The Ovalisation of Thin-walled Circular Tubes Subjected to Bending

C. Thinvongpituk, S. Poonaya, S. Choksawadee and M. Lee

***Abstract*— Circular tubes have been widely used as structural members in many engineering applications. Therefore, its collapse behavior has been studied for many decades, focusing on its energy absorption characteristics and collapse mechanism. In order to predict the collapse behavior of tubes, one could rely on the use of finite element codes or experiments. These tools provide results with high accuracy but costly and require extensive running time. Therefore, an approximated model of tubes collapse mechanism is an alternative especially for the early step of design. This paper is also aimed to develop a closed-form solution to predict the response of circular tube subjected to pure bending, focusing on the ovalisation regime. New ovalisation model was developed to include the effect of curvature into account. In order to compare, the experiment was conducted with a number of tubes having various D/t ratios. In addition, the available predictions from other investigators were also presented and compared. Good agreement was found between the theoretical predictions and experimental results. In addition, the present model provides more accurate result compared to some available theories.**

***Index Terms*—Bending, Circular tube, Plasticity, Ovalisation**

I. Introduction

Many researchers have been investigating the collapse mechanism and energy absorption capacity of many structures, majority focusing on thin-walled structures such as shell, tubes, stiffeners and stiffened sandwich panels. These structures have been identified as a very efficient impact energy absorbing system and called "energy absorber". The study of deformation in energy absorber accounts for various parameters such as; geometrical shape, mode of collapse, strain hardening and strain rate effect. In general, there are several approaches to determine the energy absorption of structural members; by using finite element analysis, experiments and theoretical analysis. Although finite element analysis and experimental approaches can provide accurate results, it is costly and requires extensive running time. Therefore, the theoretical analysis is an alternative for the early step of design.

Theoretical analysis of the collapse can be made by using hinge line method. When thin-walled members are crushed by any load, the collapse strength is reached. Then, plastic deformations are occurred over some folding lines and are called "hinge lines". When hinge lines are completed around the structure, global or local collapse will progress. Some examples of studies on collapse mechanism are as followed;

D. Kecman [1] studied the deep bending collapse of thin-walled rectangular columns and proposed a simple failure mechanism consisting of stationary and rolling plastic hinge line. The analytical solution was achieved using limit analysis techniques. L.C. Zhang and T.X. Yu., [2] studied the ovalisation of a tube with an arbitrary cross section and one symmetric plane to obtain a full moment-curvature response. Their analysis indicated that the flattening of tube increases nonlinearly as the longitudinal curvature increases. However, this phenomenon is to limited at some maximum values. T. Wierzbicki and S.U. Bhat., [3] derived a closed-form solution to predict the pressure necessary to initiate and propagate a moving hinge on the tube. The calculations were performed using a rigid-plastic material and a simple moving hinge model was assumed to occur along hinge line. The deformation of a ring was modeled into a "dumbbell" shape. The analytical results agreed well with the experiments. T. Wierzbicki and M.S. Suh., [4] conducted a theoretical analysis of the large plastic deformations of tubes subjected to combined load in the form of lateral indentation, bending moment and axial force. The model is effectively decoupling the 2-D problem into a set of 1- D problems. The theoretical results gave good correlation with existing experimental data. S.J. Cimpoeru and N.W. Murray., [5] presented empirical equations of the moment-rotation relation of a square thin-walled tube subject to pure bending where the width-to-thickness ratio less than 26. Results from the empirical model were compared with the analytical model of Kecman [1]. T. Wierzbicki et. al., [6] studied the collapse mechanism of thin-walled prismatic columns subjected to bending by using the concept of basic folding mode. They developed the collapse mechanism by adding the toroidal and rolling deformation in the compressive model. Close-form solutions were derived for the moment-rotation characteristic of square column in the post failure range. The stress profiles in the most general case of a floating neutral axis were also shown. The simplified analytical solution was shown to predict the moment-rotation relationship with an absolute error not greater than 7%. T. Wiezbriki and Sinmao.,[7] studied the simplified model of circular tube in pure bending, which was valid for large and very large sectional distortion. Good agreement with numerical solution (ABAQUS) was obtained. T.H.Kim and S.R.Reid., [8] modified the mechanism model of Wierzbicki el al. [6] ,and suggested that the toroidal deformation and conical rolling should be defined differently from the case of axial compression to satisfy the bending kinematics condition. Good agreement

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was found between the model and the experiment. M. Elchalakani el al., [9] predicted the response of a circular steel tube under pure bending. They included the effect of ovalisation along the length of the tube into the model. Work dissipated through the toroidal and the rolling hinges was ignored. The hinge mechanism was assumed straight and inextensible. Good agreement between analytical result and experiment was achieved. In another report, M. Elchalakani el al., [10] presented a closed-form solution of the post-bucking collapse of the slender circular hollow section with D/t >85 subjected to pure bending. Their theoretical analysis closely matched with the experimental results. The main objective of this paper was to develop a close-form solution for the ovalisation of thin-walled circular tube subjected to bending using a rigid plastic mechanism analysis. The model was derived to determine the ultimate moment. The experiment was also conducted with a number of circular tubes. The theoretical result was compared with experimental and with some available predictions. Good agreement between them was achieved.

