Influence of Chemical Reaction on Mixed Convection of Non-Newtonian Fluids Along Non-isothermal Horizontal Surface in Porous Media

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***Abstract*— The effect of chemical reaction on mixed convection heat and mass transfer in non-Newtonian fluid a long non-isothermal horizontal surface embedded in a saturated porous medium has been studied. The mixed convection regime is divided into two regions, namely, the forced convection dominated regime and the free convection dominated regime. The two solutions are matched. Numerical calculations are carried out for various values of dimensionless parameters and an analysis of the results obtained shows that the flow field is influenced appreciably by the chemical reaction, the buoyancy ratio, the viscosity index and the power-law of the wall temperature parameter. Effects of these parameters on the transport behaviors are investigated methodically and typically results are illustrated to reveal the tendency of the solutions. The results are presented graphically and the conclusion is drawn that the flow field and other quantities of physical interest are significantly influenced by these parameters.**

***Keywords*— non-Newtonian fluids, mixed convection, chemical reaction, non-isothermal surfaces, porous media.**

I. INTRODUCTION

Combined heat and mass transfer problems with chemical reaction are of importance in many processes and have, therefore, received a considerable amount of attention in recent years. The study of heat and mass transfer with chemical reaction is of great practical importance to engineers and scientists because of its almost universal occurrence in many branches of science and engineering. Possible applications of this type of flow can be found in many industries. For example, in the power industry. Kandasamy et al. [1,2] presented an approximate numerical solution of chemical reaction, heat and mass transfer on MHD flow over a vertical stretching surface with heat source and thermal stratification effects and effects of chemical reaction, heat and mass transfer along a wedge with heat source and concentration in the presence of suction or injection.

Abo-Eldahab and Salem [3] studied the MHD flow, heat and mass transfer of non-Newtonian fluid over a continuously moving cylinder in the presence of uniform magnetic field. Gorla et al. [6] presented a nonsimilar boundary layer analysis for the problem of mixed convection in power-law type non-Newtonian fluids along horizontal surfaces with variable wall temperature distribution. Chamkha [7] studied the analytical solutions for heat and mass transfer by laminar flow of a Newtonian, viscous, electrically conducting and heat generating/absorping fluid on a continuously moving vertical permeable surface in the presence of a magnetic field and first order chemical reaction. Numerical analysis of free convection coupled heat and mass transfer is presented for non- Newtonian power law fluids with yield stress flowing over a vertical flat plate embedded in a fluid-saturated porous medium by Jumah and Mujumdar [9].

A numerical study of the laminar mixed free-forced convection of non-Newtonian power law fluid with mass transfer presented by Eldabe et al. [10]. Devi and Kandsamy [12] studied the chemical reaction, heat and mass transfer over an accelerating surface with heat source and thermal stratification in the presence of suction and injection. Seddeek et al. [14] examined the effect of chemical reaction and variable viscosity on hydromagnetic mixed convection heat and mass transfer for Hiemenz flow through porous media has been studied in the presence of radiation and magnetic field. The influence of reaction rate on the transfer of chemically reactive species in the laminar visco-elastic fluid flow immersed in a porous medium over a stretching sheet studied by Prasad et al. [13]. Ibrahim et al. investigated the effect of the chemical reaction and radiation absorption on the unsteady MHD free convection flow past a semi infinite vertical permeable moving plate with heat source and suction. The influence of chemical reaction on heat and mass transfer by natural convection from vertical surfaces was included by Postelnicu [11] in porous media considering Soret and Dufour effects.

The present work has been undertaken in order to analyze the chemical reaction effects on mixed convection from a horizontal plate in non-Newtonian fluid saturated porous media. The temperature and concentration differences between the plate surface and the outer fluid were assumed to vary as a power-law function of the distance a long the plate. The flow is assumed steady, laminar, and incompressible.

