Transient Free Convection Flow Between Two Long Vertical Parallel Plates with Constant Temperature and Mass Diffusion

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Abstract-Transient free convection flow of a viscous and incompressible fluid between two infinite vertical parallel plates in the presence of constant temperature and mass diffusion has been investigated analytically. The method of Laplace transform is used to solve the dimensionless governing partial differential equations. The velocity, temperature and concentration profiles have been presented for different parameters like Prandtl number, Schmidt number and for multiple buoyancy effects aiding and opposing. The values of the skin-friction and volume flux are tabulated. The transient solution approaches the steady state when the non-dimensional time becomes comparable with the actual Schmidt and Prandtl numbers.

Keywords-Transient free convection, Vertical parallel plates, Heat transfer, Mass transfer, Asymmetric heating.

I. INTRODUCTION

Free convection flows in vertical channels have been studied extensively because of its importance in many engineering applications. Ostrach [1] initiated the study of fully developed free convection between two vertical walls with constant temperature. The first exact solution for free convection in a vertical parallel plate channel with asymmetric heating for a fluid with constant properties was presented by Aung [2]. Ostrach [3], Bodoia and Osterle [4], Aung et al. [5], Miyatake and Fujii [6-8], Miyatake et al. [9], Lee and Yan [10], Higuera and Ryazantsev [11], Camp et al. [12], Pantokratoras [13] have presented their results for a steady free convection flow between vertical parallel plates under different conditions on the wall temperature. The combined effect of thermal and mass buoyancy forces on laminar free convection flows between vertical parallel plate channels has received less attention. This effect is found to be important in many engineering situations, such as in the design of heat exchangers, nuclear reactors, solar energy collectors, thermo protection systems and many chemical processes. Yan et al. [14] have studied the effect of latent heat transfer associated with the liquid films vaporization on the heat transfer in the natural convection flows driven by the combined buoyancy forces of thermal and mass diffusion. Nelson and Wood [15-17] have presented numerical analysis of developing laminar flow between vertical parallel plates for combined heat and mass transfer natural convection with uniform wall temperature/concentration and uniform heat/mass flux boundary conditions. They also have presented an analytical solution for the fully developed combined heat and mass transfer natural convection between vertical parallel plates with asymmetric boundary conditions. Lee [18] performed a combined numerical and theoretical investigation of laminar natural convection heat and mass transfer in open vertical parallel plates with unheated entry and unheated exit for various thermal and concentration boundary conditions. Desrayaud and Lauriat [19] have examined the heat and mass transfer analogy for condensation of humid air in a vertical parallel plate channel. These papers discuss the steady free convection flows by considering different physical situation of transport processes.

However, very few papers deal with unsteady flows in vertical parallel plate channel. Transient considerations may be important if a cooling arrangement is to be designed using parallel plates. Thus the knowledge of the transient and the steady-state components is significant to understand the exact nature of these situations. Singh et al. [20] have studied the transient free convection flow of a viscous incompressible fluid in a vertical parallel plate channel when the walls are heated asymmetrically. Narahari et al. [21] have studied the transient free convection flow between two vertical parallel plates with constant heat flux at one boundary and the other maintained at a constant temperature. Jha et al. [22] have studied the transient free convection flow in a vertical channel as a result of symmetric heating of the channel walls. Recently, Singh and Paul [23] have presented an analysis for the transient free convective flow of a viscous and incompressible fluid between two vertical walls as a result of asymmetric heating or cooling of the walls. But the transient free convection flow between two infinite vertical parallel plates with constant temperature and mass diffusion at one boundary has not been studied in the literature, hence the motivation. In Sect. 2, the mathematical analysis is presented and in Sect. 3, the conclusions are summarized.

II. MATHEMATICAL ANALYSIS

Here an unsteady flow of a viscous incompressible fluid between two vertical parallel plates with constant temperature and mass diffusion is considered. The x'-axis is taken along one of the plates in the vertically upward

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> direction and the y'-axis is taken normal to the plates. Initially, at time $t' \le 0$, the two plates and the fluid are assumed to be at the same temperature T'_d and concentration C'_d . At time t' > 0, the temperature and concentration of the plate at y' = 0 are raised to T'_w and C'_w respectively, causing the flow of free convection currents. Then the flow can be shown to be governed by the following equations under usual Boussinesq's approximations:

$$\frac{\partial u'}{\partial t'} = g\beta(T' - T'_d) + g\beta^*(C' - C'_d) + \nu \frac{\partial^2 u'}{\partial {y'}^2}$$
(1)

$$\rho C_{p} \frac{\partial T'}{\partial t'} = k \frac{\partial^{2} T'}{\partial {y'}^{2}}$$
(2)

$$\frac{\partial C'}{\partial t'} = D \frac{\partial^2 C'}{\partial {y'}^2}$$
(3)