II. Theoretical predictions

In general, the collapse mechanism of tube can be divided into three phases which are elastic behavior, ovalisation plateau, and structural collapse. Each phase behaves in different deformation modes. Many investigators have been attempting to develop the collapse models of them by focusing on their moment-rotation relationship individually. In the ovalisation phase, the circular cross-section of tube subjected to bending is starting to deform in oval shape. In general the bending moment in this phase is assumed constant and ultimate. S. Ueda [11] proposed an interesting report which performed the analytical method of moment-curvature relationship by considering the strains developed at the surface of tube under a constant-moment. In the ovalisation regime, he assumed that an initially circular cross section is deformed to an elliptical cross section. The ultimate bending moment was obtained by integrating stress over the cross section. His ultimate moment is expressed as:

$$M_u = \sigma_y Z_p + \left(\sigma_u - \sigma_y\right) Z_e \qquad (1)$$

where σ_y is the yield stress, σ_u is the ultimate tensile stress, $Z_p = \frac{4}{3}\left(R_o^3 - R_i^3\right)$ is the plastic bending section modulus , $Z_e = \left(\pi / 4R_o\right)\left(R_o^4 - R_i^4\right)$ is the elastic bending section modulus, R_o is the outer radius of tube, and R_i is the inner radius of tube.

Recently, M. Elchalakani et al. [13] also determined the ultimate moment of circular hollow section by approximating the ovalised section as an elliptical shape. Their experimental observation suggested that the ovalisation starts when major axis reaches 1.10d and the minor axis reaches 0.9d . The solution for their ultimate moment is shown in (2).

$$M_u = S_{ovalised}\sigma_y = \frac{4}{3}\left(R_v^2 R_h - R_{vi}^2 R_{hi}\right)\sigma_y \qquad (2)$$

where $S_{ovalised}$ is the plastic section modulus of an ovalised tube, σ_y is the measured yield stress of an ovalised tube. $R_h = \frac{D_h}{2} = 0.55D_o$ and $R_v = \frac{D_v}{2} = 0.45D_o$ are the external horizontal and vertical radii of an ovalised tube, respectively. The internal horizontal and vertical radii are $R_{hi} = \left(R_h - t\right)$ and $R_{vi} = \left(R_v - t\right)$, respectively, and t is the thickness of tube.

The present paper aims to propose a new model for sectional ovalisation by developing the model of those two literatures [11, 13]. In order to simplify the problem, the following assumptions are made;

1. The material is ductile, rigid-perfectly plastic, isentropic, homogeneous and material compatibility condition is maintained.
2. The tube circumference is inextensible.
3. Shear deformation and twist of the deformed tube are neglected.
4. The collapse mechanism formed in the ovalisation of tube is shown in Fig. 1.
5. The initial mean radius of tube (R) is the tube's cross section radius at the beginning of the plastic hinge formation.
6. The tube does not elongate or contract in axial direction.
7. The radius R_1 (see Fig. 1) is constant and equal to the outside radius of tube (R) during large deformation of the cross-section.

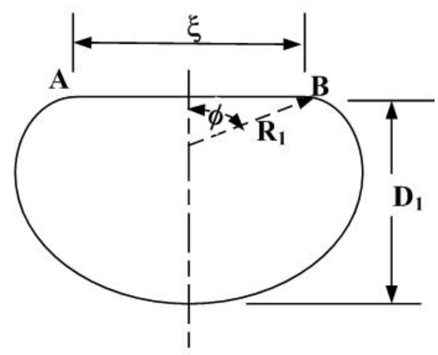

Fig.1 The model of ovalisation of tube due to bending

New ovalisation model proposed here is shown in Fig. 1. The curvature of radius R_1 which is formed at both ends of flattening sides were taken into account. Although the behavior of material exhibits as slight hardening, the bending moment is assumed constant during the increment of bending rotation. The rolling hinge of the circumferential cross-section is ignored. Then, the ultimate moment of an ovalised tube and the corresponding angle of rotation are determined by integrating the stress over the cross section.

