time advancement is given and later the results of the particle-laden channel flow are presented.

B. Experimental Solution

Several experimental work on dispersed two-phase turbulent flows have been reported since the

pioneering work by Hetsroni & Sokolov (1971) who studied the suppression of turbulence by adding droplets in a turbulent jet. Such turbulence modulation occurs in wall bounded flows, too, as studied by Zisselmar & Molerus (1979) and Maeda et al. (1980) to name a few. Hetsroni (1989) and Gore & Crowe (1989) summarized the data from the available experiments, and proposed criteria between the enhancement and the suppression of turbulence, which were later validated theoretically by Yuan & Michaelides (1992). The mechanisms of these phenomena, however, were not understood by the experiments as most of them lack detailed information. During recent few years, the focus of experiments has moved to such detailed statistics and topological structure of the flow. For example, in the experiments of gas-particle flows in a channel by Fessler et al. (1994) it was found that the particles with finite, small inertia tend to concentrate in the regions with low shear in the streaky turbulent structure near the wall (Fukagata [8]).
Kulick et al. (1994) presented detailed particle velocity statistics in gas-particle turbulent channel flow, according to which the particle phase has more complicated velocity statistics than fluid. Crowe (2000) revisited the turbulence modulation problem and proposed a model of energy production and dissipation due to presence of particles based on the recent experimental data in a channel (Kulick et al, 1994) and pipes (Hosokawa et al., 1998; Savolainen et al., 1998; Varaksin et al, 1998). Apart from the physical aspects on the dispersed two-phase turbulent flows, the measurement technique seems to be a big issue for these experiments.

II. SIMULATION

The method used to solve the incompressible Navier-Stokes equations is a subset of the pressure correction method independently by Chorin [9] and Temam ([10],[11]). The governing equations for an incompressible flow are as follows,

$$\frac{\partial u_i^f}{\partial x_i} = 0 \qquad (1)$$

$$\frac{\partial u_i^f}{\partial t} + \frac{\partial u_i^f u_j^f}{\partial x_j} = -\frac{1}{\rho}\frac{\partial P}{\partial x_i} + \nu\left(\frac{\partial^2 u_i^f}{\partial x_j^{\,2}} + \frac{\partial^2 u_j^f}{\partial x_j \partial x_i}\right) \qquad (2)$$

The extension to a multi-stage Runge-Kutta method is straight forward as the same procedure is followed at every stage. Where *xi*'s are the Cartesian coordinates, and *ui*'s are the corresponding velocity components.
However, to save some computational time, the compact scheme is only used for the convective fluxes in the streamwise and spanwise directions. The viscous and convective fluxes in the normal directions are calculated with a standard second order method. The viscous fluxes in the spanwise and streamwise directions are obtained with a classical (not compact) fourth order method. large-eddy simulations were performed at a Reynolds number, based on friction velocity and channel half-width, of $\mathrm{Re}_\tau = \frac{u_\tau \delta}{\nu} =$ 180. flow was resolved using $65\times65\times65$ grid points in the x, y and z directions. The streamwise, normal and spanwise dimensions are $4\pi\delta \times 2\delta \times \frac{4}{3}\pi\delta$, with δ the channel half-width. The grid spacing in wall coordinates in the x and z directions is $\Delta x+ = \Delta x\, u_\tau/\nu = 35$ and $\Delta z+ = \Delta z\, u_\tau/\nu = 12$. A non-uniform mesh with hyperbolic tangent distribution is used in the wall-normal direction. The first mesh point away from the wall is at $y+ = \Delta y\, u_\tau/\nu = 0.49$ originally applied by Harlow and Welch [12] for the computation of free surface incompressible flows. It is called fractional step or projection method, developed and the maximum spacing (at the centerline of the channel) is 13.8 wall units. A Smagorinsky model with damping near the wall is used for the subgrid scale modeling. The Smagorinsky constant is equal to C = 0.007. The overall accuracy of the method is then $O(\Delta x)4 + O(\Delta z)4 + O(\Delta y)2$. The turbulence intensities and mean velocity profile are shown in figures (1), and (2). The results are compared with the DNS data of Kim *et al.* [13].