The initial and boundary conditions are as follows:

$$t' \le 0: \quad u' = 0, \quad T' = T'_d, \quad C' = C'_d \quad \text{for} \quad 0 \le y' \le d ,$$

$$t' > 0: \quad u' = 0, \quad T' = T'_w, \quad C' = C'_w \quad \text{at} \quad y' = 0 ,$$

$$u' = 0, \quad T' = T'_d, \quad C' = C'_d \quad \text{at} \quad y' = d .$$
(4)

Here u' is the velocity of the fluid, g the acceleration due to gravity, β volumetric coefficient of thermal expansion, t' time, d the distance between two vertical plates, T' the temperature of the fluid, T'_d temperature of the plate at y' = d, β^* volumetric coefficient of concentration expansion, C' species concentration in the fluid, C'_d species concentration at the plate y' = d, v the kinematic viscosity, y' the coordinate axis normal to the plates, ρ the density, C_p the specific heat at constant pressure, k the thermal conductivity of the fluid, D the mass diffusion coefficient, T'_w temperature of the plate at y' = 0, C'_w species concentration at the plate y' = 0.

We now introduce the following non-dimensional quantities:

$$y = \frac{y'}{d}, \quad t = \frac{t'v}{d^2}, \quad u = \frac{u'v}{d^2g\beta(T'_w - T'_d)} = \frac{u'd}{v \ Gr},$$

$$Gr = \frac{g\beta(T'_w - T'_d)d^3}{v^2}, \quad \theta = \frac{T' - T'_d}{T'_w - T'_d}, \quad \Pr = \frac{\mu C_p}{k},$$

$$C = \frac{C' - C'_d}{C'_w - C'_d}, \quad Gm = \frac{g\beta^*(T'_w - T'_d)d^3}{v}, \quad Sc = \frac{v}{D},$$

$$N = \frac{Gm}{Gr}.$$
(5)

Where *u* the dimensionless velocity, *y* dimensionless coordinate axis normal to the plates, *t* dimensionless time, θ the dimensionless temperature, *C* the dimensionless concentration, *Gr* thermal Grashof number, *Gm* mass Grashof number, μ the coefficient of viscosity, Pr the Prandtl number, Sc the Schmidt number, and *N* is the buoyancy ratio parameter. Then in view of equations (5), equations (1) – (4) reduce to the following non-dimensional form of equations:

$$\frac{\partial u}{\partial t} = \theta + NC + \frac{\partial^2 u}{\partial y^2} \tag{6}$$

$$\Pr\frac{\partial\theta}{\partial t} = \frac{\partial^2\theta}{\partial y^2} \tag{7}$$

$$\operatorname{Sc}\frac{\partial C}{\partial t} = \frac{\partial^2 C}{\partial y^2}$$
(8)

The initial and boundary conditions are

$$t \le 0: \quad u = 0, \quad \theta = 0, \quad C = 0 \quad \text{for} \quad 0 \le y \le 1,$$

$$t > 0: \quad u = 0, \quad \theta = 1, \quad C = 1 \quad \text{at} \quad y = 0,$$

$$u = 0, \quad \theta = 0, \quad C = 0 \quad \text{at} \quad y = 1.$$
(9)

The solutions to Eqs. (6) - (8) satisfying the initial and boundary conditions (9) are derived by the usual Laplace-transform technique as follows:

Case I: Sc
$$\neq 1$$

$$u(y,t) = \frac{(\text{Sc}-1) + N(\text{Pr}-1)}{2(\text{Pr}-1)(\text{Sc}-1)} \sum_{n=0}^{\infty} \left[(a^{2} + 2t) \text{erfc}\left(\frac{a}{2\sqrt{t}}\right) - 2a\sqrt{(t/\pi)} e^{-a^{2}/4t} - (b^{2} + 2t) \text{erfc}\left(\frac{b}{2\sqrt{t}}\right) + 2b\sqrt{(t/\pi)} e^{-b^{2}/4t} \right]$$