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II. Mathematical Analysis

Consider the mixed convection in a porous medium saturated with non-Newtonian fluid on a horizontal plate, which is heated and has a variable wall temperature. The properties of the fluid and the porous medium are assumed to be constant and isotropic. The Darcy model is considered which is valid under conditions of small pores of porous medium and flow velocity. The axial and normal coordinates are x and y, and the corresponding flow velocities are u and v respectively. The temperature of the ambient medium is T_∞ and the wall temperature is T_w. The flow along the horizontal flat plate contains a species A slightly soluble in the fluid B, the concentration at the plate surface is C_w and the solubility of A in B far away from the plate is C_∞. The governing equations describing the conservation of mass, momentum, energy and concentration under the Boussinesq and boundary layer approximations can be written as follows [6]:

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0, \tag{1}$$

$$\frac{\partial u^n}{\partial y}=-\frac{\rho Kg}{\mu}\left(\beta_T\frac{\partial T}{\partial x}+\beta_C\frac{\partial C}{\partial x}\right), \tag{2}$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\alpha\frac{\partial^2 T}{\partial x^2}, \tag{3}$$

$$u\frac{\partial C}{\partial x}+v\frac{\partial C}{\partial y}=D\frac{\partial^2 C}{\partial x^2}-K_1C^m. \tag{4}$$

Where u and v are the velocity components along x and y directions, respectively, n is the power-law viscosity index, K is the permeability for the porous medium, ρ is the density, α and D are thermal diffusivity and the mass diffusivity, respectively, T is the temperature of the fluid, C is the concentration of the species of the fluid, β_T is the thermal expansion coefficient, β_C is the concentration expansion coefficient, m is the reaction order, K_1 is the dimensional chemical reaction parameter, T_w and C_w are the surface temperature and concentration, respectively, T_∞ and C_∞ are the free stream temperature and concentration, respectively, the boundary conditions of the problem are

$$\begin{aligned}&y=0: v=0, T=T_\infty+Px^\lambda, C=C_\infty+Qx^\lambda,\\ &y\to\infty: u=U_\infty, T\to T_\infty, C\to C_\infty.\end{aligned} \tag{5}$$

Where P, Q are prescribed constants and λ is the power index of the wall temperature and concentration. Both the wall temperature and concentration are assumed to have the same power index λ.

A. Forced Convection Dominated Regime

By defining a stream function $\psi(x,y)$ the continuity equation is automatically satisfied where

$$u=\frac{\partial\psi}{\partial y}\text{ and } v=-\frac{\partial\psi}{\partial x}.$$

Equations (1-5) are now nondimensionalized using the following quantities:

$$\eta=\frac{y}{x}Pe_x^{\frac{1}{2}},\ \xi=\frac{Ra_x^{\ n}}{Pe_x^{\frac{2n+1}{2}}},\ \psi=\alpha Pe_x^{\frac{1}{2}}f(\xi,\eta),$$

$$\theta=\frac{T-T_\infty}{T_w-T_\infty},\ \phi=\frac{C-C_\infty}{C_w-C_\infty}, \tag{6}$$

$$Pe_x=\frac{U_\infty x}{\alpha},\ Ra_x=\frac{x}{\alpha}\left(\frac{\rho Kg\beta_T\Delta T_w}{\mu}\right)^{\frac{1}{n}}.$$

The governing equations and boundary conditions become

$$n(f')^{n-1}f''=-\xi\left[\lambda(\theta+N\phi)+\frac{2\lambda-1}{2}\xi\left(\frac{\partial\theta}{\partial\xi}+N\frac{\partial\phi}{\partial\xi}\right)-\frac{\eta}{2}(\theta'+N\phi')\right], \tag{7}$$

$$\theta''-\lambda f'\theta+\frac{f\theta'}{2}=\frac{2\lambda-1}{2}\xi\left(f'\frac{\partial\theta}{\partial\xi}-\theta'\frac{\partial f}{\partial\xi}\right), \tag{8}$$

$$\frac{1}{Le}\phi''-\lambda f'\phi+\frac{f\phi'}{2}-\gamma\phi^m=\frac{2\lambda-1}{2}\xi\left(f'\frac{\partial\phi}{\partial\xi}-\phi'\frac{\partial f}{\partial\xi}\right), \tag{9}$$

$$\begin{aligned}&f(\xi,0)=(1-2\lambda)\xi\frac{\partial f}{\partial\xi}(\xi,0)\ \text{ or}\\ &f(\xi,0)=0,\ \theta(\xi,0)=1,\ \phi(\xi,0)=1,\\ &f'(\xi,\infty)=1,\ \theta(\xi,\infty)=0,\ \phi(\xi,\infty)=0,\end{aligned} \tag{10}$$

where Le and N are Lewis and sustentation parameter, respectively

$$Le=\frac{\alpha}{D},\ N=\frac{\beta_C(C_w-C_\infty)}{\beta_T(T_w-T_\infty)}. \tag{11}$$