From Fig.1, the geometry of circumferential cross-section of tube is assumed inextensible and can be expressed as (3),

$$\frac{\xi}{2} + R_1\left(\pi - \phi\right) = \pi R \qquad (3)$$

where $\frac{\xi}{2} = R_1 \sin(\phi)$ and R is the initial radius of tube.

The bending moment in a tube can be obtained by integrating the stress over the circumferential cross section which is expressed in (4),

$$M \quad = \quad \int_A \sigma \, zdA \tag{4}$$

where $dA = tds$ is the cross- sectional area of an element of the tube, t is the thickness of tube, z is the distance from the neutral axis of a sectional ovalisation to the circumferential area and ds is the length of the circumferential cross section.

By integrating (4), the expression for moment can be obtained as shown in (5).

$$M = 2\sigma_0 t.\int_0^{\pi R} zds$$

$$M = \sigma_0 t R_1^2 \left(\sin(2\phi) + 2\sin(\phi)\right) \tag{5}$$

where $R_1 = \frac{\pi R}{\pi - \phi + \sin(\phi)}$, for large deformation the R_1 is equal to the outside radius of tube, R.

The ultimate bending moment can be determined by minimizing the bending moment in (5) with respect to the deformation angle ϕ. The ultimate bending moment is finally obtained as expressed in (6)

$$M_u = 3\sigma_0 t R^2 \tag{6}$$

where σ_0 is the ultimate stress of material, t is the thickness of tube, and R is the outside radius of tube.

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III. Experiments

A. *Specimen preparation*

In order to verify the proposed model, the experiment is conducted with 18 tubes (UB1 to UB6) of mild steel with different diameter to thickness ratios. The nominal diameter to thickness ratios ranges from 21.16 to 42.57 and the length of each specimen is 1,500 mm. The material properties are determined by using the tensile coupons tested according to the British Standard BSEN 10 002-1:1990 [14]. Results from tensile tests are shown in Table 1.

TABLE I
DIMENSIONS AND MATERIAL PROPERTIES OF SPECIMENS

Specimen No.	Diameter (mm)	Thickness (mm)	D/t	Modulus of Elasticity E (GPa)	Yield Stress σ_y (MPa)	Ultimate Stress σ_u (MPa)	Yield Angle θy (deg)	Yield Moment M_y (kN)
UB1	59.25	2.80	21.16	128	330	383	2.99	2.21
UB2	59.00	2.30	25.65	160	270	314	1.96	1.51
UB3	46.85	1.80	26.03	173	320	355	3.17	1.01
UB4	59.35	1.80	32.97	178	354	370	2.3	1.61
UB5	58.55	1.60	36.59	128	257	295	2.44	1.02
UB6	74.50	1.75	42.57	133	306	380	2.41	2.45

B. *Test setup and procedure*

The experimental setup is designed to obtain a pure bending moment over middle span of the specimen. The influence of shear and axial forces should be avoided or minimized as much as possible. To meet this requirement, S.J. Cimpoeru et al [5] introduced a machine that is able to apply a pure bending moment without imposing shear or axial forces. A machine based on that concept has been built at Ubonratchathani University to apply a pure bending test on those 18 specimens. The diagram of this machine is shown in Fig.2.

As can be seen from the diagram in Fig.2, the machine consists of two load application wheels on its left and right ends. These two wheels are connected to the tensile testing machine via two connecting rods. The tested tube is placed on the load application wheels and locked with two bolts on each side. As the tensile machine pulls the connecting rods upward, the wheels start to rotate and apply pure bending moment on tested specimen. Fig. 3 shows an experimental set up and various views of deformed specimen. The experimental ovalisation shape is found similar to the proposed model shown in Fig.1.