$$+ \frac{1}{2(\text{Pr}-1)} \sum_{n=0}^{\infty} \left[(b^{2} \text{Pr}+2t) \text{erfc}\left(\frac{b\sqrt{\text{Pr}}}{2\sqrt{t}}\right) - 2b\sqrt{\text{Pr}(t/\pi)} e^{-b^{2} \text{Pr}/4t} - (a^{2} \text{Pr}+2t) \text{erfc}\left(\frac{a\sqrt{\text{Pr}}}{2\sqrt{t}}\right) + 2a\sqrt{\text{Pr}(t/\pi)} e^{-a^{2} \text{Pr}/4t} \right]$$

$$+ \frac{N}{2(\text{Sc}-1)} \sum_{n=0}^{\infty} \left[(b^{2} \text{Sc}+2t) \text{erfc}\left(\frac{b\sqrt{\text{Sc}}}{2\sqrt{t}}\right) - 2b\sqrt{\text{Sc}(t/\pi)} e^{-b^{2} \text{Sc}/4t} - (a^{2} \text{Sc}+2t) \text{erfc}\left(\frac{a\sqrt{\text{Sc}}}{2\sqrt{t}}\right) - 2b\sqrt{\text{Sc}(t/\pi)} e^{-b^{2} \text{Sc}/4t} - (a^{2} \text{Sc}+2t) \text{erfc}\left(\frac{a\sqrt{\text{Sc}}}{2\sqrt{t}}\right)$$

$$+ 2a\sqrt{\operatorname{Sc}(t/\pi)} e^{-a^2\operatorname{Sc}/4t}$$
(10)

Where a = 2n + y, b = 2n + 2 - y.

$$\theta(y,t) = \sum_{n=0}^{\infty} \left[\operatorname{erfc}\left(\frac{a\sqrt{\Pr}}{2\sqrt{t}}\right) - \operatorname{erfc}\left(\frac{b\sqrt{\Pr}}{2\sqrt{t}}\right) \right]$$
(11)

$$C(y,t) = \sum_{n=0}^{\infty} \left[\operatorname{erfc}\left(\frac{a\sqrt{\operatorname{Sc}}}{2\sqrt{t}}\right) - \operatorname{erfc}\left(\frac{b\sqrt{\operatorname{Sc}}}{2\sqrt{t}}\right) \right]$$
(12)

Case II: Sc = 1

$$u(y,t) = \frac{1}{2(\Pr-1)} \sum_{n=0}^{\infty} \left[(a^{2} + 2t) \operatorname{erfc} \left(\frac{a}{2\sqrt{t}} \right) - 2a\sqrt{(t/\pi)} e^{-a^{2}/4t} - (b^{2} + 2t) \operatorname{erfc} \left(\frac{b}{2\sqrt{t}} \right) + 2b\sqrt{(t/\pi)} e^{-b^{2}/4t} - (a^{2} \operatorname{Pr} + 2t) \operatorname{erfc} \left(\frac{a\sqrt{\Pr}}{2\sqrt{t}} \right) + 2a\sqrt{\operatorname{Pr}(t/\pi)} e^{-a^{2}\operatorname{Pr}/4t} + (b^{2} \operatorname{Pr} + 2t) \operatorname{erfc} \left(\frac{b\sqrt{\Pr}}{2\sqrt{t}} \right) - 2b\sqrt{\operatorname{Pr}(t/\pi)} e^{-b^{2}\operatorname{Pr}/4t} \right] - \frac{N}{2} \sum_{n=0}^{\infty} \left[ay \operatorname{erfc} \left(\frac{a}{2\sqrt{t}} \right) - 2y\sqrt{(t/\pi)} e^{-a^{2}/4t} - b^{2} \operatorname{erfc} \left(\frac{b}{2\sqrt{t}} \right) + 2b\sqrt{(t/\pi)} e^{-b^{2}/4t} + 2c(n+1) \operatorname{erfc} \left(\frac{c}{2\sqrt{t}} \right) - 4(n+1)\sqrt{(t/\pi)} e^{-c^{2}/4t} \right]$$
(13)

Where c = 2n + 2 + y.

$$C(y,t) = \sum_{n=0}^{\infty} \left[\operatorname{erfc}\left(\frac{a}{2\sqrt{t}}\right) - \operatorname{erfc}\left(\frac{b}{2\sqrt{t}}\right) \right]$$
(14)