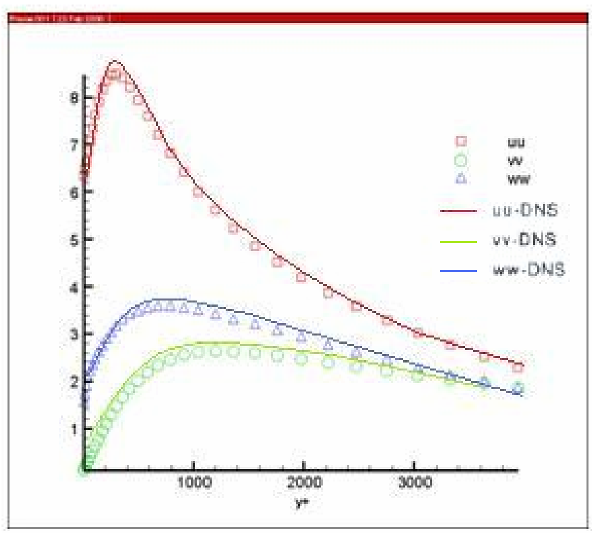

Figure 1: Turbulence intensities for channel flow

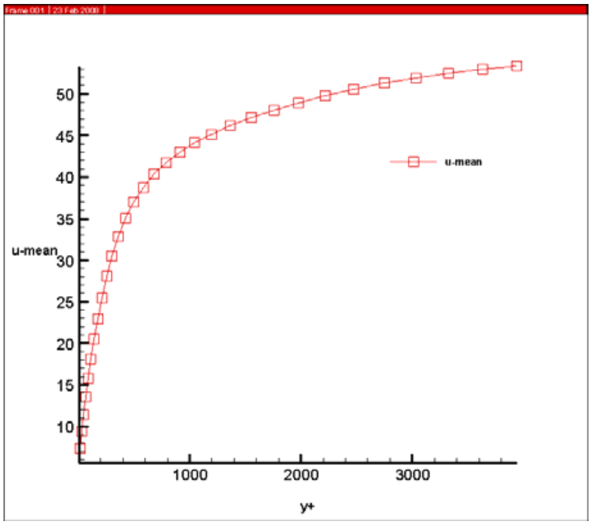

Figure 2: Mean streamwise velocity of channel flow

III. CALCULATION OF THE PARTICLE TRAJECTORIES

The particle tracking routines can be added to any LES solver without major modification of the original routines. The time step used for the particles must be equal to the time step of the carrier flow. The three components of the particles velocities as well as their x,y and z coordinates are updated at every iteration. The particle equation of motion used in the simulations describes the motion of particles with densities substantially larger than that of the surrounding fluid and diameters small compared to the Kolmogorov scale. The total force F_i of a single particle in a uniform flow field can be generally expressed by,

$$F_i = C_D A_p \frac{\rho_f}{2} |V - U| (u_i - v_i) + g\delta_{i1}(\rho_p V_p)$$

$$i = 1,2,3 \tag{3}$$

Where A_p is the exposed frontal area of the particle to the direction of the incoming flow, and V_p is the volume of the particle. For a spherical particle $A_p = \pi d^2/4$ and $V_p = 4/3\pi(d/2)^3$, with d the particle diameter. v_i is the velocity of the particle and u_i is the velocity of the fluid at the particle position (Lessani [14]). The body force $g\delta_{i1}(\rho_p V_p)$ acts along vertical direction of the wall. The fluid and particle densities are denoted ρ_f and ρ_p, respectively. The particle acceleration $\frac{dV_i}{dt} = \frac{F_i}{\rho_p V_p}$ can finally be written as,

$$\frac{dV_i}{dt} = \frac{\rho_f}{\rho_p} \frac{3}{4} \frac{C_D}{d} |V - U| (u_i - v_i) + g\delta_{i1}$$

$$i = 1,2,3 \tag{4}$$

Equation (4) may also be written in the following form,

$$\frac{dV_i}{dt} = \frac{1}{\tau_p}(u_i - v_i) + g\delta_{i1} \tag{5}$$

If the Drag coefficient is defined with the Stokes's drag law $C_D = \frac{24}{\mathrm{Re}_p}$ and $\mathrm{Re}_p = \frac{|V-U|d}{\nu}$, the Stokes relaxation time τ_p may be written as,