We notice that N is positive for thermally assisting flows, negative for thermally opposing flows and zero for thermal-driven flows. Further, in order to get similarity solutions, the constant dimensionless chemical reaction parameter $\gamma=\frac{K_1}{\alpha}\frac{x^2}{Ra_x}$ was introduced in Eq. (9). We notice that the primes in the above equations denote partial differentiations with respect to η. The presence of $\frac{\partial}{\partial\xi}$ in these equations makes them nonsimilar. The case of $\xi=0$ corresponds to pure forced convection, the limiting case of

$\xi = \infty$ corresponds to pure free convection region. The velocity components u and v in the x and y directions are given by the expressions,

$$u = U_\infty f'(\xi,\eta),$$

$$v = -\frac{\alpha}{x} Pe_x^{\frac{1}{2}} \left[\frac{f(\xi,\eta)}{2} - \frac{\eta}{2} f'(\xi,\eta) + \frac{2\lambda - 1}{2} \xi \frac{\partial f}{\partial \xi}(\xi,\eta) \right] \quad (12)$$

B. Free Convection Dominated Regime

For Free convection dominated regime the following dimensionless variables are introduced in the transformation

$$\eta = \frac{y}{x} Ra_x^{\frac{n}{2n+1}},\ \xi = \frac{Pe_x}{Ra_x^{\frac{2n}{2n+1}}},\ \psi = \alpha Ra_x^{\frac{n}{2n+1}} f(\xi,\eta),$$

$$\theta = \frac{T - T_\infty}{T_w(x) - T_\infty},\ \phi = \frac{C - C_\infty}{C_w(x) - C_\infty}. \quad (13)$$

Substituting Eq. (13) into the governing Eqs. (1-5) leads to

$$n(f')^{n-1} f'' + \lambda(\theta + N\phi) + \frac{\lambda - n - 1}{2n+1} \eta(\theta' + N\phi') + \frac{1-2\lambda}{2n+1} \xi \left(\frac{\partial \theta}{\partial \xi} + N \frac{\partial \phi}{\partial \xi} \right) = 0, \quad (14)$$

$$\theta'' + \frac{\lambda + n}{2n+1} f\theta' - \lambda f'\theta = \frac{1-2\lambda}{2n+1} \xi \left(f' \frac{\partial \theta}{\partial \xi} - \theta' \frac{\partial f}{\partial \xi} \right), \quad (15)$$

$$\frac{1}{Le}\phi'' + \frac{\lambda + n}{2n+1} f\phi' - \lambda f'\phi - \gamma\xi\phi^m = \frac{1-2\lambda}{2n+1} \xi \left(f' \frac{\partial \phi}{\partial \xi} - \phi' \frac{\partial f}{\partial \xi} \right), \quad (16)$$

$$(\lambda + n) f(\xi,0) = (2\lambda - 1)\xi \frac{\partial f}{\partial \xi}(\xi,0) \text{ or}$$

$$f(\xi,0) = 0,\ \theta(\xi,0) = 1,\ \phi(\xi,0) = 1, \quad (17)$$

$$f'(\xi,\infty) = \xi,\ \theta(\xi,\infty) = 0,\ \phi(\xi,\infty) = 0,$$

Note that in order to get similarity solutions, the constant dimensionless chemical reaction parameter $\gamma = \frac{K_1}{\alpha} \frac{x^2}{Pe_x}$.

The case of $\xi = 0$ corresponds to pure free convection and the limiting case of $\xi = \infty$ corresponds to pure forced convection. The velocity components u and v in the x and v directions are given by the expressions,

$$u = \left(\frac{\alpha}{x} \right) Ra_x^{\frac{2n}{2n+1}} f'(\xi,\eta),$$

$$v = -\left(\frac{\alpha}{x} \right) Ra_x^{\frac{n}{2n+1}} \left[\frac{\lambda + n}{2n+1} f(\xi,\eta) - \frac{\lambda - n - 1}{2n+1} \eta f'(\xi,\eta) + \frac{1-2\lambda}{2n+1} \xi \frac{\partial f}{\partial \xi}(\xi,\eta) \right] \quad (18)$$

III. Results and discussions

Equations (7-9) must be solved along with the boundary conditions in Eq. (10). Since analytical solutions do not exist, one has to use numerical techniques. In this study, a version of finite differences method to solve ordinary differential equations was used. Our results have been checked against those obtained in [6] for no mass transfer and chemical reaction. In order to get a clear insight of the physical problem, numerical results are displayed when $Le = 1.0$, and with the help of graphical illustrations.