The series in Eqs. (10) - (14) can be shown to be absolutely convergent because of the presence of standard mathematical functions. The numerical values of the velocity, temperature, concentration, skin-friction and volume flow rate are

computed for different parameters like Prandtl number, Schmidt number, buoyancy ratio and time. The buoyancy ratio parameter N represents the ratio between mass and thermal buoyancy forces. When N = 0, there is no mass transfer and the buoyancy force is due to the thermal diffusion only. N > 0 means that mass buoyancy force acts in the same direction of thermal buoyancy force, while N < 0 means that mass buoyancy force acts in the opposite direction. Since the two most commonly occurring fluids are atmospheric air and water, the results are limited to Prandtl numbers of 0.71 (air) and 7.0 (water). The effect of buoyancy ratio N for both aiding and opposing flows are shown in Fig. 1. It is observed that the velocity increases in the presence of aiding flows whereas it decreases in the presence of opposing flows. It is also observed that the velocity increases with increasing the time.

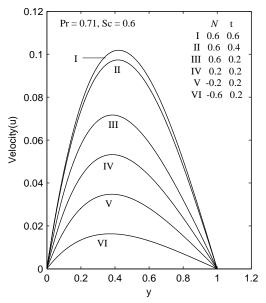


Fig. 1. Velocity profiles for different N and t

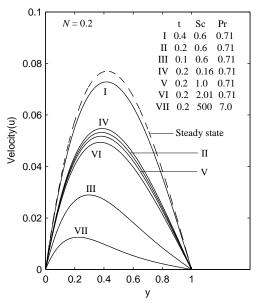


Fig. 2. Velocity profiles for different Sc and t

To derive the solutions for steady state, we put $\partial()/\partial t = 0$ in Eqs. (6) – (8) which then reduces to

$$0 = \theta + NC + \frac{\partial^2 u}{\partial y^2}$$
(15)

$$0 = \frac{\partial^2 \theta}{\partial y^2} \tag{16}$$

$$0 = \frac{\partial^2 C}{\partial y^2} \tag{17}$$

These are solved using the boundary conditions (9) and these steady-state velocity, temperature and concentration profiles are computed and shown in Figs. 2 to 4 as dotted lines. When

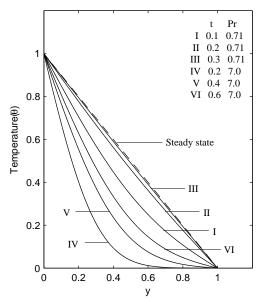


Fig. 3. Temperature profiles

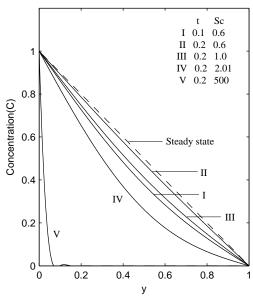


Fig. 4. Concentration profiles

computing steady-state solutions for velocity, temperature and concentration from Eqs. (6) – (8), it is observed that for t = 1.0,500 the values of u for fixed N, θ and C for Sc = 0.6,500; Pr = 0.71,7.0 respectively coincides with those derived from the solution of Eqs. (15) – (17). Hence the transient solution approaches the steady-state when the non-dimensional time becomes comparable with the actual Schmidt and Prandtl numbers.

In Fig. 2, the velocity profiles are shown for different values of Schmidt number and time. It is observed that an increase in Schmidt number leads to a fall in the velocity. Also, the velocity increases with increasing time.

The temperature profiles are shown in Fig. 3 for different values of Prandtl number and time. From this figure it is evident that the temperature increases with increasing time but it falls owing to an increase in the Prandtl number.

The numerical values of the concentration profiles are computed from Eqs. (12) and (14) and these values are depicted in Fig. 4 for different values of Schmidt number and time. The effect of Schmidt number is very important in concentration field. It is observed that the concentration increases with increasing time but decreases with increasing the value of the Schmidt number.

