

Spiking Neurons: Is coincidence-factor enough for comparing responses with fluctuating membrane voltage?

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Abstract— Similarity between two spike trains is generally estimated using a ‘coincidence factor’. This factor relies on counting coincidences of firing-times for spikes in a given time window. However, in cases where there are significant fluctuations in membrane voltages, this uni-dimensional view is not sufficient. Results in this paper show that a two-dimensional approach taking both firing-time and the magnitude of spikes is necessary to determine similarity between spike trains. It is observed that the difference between the lower-bound limit of faithful behaviour and the reference inter-spike interval (ISI) reduces with the increase in the ISI of the input spike train. This indicates that spike trains generated by two highly-varying currents have a high coincidence factor thus indicating higher similarity – a limitation imposed due to a one-dimensional comparison approach. These results are analysed based on the responses of a Hodgkin-Huxley neuron, where the synaptic input induces fluctuations in the output membrane voltage. The requirement for a two-dimensional analysis is further supported by a clustering algorithm which differentiates between two visually-distinct responses as opposed to coincidence-factor.

Index Terms—coincidence factor, comparison, fluctuations, synaptic current.

I. INTRODUCTION

The responses of a neuron to various types of stimuli have been studied extensively over the past years [1]-[9]. Stimulus-dependent behaviour of neurons has already been pursued to understand the spiking responses and it is thought that either the firing rate or firing time of individual spikes carries specific information of the neuronal response [3], [10]-[16]. The response of the neurons studied above has a constant magnitude whose variance is very low. In this paper, the neural responses fluctuate and a one-dimensional analysis based on firing times is shown to be insufficient for comparison.

A supra-threshold static current stimulus is sufficient to induce a spiking behaviour in the neuron. The magnitude of these action potentials is considered to be almost the same

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and their variance is thus ignored. Such responses have been studied and models to depict their spiking behaviour have been proposed and implemented [17]-[28]. On the other hand, a synaptic current is used to stimulate the same neuron [3]. This synaptic current comprises of a static and a pulse component and is of particular interest as it induces fluctuations in the membrane voltage. These responses can be compared by their firing times [18], [20], [23]-[26] using a measure of comparison known as coincidence-factor. Here, the generality of this approach is investigated for a Hodgkin-Huxley (H-H) neuron [29] for which a synaptic current induces membrane fluctuations.

In this paper, neural responses are generated by changing the Inter-Spike-Interval (ISI) of the stimulus. These responses are subsequently compared and a coincidence factor is obtained. This coincidence-factor does not have a consistent trend over the entire simulation time window. It is observed that the lower-bound limit for faithful behaviour of coincidence factor shifts towards the right with the increase in the ISI of the stimulus. This indicates the spike trains generated by two highly-varying currents have a high coincidence factor thus indicating higher similarity. Conversely, if the responses have a very high similarity, then the input currents should be very similar. From the reverse-engineering view these two currents would be considered as same, which is clearly not the case as shown in the later sections. This is shown to be a drawback of a one-dimensional consideration of the coincidence-factor approach. Elsewhere, [30], [31] also would not be able to explain this phenomenon as their approach too focuses on firing time alone. Thus in order to differentiate spike trains with fluctuating membrane voltages, a two dimensional analysis is necessary as the magnitude of the action potentials also play a vital role.

II. NEURONAL MODEL AND SYNAPSE

A. The neuron model

The computational model and stimulus for an H-H neuron is replicated from [3]. The differential equations of the model are the result of non-linear interactions between the membrane voltage V and the gating variables m , h and n . for Na^+ and K^+ .

$$C \frac{dv}{dt} = \left. \begin{aligned} & -g_{Na} m^3 h (V - V_{Na}) - g_K n^4 (V - V_K) \\ & - g_L (V - V_L) + I_i \end{aligned} \right\} \quad (1)$$

$$\left. \begin{aligned} \frac{dm}{dt} &= -(\alpha_m + \beta_m)m + \alpha_m \\ \frac{dh}{dt} &= -(\alpha_h + \beta_h)h + \alpha_h \\ \frac{dn}{dt} &= -(\alpha_n + \beta_n)n + \alpha_n \end{aligned} \right\} \quad (2)$$

$$\left. \begin{aligned} \alpha_m &= 0.1(V+40)/[1-e^{-(V+40)/10}] \\ \alpha_h &= 0.07e^{-(V+65)/20} \\ \alpha_n &= 0.01(V+55)/[1-e^{-(V+55)/10}] \\ \beta_m &= 4e^{-(V+65)/18} \\ \beta_h &= 1/[1+e^{-(V+35)/10}] \\ \beta_n &= 0.125e^{-(V+65)/80} \end{aligned} \right\} \quad (3)$$