$$\tau_p = \frac{\rho_p d^2}{18\nu\rho_f} \tag{6}$$

Previous computations of particle-laden turbulent channel flow have shown that the particle Reynolds number Re_p does not necessarily remain small (Rouson *et al.* [15]). Therefore, an empirical relation for C_D from (Clift *et* al. [16]) valid for particle Reynolds number up to about 40 was employed,

$$C_D = \frac{24}{\mathrm{Re}_p}(1 + 0.15\,\mathrm{Re}_p^{0.687}) \tag{7}$$

The particle Reynolds number of the present calculations never exceeds this maximum value. For particles with material densities large compared to the fluid the other forces are negligible compared to the drag. The effect of lift force, while relevant to problems of particle deposition, is less significant to this work and therefore the effect of shear-induced lift in the equation of motion has been neglected. The volume fraction of particles is assumed small enough such that particle-particle interactions are negligible. Once the velocities of the particles at the new time level are calculated with equation (4), Positions will be obtained by solving the following equation:

$$\frac{dx_{i,p}}{dt} = V_i \qquad i = 1,2,3 \tag{8}$$

Where $x_{i,p}$ is the particle position. Equations (4) and (8) are integrated in time using a second-order Adams-Bashforth method (Lessani [14]).

IV. PARTICLE VELOCITY STATISTICS

To be able to draw the profiles of the particle velocities, the cell-averaged values of the particles are needed. The instantaneous cell-averaged velocity

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vector of the particles is,

$$\overline{V_p} = \frac{\sum_{s=1}^{N_{part}} V_s}{N_{part}} \tag{9}$$

where $\overline{V_p}$ is the cell averaged velocity vector of the particles, V_s is the velocity vector of a single particle inside the cell, and N_{part} is the total number of particles in the cell. Once $\overline{V_p}$ is obtained the time or/and space averaging can be performed and, in the same way that the turbulence statistics of the carrier flow are calculated, the particle fluctuations can be calculated. The particle fluctuating velocity vector, $v_p{}'$ can be defined as,

$$v_p{}' = \left\langle \overline{V_p} \right\rangle - \overline{V_p}$$

(10)

Where $\langle \ldots \rangle$ denotes the time or/and space averaging. For example, if we denote the components of the cell averaged velocity vector of the particles as, $\overline{V_p} = (\overline{V_{1p}}, \overline{V_{2p}}, \overline{V_{3p}})$ and the fluctuating part as, $v_p{}' = (v_{1p}{}', v_{2p}{}', v_{3p}{}')$ .Then $\left\langle v_{1p}{}' v_{2p}{}' \right\rangle$ is calculated as follows,

$$\left\langle v_{1p}{}' v_{2p}{}' \right\rangle = \left\langle \overline{V_{1p}}\,\overline{V_{2p}} \right\rangle - \left\langle \overline{V_{1p}} \right\rangle \left\langle \overline{V_{2p}} \right\rangle \tag{11}$$

V. PARTICLE-LADEN CHANNEL FLOW

Once a time-averaged steady state solution has been obtained for the Eulerian velocity field the carrier flow, the particles are assigned random locations throughout the channel. Starting with this initial solution, the flow and particles are advanced in time simultaneously until a time-averaged steady state is reached for the particles. The development time, i.e., the time required for particles to become independent of their initial conditions, depends on the particle response time (Lessani [14]). In the present calculations, the development time is set to $\frac{6\delta}{u_\tau}$ for all particles. The particle response time $\tau_p = \frac{\rho_p d^2}{18\nu\rho_f}$,the radius of the particle r_p, and the particle to fluid density ratio $\frac{\rho_p}{\rho_f}$ are shown in table 1,

$\frac{\tau_p}{\delta / u_\tau}$	τ_p	$\frac{r_p}{\delta}$	r_p	$\frac{\rho_p}{\rho_f}$
4.32×10^{-2}	8.32	10^{-4}	$10^{-6} m$	3.811

VI. RESULTS AND DISCUSSION

The results for Al_2O_3 particles have been presented as below:
(For better understanding diagrams for experimental and numerical solution are shown in one figure)