We now study the skin-friction, which is given in non-dimensional form by

Case I: Sc $\neq 1$

$$\begin{aligned} \tau_0 &= \frac{\tau'_0}{dg\beta(T'_w - T'_d)} = \frac{du}{dy} \bigg|_{y=0} \\ &= \frac{2((\mathrm{Sc} - 1) + N(\mathrm{Pr} - 1))}{(\mathrm{Pr} - 1)(\mathrm{Sc} - 1)} \sum_{n=0}^{\infty} \left[n \operatorname{erfc}\left(\frac{n}{\sqrt{t}}\right) \\ &\quad -\sqrt{(t/\pi)} \ e^{-n^2/t} + (n+1) \operatorname{erfc}\left(\frac{n+1}{\sqrt{t}}\right) \\ &\quad -\sqrt{(t/\pi)} \ e^{-(n+1)^2/t} \right] \\ &\quad -\frac{2}{(\mathrm{Pr} - 1)} \sum_{n=0}^{\infty} \left[n \operatorname{Pr} \operatorname{erfc}\left(\frac{n\sqrt{\mathrm{Pr}}}{\sqrt{t}}\right) - \sqrt{\mathrm{Pr}(t/\pi)} \ e^{-n^2 \mathrm{Pr}/t} \\ &\quad + (n+1) \operatorname{Pr} \operatorname{erfc}\left(\frac{(n+1)\sqrt{\mathrm{Pr}}}{\sqrt{t}}\right) \\ &\quad -\sqrt{\mathrm{Pr}(t/\pi)} \ e^{-(n+1)^2 \mathrm{Pr}/t} \right] \\ &\quad -\frac{2N}{(\mathrm{Sc} - 1)} \sum_{n=0}^{\infty} \left[n \operatorname{Sc} \operatorname{erfc}\left(\frac{n\sqrt{\mathrm{Sc}}}{\sqrt{t}}\right) - \sqrt{\mathrm{Sc}(t/\pi)} \ e^{-n^2 \mathrm{Sc}/t} \end{aligned}$$

+
$$(n + 1)$$
 Sc erfc $\left(\frac{(n + 1)\sqrt{Sc}}{\sqrt{t}}\right)$
- $\sqrt{Sc(t/\pi)} e^{-(n+1)^2 Sc/t}$ (18)

and

$$\begin{aligned} \tau_{1} &= -\frac{du}{dy} \Big|_{y=1} \\ &= \frac{-2((\text{Sc}-1) + N(\text{Pr}-1))}{(\text{Pr}-1)(\text{Sc}-1)} \sum_{n=0}^{\infty} \left[(2n+1) \operatorname{erfc} \left(\frac{2n+1}{2\sqrt{t}} \right) \right. \\ &\left. -2\sqrt{(t/\pi)} \, e^{-(2n+1)^{2}/4t} \right] \\ &\left. + \frac{2}{(\text{Pr}-1)} \sum_{n=0}^{\infty} \left[(2n+1) \operatorname{Pr} \operatorname{erfc} \left(\frac{(2n+1)\sqrt{\text{Pr}}}{2\sqrt{t}} \right) \right. \\ &\left. -2\sqrt{\text{Pr}(t/\pi)} \, e^{-(2n+1)^{2} \operatorname{Pr}/4t} \right] \\ &\left. + \frac{2N}{(\text{Sc}-1)} \sum_{n=0}^{\infty} \left[(2n+1) \operatorname{Sc} \operatorname{erfc} \left(\frac{(2n+1)\sqrt{\text{Sc}}}{2\sqrt{t}} \right) \right] \\ &\left. -2\sqrt{\text{Sc}(t/\pi)} \, e^{-(2n+1)^{2} \operatorname{Sc}/4t} \right] \end{aligned}$$
(19)

Case II: Sc = 1

$$\tau_{0} = \frac{2 - N(\Pr-1)}{(\Pr-1)} \sum_{n=0}^{\infty} \left[n \operatorname{erfc}\left(\frac{n}{\sqrt{t}}\right) - \sqrt{(t/\pi)}e^{-n^{2}/t} + (n+1)\operatorname{erfc}\left(\frac{n+1}{\sqrt{t}}\right) - \sqrt{(t/\pi)}e^{-(n+1)^{2}/t} \right] - \frac{2}{(\Pr-1)} \left[n \operatorname{Pr}\operatorname{erfc}\left(\frac{n\sqrt{\Pr}}{\sqrt{t}}\right) - \sqrt{\Pr(t/\pi)}e^{-n^{2}\operatorname{Pr}/t} + (n+1)\operatorname{Pr}\operatorname{erfc}\left(\frac{(n+1)\sqrt{\Pr}}{\sqrt{t}}\right) - \sqrt{\Pr(t/\pi)}e^{-(n+1)^{2}\operatorname{Pr}/t} \right] - 2N\sum_{n=0}^{\infty} \left[(n+1)\operatorname{erfc}\left(\frac{n+1}{\sqrt{t}}\right) \right]$$
(20)