The variable V is the resting potential where as V_{Na} , V_K and V_L are the reversal potentials of the Na^+ , K^+ channels and leakage. $V_{Na} = 50mV$, $V_K = -77mV$ and $V_L = -54.5mV$. The conductance for the channels are $g_{Na} = 120mS/cm^2$, $g_K = 36mS/cm^2$ and $g_L = 0.3mS/cm^2$. The capacitance of the membrane is $C = 1\mu F/cm^2$.

B. The synaptic current

An input spike train give by (4) is considered to generate the pulse component of the external current.

$$U_i(t) = V_a \sum_n \delta(t - t_f) \quad (4)$$

where, t_f is the firing time and is defined as

$$t_{f(n+1)} = t_{f(n)} + T \quad (5)$$

$$t_{f(1)} = 0 \quad (6)$$

T represents the ISI of the input spike train and can be varied to generate a different pulse current. The spike train is injected through a synapse to give the pulse current I_P .

$$I_P = g_{syn} \sum_n \alpha(t - t_f)(V_a - V_{syn}) \quad (7)$$

g_{syn} , V_{syn} are the conductance and reversal potential of the synapse. [32] define the α -function as

$$\alpha(t) = (t/\tau)e^{-t/\tau} \Theta(t), \quad (8)$$

where, τ is the time constant of the synapse and $\Theta(t)$ is the Heaviside step function. $V_a = 30mV$, $\tau_{syn} = 2ms$, $g_{syn} = 0.5mS/cm^2$ and $V_{syn} = -50mV$.

C. The total external current

The total external current applied to the neuron is a combination of static and pulse component

$$I_i = I_S + I_P + \varepsilon \quad (9)$$

where, I_S is the static and I_P is the pulse current, ε is the random Gaussian noise with zero mean and standard deviation $\sigma = 0.025$. [3] has ignored the noise in the external current and the current consists of only 2 terms. However, the

presence of noise is necessary in the simulation of a biological activity and hence added.

III. COMPARISON OF TWO SPIKE TRAINS

A. Responses of the neuron

The static component I_S of the external current is set at $25\mu A$. The H-H neuron is stimulated with a current $I_i = I_S + I_P + \varepsilon$ and its response is recorded. The fluctuations in the membrane are due to the specific nature of the input current. The amplitude of the action potential in Fig.1 is not constant and the standard deviation is $\sigma_{Amp} = 3.0978$. Hence, the amplitude of the response is not ignored. This is one major difference between [3], [30], [31] and our work. The synaptic time constant of 2ms defines the shape of the pulse current. We therefore choose a 2ms bound for coincidence detection. The simulation activity is divided into three sets of ISIs. Each set has a corresponding reference ISI (T_{ref}). The first set compares responses generated using stimulus ISI between 14-16ms while the second set compares responses of ISIs between 13-15ms and the third set caters to 15-17ms. Reference ISIs for the sets are 15ms, 14ms and 16ms respectively. Neural responses are recorded for various ISIs within a set and compared with the response of the reference ISI for that set. For set 1, the reference spike train is generated with $T=15ms$ and compared with responses generated with $T=14-16ms$. Coincidence factors are calculated to estimate the similarity between these responses.