The dimensionless velocity profiles for different values of temperature exponent parameter $\lambda = 0.0,\ 0.5,\ 1.0$ and nonsimilar parameter ξ with constant chemical reaction parameter γ, viscosity index n, order of chemical reaction parameter m and sustentation parameter N are presented in Figs. (1-3). It is observed that the slip velocity at the porous surface $f'(\xi,0)$ increases as the temperature exponent parameter and nonsimilar parameter increase. The surface temperature gradient and hence the heat transfer rate increases as ξ increases this is noted through Figs. (4-6).

The dimensionless concentration profiles for different values of temperature exponent parameter and nonsimilar parameter with constant chemical reaction parameter, viscosity index, order of chemical reaction parameter and sustentation parameter are demonstrated in Figs. (7-9). It is clear that the concentration of the fluid decreases with the increase of temperature exponent parameter and increase of nonsimilar parameter.

Fig. (10-12) depict the dimensionless concentration profiles for different values of chemical reaction parameter $\gamma = 0.1,\ 1.0,\ 4.0$ and order of chemical parameter with constant temperature exponent parameter, viscosity index, nonsimilar parameter and sustentation parameter. It is observed that the concentration of the fluid increases with the increase of order of chemical reaction parameter and decrease of chemical reaction parameter.

The dimensionless velocity profiles for different values of sustentation parameter $N = 0.5,\ 1.5,\ 2.5$ with constant chemical reaction parameter, viscosity index, order of chemical reaction parameter and nonsimilar parameter are depicted in Fig. (13). It is observed that the velocity of the fluid increases with the increase of sustentation parameter.

Fig. (14) and Fig. (15) demonstrate the dimensionless temperature and concentration profiles for different values of sustentation parameter $N = 0.5,\ 1.5,\ 2.5$ with constant chemical reaction parameter, viscosity index, order of chemical reaction parameter and nonsimilar parameter. It is seen that the temperature and concentration decrease with the increase of sustentation parameter.

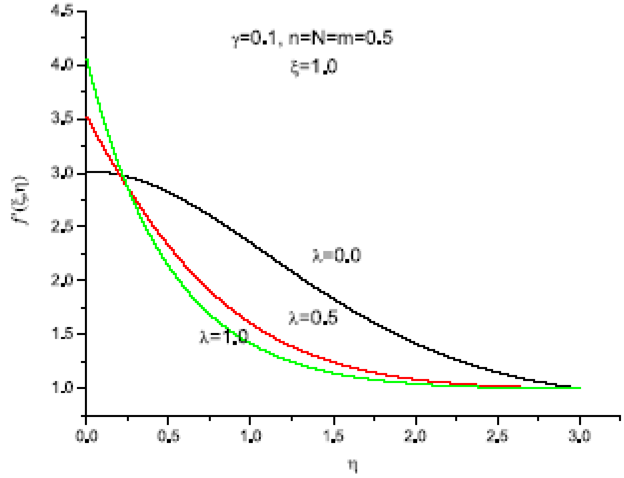

Figure 1: Velocity distribution ($\xi = 1.0$).

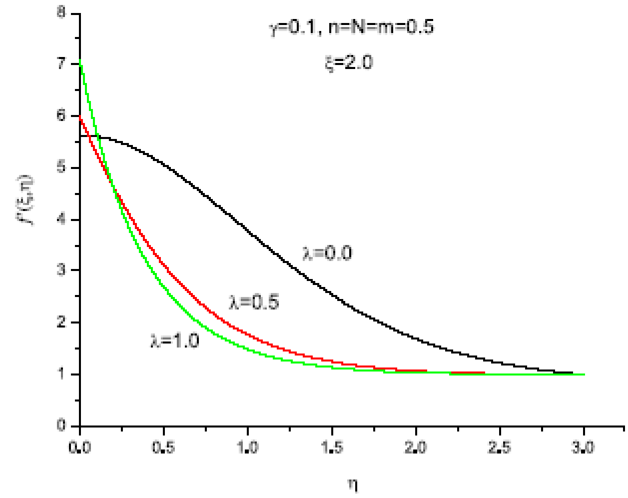

Figure 2: Velocity distribution ($\xi = 2.0$).