$$\tau_{1} = \frac{N(\Pr-1) - 2}{(\Pr-1)} \sum_{n=0}^{\infty} \left[(2n+1) \operatorname{erfc}\left(\frac{2n+1}{2\sqrt{t}}\right) - 2\sqrt{(t/\pi)}e^{-(2n+1)^{2}/4t} \right] + \frac{2}{(\Pr-1)} \sum_{n=0}^{\infty} \left[(2n+1) \operatorname{Pr}\operatorname{erfc}\left(\frac{(2n+1)\sqrt{\Pr}}{2\sqrt{t}}\right) - 2\sqrt{\Pr(t/\pi)} e^{-(2n+1)^{2}\operatorname{Pr}/4t} \right] + N \sum_{n=0}^{\infty} (n+1) \left[\operatorname{erfc}\left(\frac{2n+1}{2\sqrt{t}}\right) + \operatorname{erfc}\left(\frac{2n+3}{2\sqrt{t}}\right) \right]$$
(21)

The numerical values of τ_0 and τ_1 are evaluated and these are listed in Table I. From this table, it is observed that the skin-friction increases with increasing time but decreases with increasing the value of the Schmidt and Prandtl numbers. Physically, this is possible because fluids with high Schmidt and Prandtl numbers move slowly and hence there is less friction at the plates. Moreover, the skin-friction increases in the presence of aiding flows and decreases in the presence of opposing flows. It is also computed the steady-state value of the skin-friction by calculating τ_0 and τ_1 from Eqs. (18) and (19) for large values of time t for a fixed buoyancy ratio, for example N = 0.2, and it is seen that $\tau_0 = 0.400000$ and $\tau_1 = 0.200000$ which agree well with those computed from their steady-state solution obtained from Eq. (15).

Table I. Numerical values of τ_0 , τ_1 and Q

t	Pr	Sc	Ν	τ_0	τ_1	Q
0.2	0.71	0.16	0.2	0.32694	0.12707	0.035208
0.2	0.71	0.6	0.2	0.32183	0.12197	0.034173
0.2	0.71	2.01	0.2	0.30862	0.10953	0.031574
0.2	0.71	0.6	-0.2	0.21213	0.07890	0.022293
0.2	0.71	0.6	0.4	0.37667	0.14350	0.040113
0.2	0.71	0.6	-0.4	0.15728	0.05737	0.016352
0.4	0.71	0.6	0.2	0.38655	0.18656	0.047275
0.2	7.0	500	0.2	0.14265	0.01027	0.006459
0.2	7.0	500	0.4	0.14697	0.01029	0.006483
0.4	7.0	500	0.2	0.19922	0.03894	0.014598
0.2	7.0	500	-0.2	0.13401	0.01022	0.006410
0.2	7.0	500	-0.4	0.12969	0.01019	0.006386
Steady	state		0.2	0.40000	0.20000	0.050000

Another interesting phenomenon in this study is to understand the effects of t, Sc, Pr and N on the volume flow rate which is given by

and

$$Q = \frac{Q'v}{d^{3}g\beta(T'_{w} - T'_{d})} = \int_{0}^{1} u \, dy$$
 (22)

Where Q is the non-dimensional volume flux. We substitute for u from (10) in Eq. (22), and compute the integral numerically using Simpson's rule. The numerical values of Q are listed in Table I. It is observed from this table that the volume flux increases with increasing time and it decreases with increasing the value of the Schmidt and Prandtl numbers. It is also observed that the volume flux increases in the presence of aiding flows and decreases in the presence of opposing flows.

III. CONCLUSIONS

An exact solution of the transient free convection flow between two long vertical parallel plates with constant temperature and mass diffusion at one boundary is presented. The dimensionless governing coupled linear partial differential equations are solved by the usual Laplace-transform technique. The effect of different parameters like buoyancy ratio, Schmidt number, Prandtl number and time are studied. Conclusions of the study are as follows:

- 1. The velocity of the fluid increases in the presence of aiding flows (N > 0) and decreases with opposing flows (N < 0).
- 2. The velocity increases with increasing time and it decreases with increasing the value of the Schmidt number.
- 3. The temperature increases with increasing time but falls owing to an increase in the Prandtl number.
- 4. The concentration increases with increasing time but decreases with increasing the value of the Schmidt number.
- The skin-friction increases with increasing time but decreases with increasing the value of the Schmidt and Prandtl numbers. Also, the skin-friction increases in the presence of aiding flows and decreases with opposing flows.
- 6. The volume flux increases with increasing time and it decreases with increasing the value of the Schmidt and Prandtl numbers. Also, the volume flux increases in the presence of aiding flows and decreases with opposing flows.

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