B. Comparison of responses

The response of the neuron is specific to an input stimulus. In order to generate different input currents, we varied the ISI of the synaptic input from $T=14ms-16ms$ with $T=15ms$ as the reference ISI. Fig 2 & Fig 3 show that the response of the neuron differs with respect to both firing time and magnitude. The figures indicate that the variation in the input ISI causes the membrane voltage to fluctuate. They also show the difference in responses generated with $T=14ms$ & T_{ref} and $T=16ms$ & T_{ref}

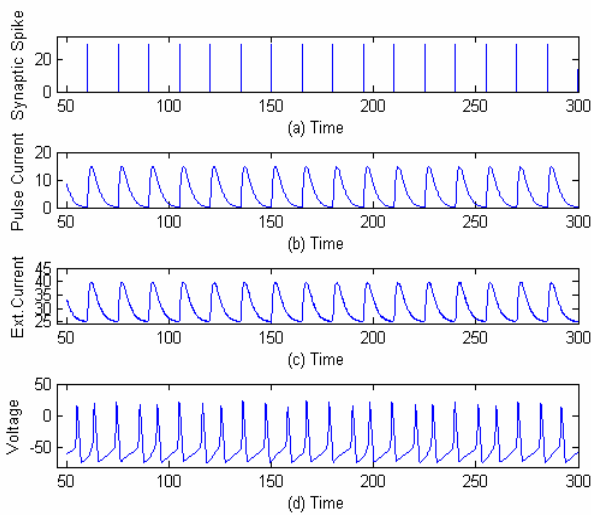


Fig 1: Response of the H-H neuron to I_i with $T=15\text{ms}$ causing fluctuations in membrane voltage. (a) The synaptic spike train input that induces a pulse current. (b) The pulse current generated. (c) The total external current $I_s + I_p + \varepsilon$ applied to the neuron. Note that there is a static offset (d) The neuronal response to the current.

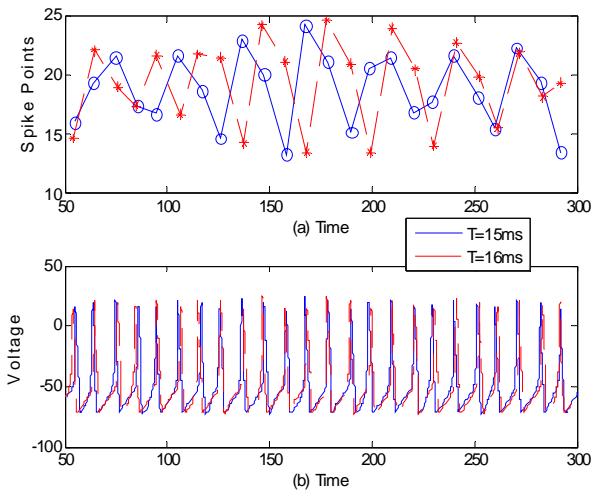


Fig 2: Comparison of responses. (a) The corresponding magnitude of spikes for the responses at $T=16\text{ms}$ and $T=15\text{ms}$. (b) The two spike trains not only differ in firing times but also in magnitudes.

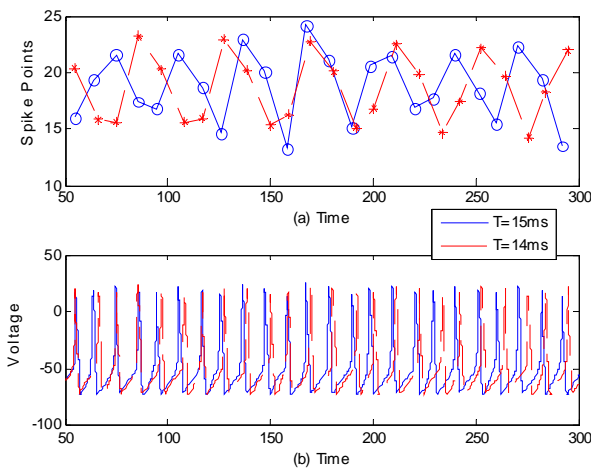


Fig 3: Comparison of responses. (a) The corresponding magnitude of spikes for the responses at $T=14\text{ms}$ and $T=15\text{ms}$. (b) The two spike trains not only differ in firing times but also in magnitudes.