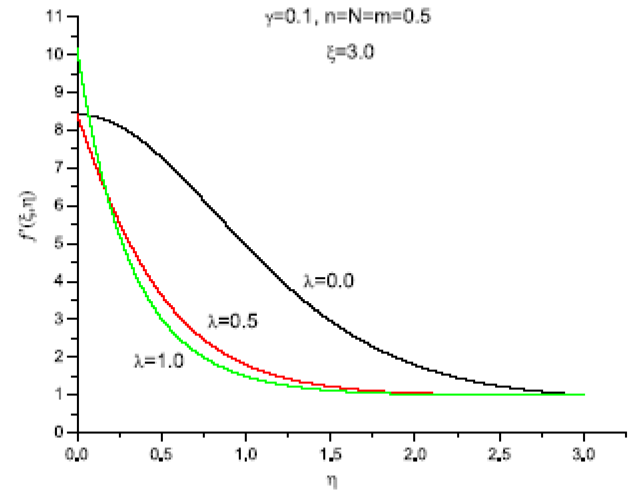

Figure 3: Velocity distribution ($\xi = 3.0$).

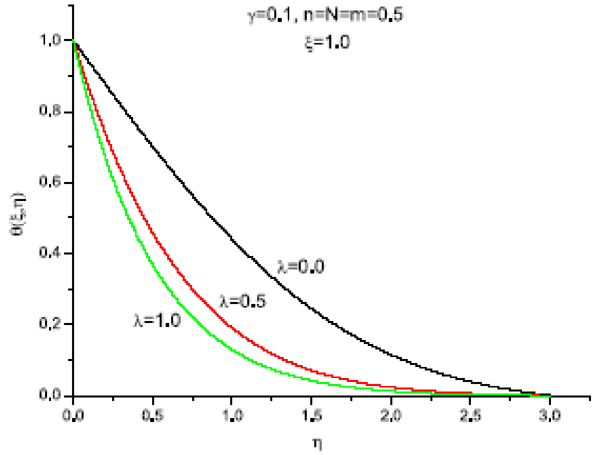

Figure 4: Temperature distribution ($\xi = 1.0$).

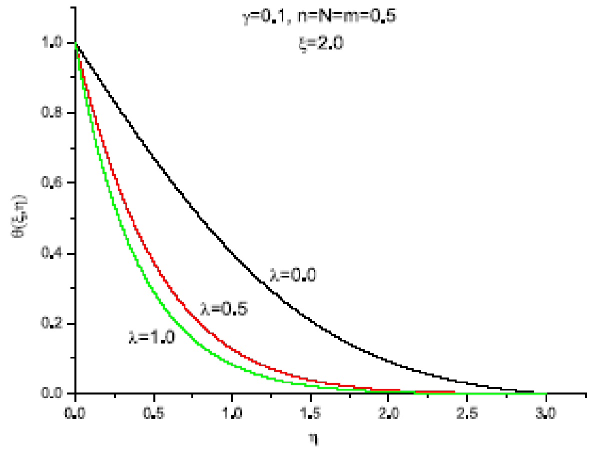

Figure 5: Temperature distribution ($\xi = 2.0$).

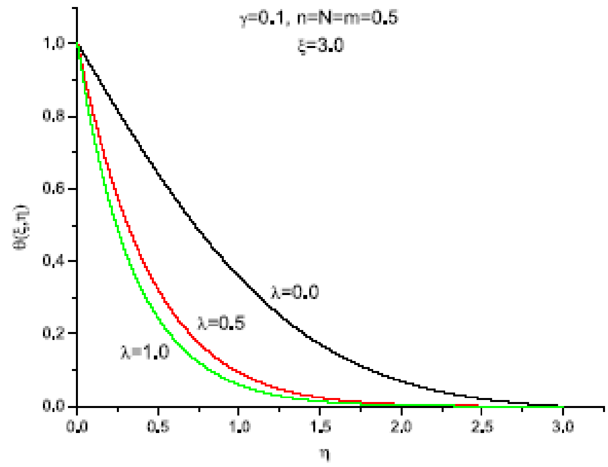

Figure 6: Temperature distribution ($\xi = 3.0$).