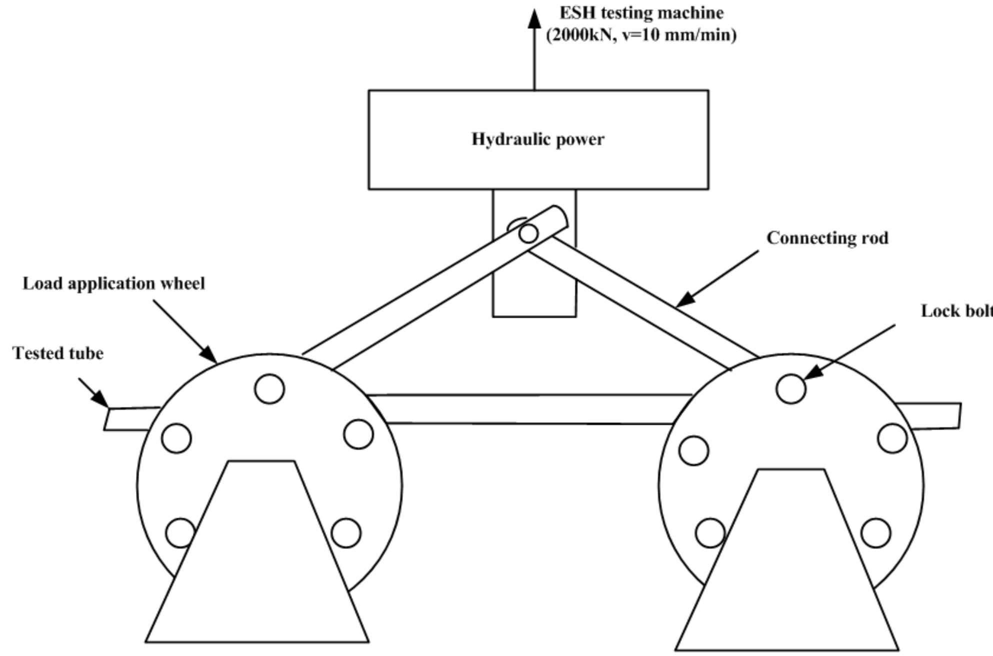


Fig. 2 The diagram of the pure bending machine used in this study

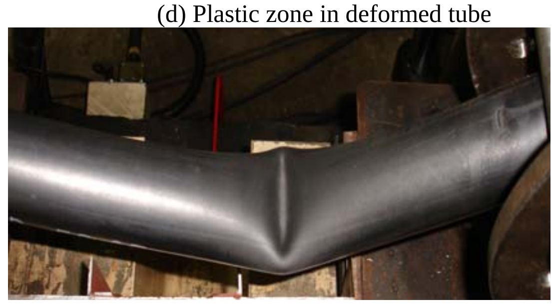
(a) The experimental setup

(b) Undeformed tube (UB1) in place

(c) Starting in to deform

(d) Plastic zone in deformed tube

(e) Final deform (side view)



(f) Final deform (Plain view)

Fig. 3 The experimental setup, undeformed and deformed specimen in various views

IV. RESULTS AND DISCUSSIONS

The ultimate moment analysed in this paper are compared with experimental results as well as with available models such as Ueda's [11] and Elchalakani's models [13]. Table 2 shows a summary of the ultimate bending moment predicted by those two models and newly derived model, compared with experimental results.

From Table 2, it can be observed that the present prediction (6), Elchalakani's and Ueda's formulae overestimate the ultimate bending moments by 1.67%, 8.7% and 11.8%, in average, respectively. The present study, which includes the curvature into account, seems to give more accurate results compared to experiments and other two predictions. However, it still overestimates the ultimate moment, especially for high D/t tubes. This can be explained that, for tubes with high D/t ratios, the plasticity does not spread linearly along the whole length as it is assumed in the analysis. In contrast, the plasticity tends to concentrate at the plastic hinge region and causes premature failure.

TABLE I I
COMPARISON OF ULTIMATE MOMENTS PREDICTED FROM SIMPLIFIED MODEL AND TEST RESULTS.

Specimen No.	D/t	Experimental Ultimate Moment M_{Exp}, kN.m	Predicted Ultimate Moment, (6)		Elchalakani's Ultimate Moment [13],		Ueda's Ultimate Moment [11],	
			M_u, kN.m	$\frac{M_u}{M_{Exp}}$	M_u, kN.m	$\frac{M_u}{M_{Exp}}$	M_u, kN.m	$\frac{M_u}{M_{Exp}}$
BC1	21.16	3.20	2.83	0.88	3.02	0.94	2.93	0.92
BC2	25.65	1.85	1.87	1.01	2.12	1.15	2.1	1.13
BC3	26.03	1.12	1.05	0.94	1.18	1.05	1.17	1.04
BC4	32.97	1.76	1.76	1.00	2.09	1.19	2.11	1.2
BC5	36.59	1.09	1.22	1.11	1.13	1.04	1.34	1.23
BC6	42.57	2.38	2.76	1.16	2.76	1.16	2.84	1.19
		average		1.0167		1.087		1.12

V. CONCLUSION

This paper provides a theoretical model to predict the ovalisation mechanism of thin-walled circular tube subjected to pure bending. The effect of curvature is taken into account for the ovalisation phase. This model predicts the ultimate moment with higher accuracy than other predictions, but seems to overestimate for high D/t tubes. The mechanism in other collapse phases of tube are also being investigated and the results will be presented in the near future.

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