C. Coincidence-factor

The coincidence-factor, as described by [18], [20] is 1 only

if the two spike trains are exactly the same and 0 if they are very dissimilar. Coincidence for an individual spike is established if its firing time is within 2ms of the firing time of the corresponding spike in the reference spike train (in this case $T=15\text{ms}$). The mathematical equations are discussed very briefly here as they are discussed in detail in [20]. The coincidence-factor is given by

$$\Gamma = \frac{N_{coinc} - \langle N_{coinc} \rangle}{1/2(N_1 + N_2) N} \quad (10)$$

where, N_1 is the number of spikes in the reference train, N_2 is the number of spikes in the train to be compared, N_{coinc} is the number of coincidences with a precision $\delta = 2\text{ms}$ between the spike trains. $\langle N_{coinc} \rangle = 2\nu\delta N_1$ is the expected number of coincidences generated by a homogeneous Poisson process with the same rate as the spike train to be compared. $N = 1 - 2\nu\delta$ is the normalising factor. For set 1, N_1 is the number of spikes in the reference spike train ($T_{ref}=15\text{ms}$) and N_2 is the number of spikes in the train to be compared ($T=14-16\text{ms}$). Fig.4 shows that the coincidence-factors for responses generated using $T=14-16\text{ms}$ do not follow a fixed pattern. The coincidence-factor (Γ) is expectedly 1 when spike train generated with $T=15\text{ms}$ is compared with the reference spike train ($T_{ref}=15\text{ms}$). However, the coincidence factor for spike trains generated at $T=16\text{ms}$ and T_{ref} is 1. This indicates that the two highly varying currents have an exactly similar response or conversely as the responses are same; the two input currents are similar, which is clearly not the case. The coincidence factor for the spike trains generated at $T=14\text{ms}$ and T_{ref} is 0.1207 indicating very low similarity. From a mathematical and signal transmission standpoint, the coincidence-factor should decrease as the input current increasingly varies from T_{ref} . However, this can only be observed between $T=14.65\text{ms}-15.25\text{ms}$ (30% of the 2ms time window). The coincidence-factor Γ increases from $T=14\text{ms}-14.5\text{ms}$ but then drops till $T=14.65\text{ms}$. Γ steadily increases to 1 when $T=15\text{ms}$ and drops for 0.25ms. There is an upward rise from $T=15.25\text{ms}-15.5\text{ms}$, a sharp drop from $T=15.5\text{ms}-15.75\text{ms}$ followed by a steep increase to $\Gamma = 1$ at $T=16\text{ms}$. Traversing from the reference the expected trajectory of the coincidence-factor breaks at $T=14.65\text{ms}$ and $T=15.25\text{ms}$.

These are therefore taken as limits for faithful behaviour of the coincidence-factor approach. However, for set 2 reference spike train is chosen as $T_{ref}=14\text{ms}$, limits of faithful behaviour change (Fig. 5). The coincidence factor steadily rises to unity, stays there for 0.5ms and drops gradually. Ideally, the coincidence-factor should be not 1 for $T=13.5\text{ms}$, 13.65ms and 13.75ms . While in set 3, Fig. 6, reference spike train chosen is at $T_{ref}=16\text{ms}$. The limits of faithful behaviour change with a change in the input current. There is a sharp rise in the coincidence factor from 15.75ms to 16ms where it reaches unity. From 16ms to 17ms the coincidence-factor executes a perfect curve as expected. From the above figures it is conclusive that the lower-bound of faithful behaviour increases with the increase in the input reference ISI. The difference between the reference ISI (T_{ref}) and the

lower-bound limit decreases with the increase in the reference ISI.

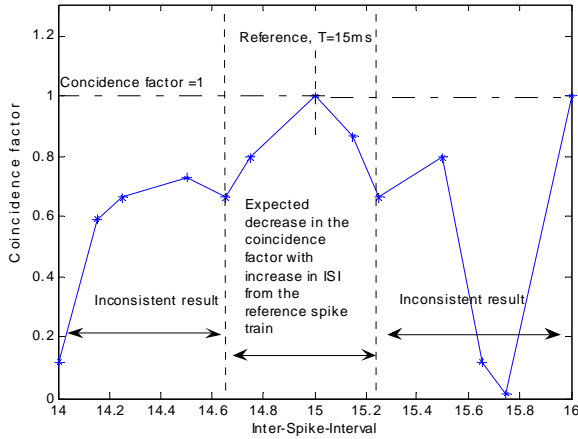


Fig 4: Coincidence-factor versus ISI. The coincidence-factor decreases expectedly between $T=15\text{ms}$ - 14.65ms and $T=15\text{ms}$ - 15.25ms . At other times the result is inconsistent and does not have a fixed pattern.