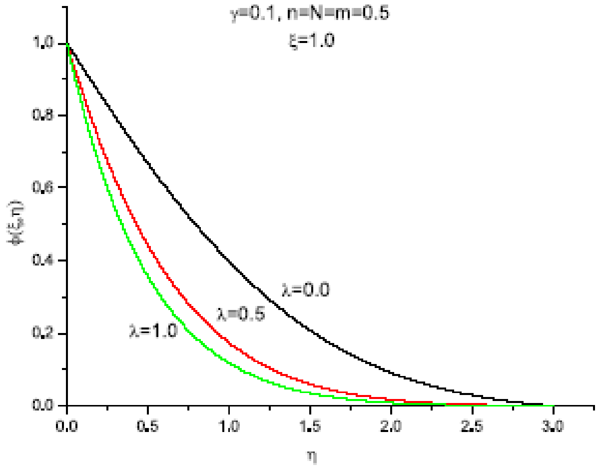

Figure 7: Concentration profiles with $\xi = 1.0$.

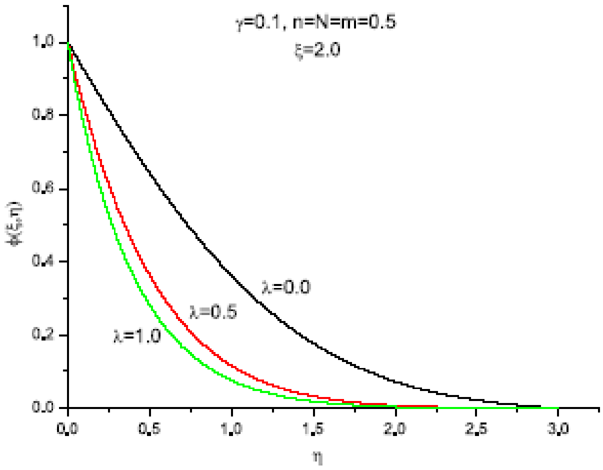

Figure 8: Concentration profiles with $\xi = 2.0$.

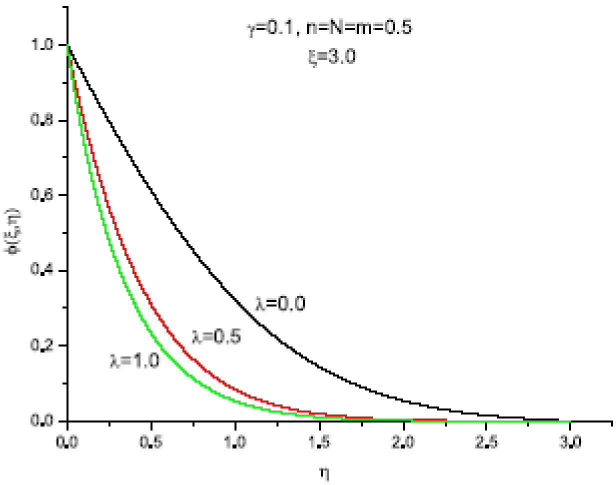

Figure 9: Concentration profiles with $\xi = 3.0$.

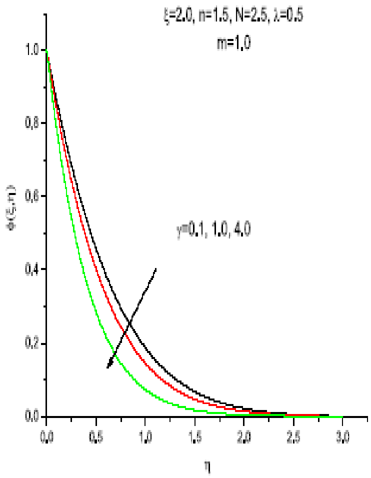

Figure 10: Chemical reaction over the concentration profiles with $m = 1.0$.

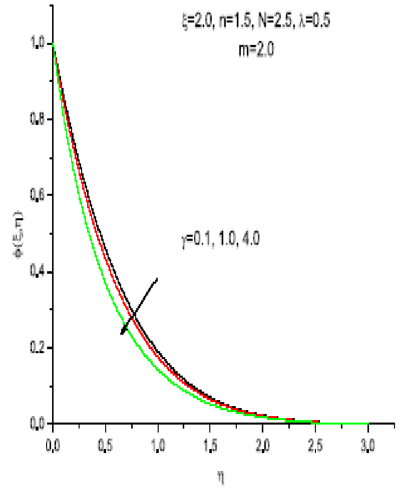

Figure 11: Chemical reaction over the concentration profiles with $m = 2.0$.