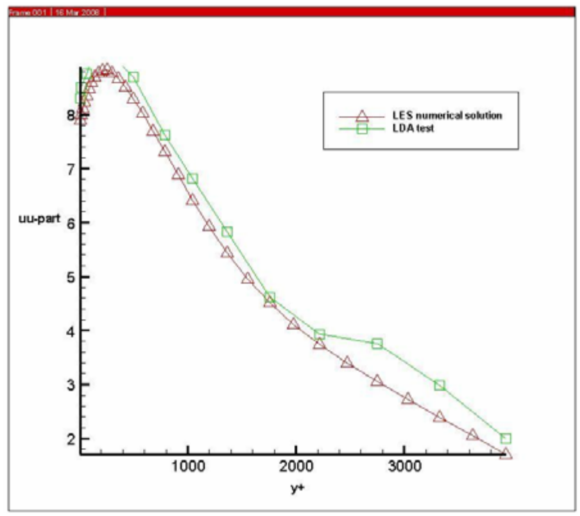

Figure 3: Root-mean-square velocity fluctuations of Al_2O_3 particles in turbulent channel flow, streamwise $u'u'$

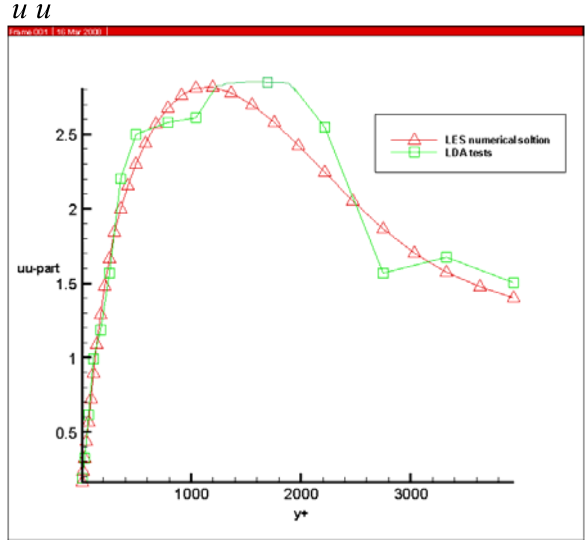

Figure 4: Root-mean-square velocity fluctuations of Al_2O_3 particles in turbulent channel flow, normal direction $v'v'$

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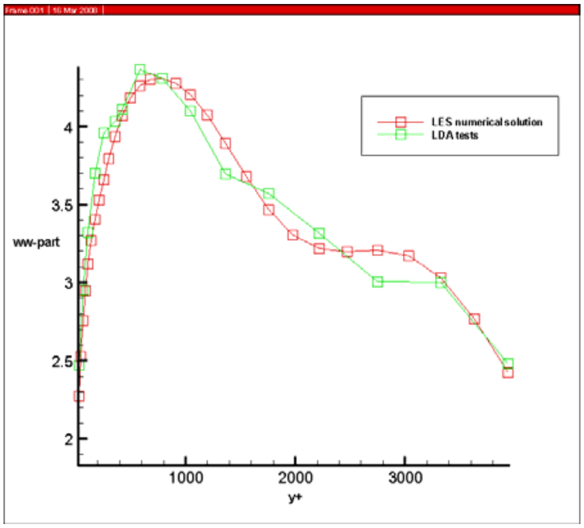

Figure 5: Root-mean-square velocity fluctuations of Al_2O_3 particles in turbulent channel flow, spanwise $w'w'$

VII. CONCLUSION

To have a faster calculation, the viscous fluxes are only calculated on the finest grid. This has another advantage that is in agreement with the basic assumption of LES which needs rather fine mesh. The effect of the residual smoothing was not clear. For the channel flow it did not bring any improvement, but for the cavity flow, helped to reduce the number of inner iterations (Lessani [14]).
Large-eddy simulation of particle-laden channel flow was carried out at $Re\tau$ =180. An incompressible finite volume solver, based on a cell-averaged approach, was used for the carrier flow calculations. The methodology of calculating the space derivatives with a finite volume compact scheme was briefly described. The velocity and location of the particles were calculated using a second order Adams-Bashforth formula. The particles were considered. The agreement between the present calculations and the existing LES data in the literature is satisfactory. The anisotropy of particle fluctuations was compared. It was demonstrated that the anisotropy of the particle fluctuations increases with the Stokes number.

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