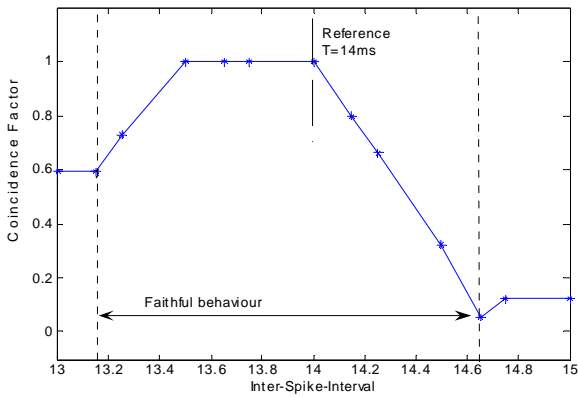


Fig 5: Coincidence-factor versus ISI. The coincidence-factor has a faithful behaviour between $T=13.15\text{ms}$ - 14.65ms .

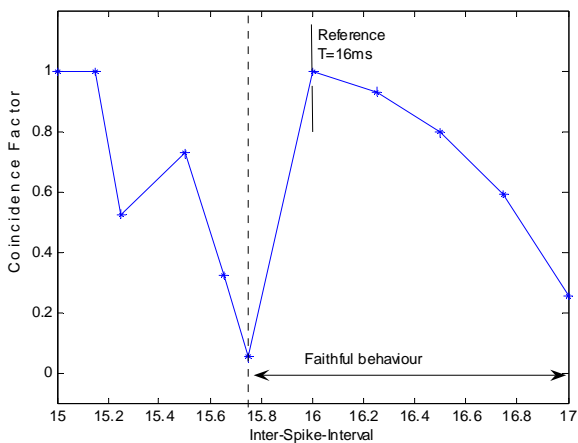


Fig 6: Coincidence-factor versus ISI. The coincidence-factor has a faithful behaviour between $T=15.75\text{ms}$ - 17ms . It executes a perfect curve after 16ms .

D. Two-dimensional analysis

The coincidence-factors over the 2ms time window show an inconsistent trend. A 1-dimensional approach of the coincidence-factor determination is thought to be the cause of this inconsistency. The coincidence-factor is highly accurate for spike trains with a constant amplitude response however; the coincidence-factor does not give a proper estimate of similarity between two spike trains with varying amplitudes. As a result, two visually distinct spike trains would still

generate a high coincidence-factor (Fig. 2 & Fig. 3). A 2-dimensional analysis of spike trains with fluctuating magnitudes can resolve this inconsistency. To support this, a simple binary clustering algorithm is used. It shows that the clustering solution for each response is unique to itself and thus eliminates ambiguity.

E. Binary clustering

The peak of each spike in the spike train is considered as an object. The object (Obj) is defined as point with its firing time and amplitude. The number of objects for each spike train is equal to the number of spikes.

$$Obj = [Firingtime, Amplitude] \quad (11)$$

We calculate the Euclidean distances between objects in each spike train using

$$d_{rs}^2 = (N_r - N_s)(N_r - N_s) \quad (12)$$

where N_r, N_s are the objects in the spike train. Once the distance between each pair of objects is determined, the objects are clustered based on the nearest neighbour approach using

$$d(r, s) = \min(\text{dist}(N_{ri} - N_{sj})) \quad (13)$$

$$i \in (1, \dots, n_r), j \in (1, \dots, n_s)$$

where n_r, n_s is the total number of objects in the respective clusters. The binary clusters are plotted to form a hierarchical tree whose vertical links indicate the distance between two objects linked to form a cluster. A number is assigned to each cluster as soon as it is formed. Numbering starts from $(m+1)$, where m =initial number of objects, till no more clusters can be formed.

We investigated the case described in section 3.3 for the response generated at $T_{ref}=15\text{ms}$ and $T=16\text{ms}$. The coincidence-factor for these two responses is 1 (Fig. 4) and indicates an exact match between the two. The clustering solution shows that these two responses are actually different from each other by a margin not captured by the coincidence-factor (Fig. 7 & Fig. 8). A comparison of the clustering solutions shows that the shape, form, height as well as linkage is different for the two spike trains. It indicates that the two spike trains are inherently different by a margin not captured by the coincidence factor. The results hence prove that the two spike trains are not exactly the same. We therefore believe that though determining coincidence-factor is important, a two-dimensional analysis is necessary for a response with a fluctuating membrane voltage.