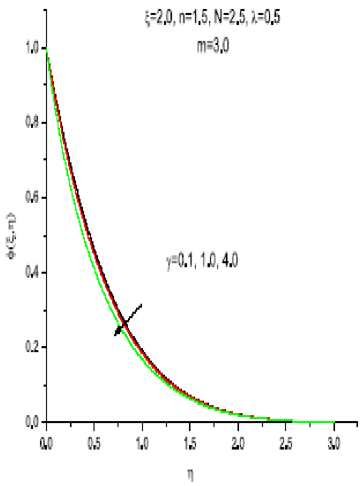

Figure 12: Chemical reaction over the concentration profiles with $m = 3.0$.

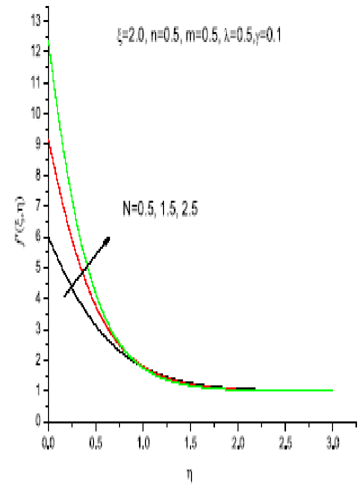

Figure 13: Velocity distribution for different values of sustentation parameter N.

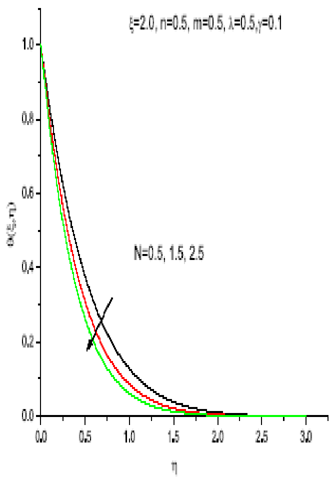

Figure 14: Temperature distribution for different values of sustentation parameter N.

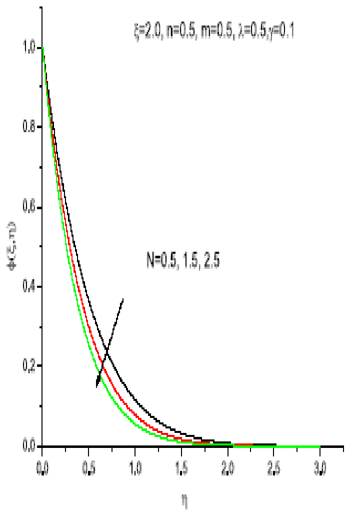

Figure 15: Concentration profiles for different values of sustentation parameter N.

IV. Conclusions

This paper studied the effect of chemical reaction for the mixed convection in non-Newtonian fluids along a non-isothermal horizontal plate embedded in fluid-saturated porous medium. The governing equations are approximated to a system of non-linear ordinary differential equations. The flow regime was divided into forced convection dominated and natural convection dominated regions. In the forced convection dominated region, $\xi = \frac{{Ra_x}^n}{{Pe_x}^{\frac{2n+1}{2}}}$ characterizes the buoyancy effect on forced convection where as $\xi = \frac{Pe_x}{{Ra_x}^{\frac{2n}{2n+1}}}$ is a measure of the effect of forced flow on free convection.

Numerical calculations are carried out for various values of the dimensionless parameters of the problem using a finite differences scheme. Flow, temperature and concentration fields have been calculated for various combinations of the problem parameters ξ, λ, N, m, γ, n.

List of symbols

f	dimensionless stream function
g	acceleration due to gravity
k	thermal conductivity
K	permeability for the porous medium
L	plate length
n	viscosity index
Pe	Peclet number
Ra	Rayleigh number
T	temperature
C	concentration
K_1	dimensional chemical reaction parameter
Le	Lewis number
N	sustentation parameter
m	order of the chemical reaction
D	mass diffusivity
u, v	velocity components in x and y directions
U_∞	free stream velocity
x, y	Cartesian co-ordinates
α	thermal diffusivity
η	similarity variable
β_T	coefficient of thermal expansion
β_C	coefficient of concentration expansion
θ	dimensionless temperature
ϕ	dimensionless concentration
γ	dimensionless chemical reaction parameter
ξ	nonsimilar parameter
ρ	density of fluid
μ	consistency index for viscosity
ψ	stream function

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SUBSCRIPTS

w wall Conditions
∞ free stream conditions

SUPERSCRIPT

$'$ differentiation with respect to η

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