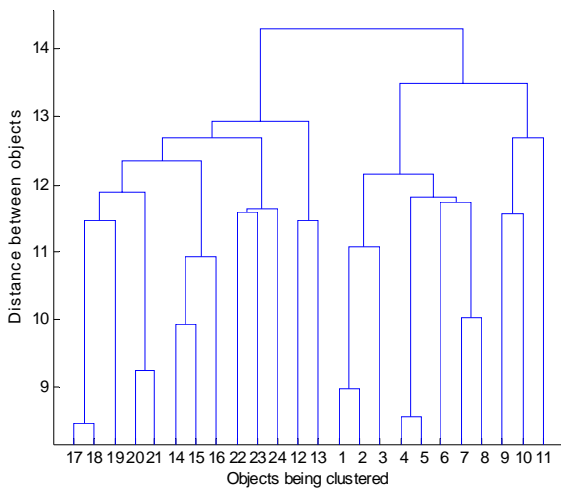


Fig 7: Clustering solution for T=15ms indicating objects being clustered

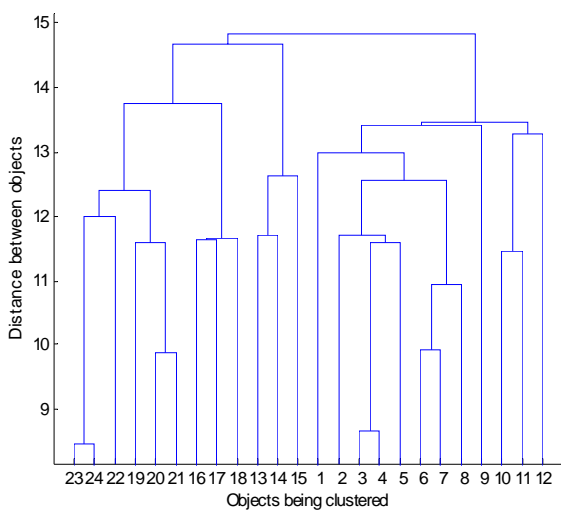


Fig 8: Clustering solution for T=16ms indicating objects being clustered

IV. CONCLUSIONS

The response of a neuron to a time-varying stimulus has been studied before and the complexity of the H-H model has led neuroscientists to develop simpler models that reconstruct the firing pattern of a biological neuron [17]-[28]. Recently, comparisons have been made between responses and similarity measures proposed [18], [20], [23]-[26], [30], [31]. However, the responses considered have a constant magnitude thereby making their analysis one-dimensional.

A synaptic current known to induce fluctuations in the membrane voltage is used to stimulate an H-H neuron [3] to verify if firing time alone is enough to differentiate between these responses. The time constant of the pulse component of the external current is 2ms and thus the simulation time window chosen as 2ms. The coincidence-factors are calculated for time windows $t_1 = 14 - 16ms$, $t_2 = 13 - 15ms$ and $t_3 = 15ms - 17ms$ with reference spike trains at T=15ms, 14ms and 16ms respectively. In all three sets of results, there is no consistent trend exhibited by the coincidence-factor. Also, the limits of faithful behaviour change and the percentage of acceptable results varies. The percentage of faithful behaviour for the three time windows is 30, 75 and 62.5 respectively. The main findings through these sets of

results are: (a) the limits of faithful behaviour change with a change in the reference ISI. (b) the lower-bound limit of faithful behaviour increases with the increase of the reference ISI. (c) the difference between the reference ISI and the lower-bound limit of faithful behaviour decreases with the increase in the reference ISI. This is shown to be due to the one-dimensional similarity measure undertaken. In order to differentiate between these responses accurately, a two-dimensional analysis is required as the magnitudes of the action potentials are vital. A simple clustering algorithm is seen to easily differentiate between two visually-distinct responses as opposed to the coincidence-factor approach. Thus a two-dimensional analysis to differentiate between such responses is necessary and we are currently working towards a more robust differentiation strategy which also quantifies the difference between responses.

The aim of using clustering technique is to exemplify the requirement of a two-dimensional analysis. We take this as a supporting claim for our future work.

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