A Novel Algorithm based on Fuzzy Optimization for Antenna Arrays used in Radio Mobile Communications

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Abstract- **In view of the rapid development in the use of mobile phones, it is becoming increasingly crucial to combat the growing threat of co-channel interference (CCI) . An adaptive antenna consisting of an array of spatially distributed antennas can be used to reject CCI . By properly combining the outputs of such antenna , it is possible to extract the individual desirable signal waveforms from the received superposition , even if they all occupy the same frequency band . The new algorithm proposed in this paper is based on the knowledge of users' location which is non - deterministic thus resulting a significant improvement in the reception of the desired signal from the co-channel signals. Such improvement is achieved through the utilization of a fuzzy algorithm in combination with an adaptive array . Computer simulations are carried out for testing the proposed technique in urban areas where there is more CCI due to the increased scattering. Simulation results are presented to demonstrate that the proposed scheme has indeed a great potential in rejecting CCI .**

Key words: **Fuzzy decision , Array antenna , MUSIC algorithm , Base station , MVDR.**

I. Introduction

In recent years, there has been an explosive increase in the use of mobile telephones. Precautions have been necessary to combat the growing threat of co-channel interference . It is shown in this paper that by properly combining the outputs of an array of spatially distributed antennas, it is possible to extract the individual desired signal waveforms from the received superposition , even if they all occupy the same frequency band.

In an urban area , it is quite likely that that no direct path will exist between the base station (BS) and the mobile station (MS). Any communication thus depends on single or multiple reflections from buildings and surrounding objects, rather than the desired line of sight (LOS). In these cases, the radio channel is subjected to *fading* and field strength variations may only be derived from measurement and computer modeling. Under these conditions, if the phase difference between diffracted and reflected waves is a whole number of wavelengths, the two waveforms will reinforce and the amplitude at the receiving antenna will increase (approximately doubled). As the mobile moves, the phase difference between the two paths changes. If the phase difference becomes an odd number of half wavelengths, the two waves cancel producing a null. Therefore as the mobile moves, there are substantial amplitude fluctuations in the received signal known as *fast fading*. Fast fading is also accompanied by a slower variation in mean signal strength called, *slow fading*. Fast fading is observed over distances of about half a wavelength and can produce signal strength variations in excess of 30dB. Slow fading is produced by movement over much longer distances, sufficient to produce gross variations in the overall path between BS and MS. A part from fading, signal reaching the BS is also subjected to various noise resulting the so called Additive White Gaussian Noise (AWGN) . It is because of the random variation in the received signal, it is a non-deterministic and thus makes the evaluation of all these undesirable effects on the signal transmitted from the BS difficult. Authors have come up with the idea of using a fuzzy decision optimization algorithm. In this paper in the section 2, signal modeling within such a mobile environment is described. This is followed in the section 3 by a brief overview of the Multiple Signal Classification (MUSIC) algorithm and associated minimum variance distortionless response (MVDR)[1,2]. Theories for the proposed method are described in section 4. Simulation and interpretation of results are covered in section 5, followed by conclusions. Processing the signals received on an array of sensors for the location of the emitter is of great interest and thus has been treated under many different special cases. One popular algorithm for estimating the location of users is the multiple signal classification (MUSIC) which provides asymptotically

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unbiased estimate of :

- Number of incident waveforms present in the directions of arrival (DOA) or emitter location
- Strengths and cross correlations among the incident waveforms
- Noise / interference strength

After recognising the direct of arrivals of the users by the MUSIC algorithm , the weights of the array can be adjusted by an estimator such MVDR. This is the Maximum Likelihood (ML) method of spectrum estimation. It finds the ML estimate of the power arriving from a point source in direction assuming all other sources as interferences. In beam –forming literature, this known as the MVDR beam former. It is also called the optimal beam former since in the absence of errors , it maximises the output signal to noise ratio (SNR) and passes the look-direction signal undistorted. For the directions of arrival (DOA) estimation problem, this method uses the array weights, which are obtained by minimising the mean output power subject to unity constraint in the look direction .

To the best of our knowledge no one has considered the use of fuzzy optimisation in the area of radio mobile communications and problems associated with the attenuation of the signal . In section 4 for the first time, we have introduced such a concept as the solution. Normally triangular membership functions in fuzzy decision consists of three fix parameters but we have modified this approach by considering the use of smart parameters. This is so because of the Electro Magnetic (EM) nature of the signals involved in this case. Co-channel angles associated with transmitted signals which form the parameters of our model are indeed smart (informative). On this bases we have developed a new and efficient theory which is described in this paper.

2. THE MODEL OF SIGNALS AND RECEIVER

Let narrow band signal of the k-*th* mobile be $m_k(t), k = 1,....p$ then the received signal by base station (BS) from the n-*th* reflector which is near the mobile is :

$$m_{k,l}(t) = B_{k,l} e^{j\psi_{k,l}} m_k(t), \qquad l = 1,2,\cdots,n \qquad (1)$$

The ' n ' sources are correlated and $j = \sqrt{-1}$. The reflected phases and coefficients are random variables with Rayleigh distribution and uniform distribution respectively

ie :

$B_{k,l}$: Rayleigh distribution
(2)

$\psi_{k,l}$: Uniform distribution in $[0,2\pi]$ (3)

The 'narrow band' assumption is made when analysing the performance of an adaptive array signal processing or estimation scheme. A vague definition of narrow band given in the literature[9] is that there essentially no decorrelation between signals received on opposite ends of the array.

The distribution of the reflected coefficients can be determined by the surrounding buildings. Let θ_k be the *k-th* angle of arrival (AOA)at the BS as shown in Fig. 1. Suppose that the reflectors around each mobile are circular and the distance of mobile and BS is very far and the assumption of point source can be considered. If the distribution of all reflectors is represented as

$$[\ \theta_k - \frac{\theta_{BW}}{2}, \theta_k + \frac{\theta_{BW}}{2}\ ]$$

then :

$$\theta_{k,1} = \theta_k + \frac{1}{2}\theta_{BW} \sin\left(\frac{2\pi l}{n-1}\right) \qquad (4)$$

for K=1, … …, p and l=1,2,…….,n

θ_{BW} is named cluster width which decreases with increase in the distance between BS and MS. Also, under these circumstances, MS can treated as a Point-Source.

Consider a uniform linear array (ULA) of *m* antennas with an inter-element spacing *d*. The θ transfer function between the angle of arrival (AOA) from the array broadside and the output of the array is represented by the steering vector :

$$a(\theta) = \left[1, e^{2\pi j d \sin(\theta) f/c}, ..., e^{2\pi j (m-1) d \sin(\theta) f/c}\right]^T \quad (5)$$

where f is the frequency of the signal incident on the array , and *c* the propagation velocity. For convenience , the array is assumed to have half – wave length spacing at the chosen operating frequency , *fo*.

In array processing a fixed phase is typically used to approximate the time delay of a signal between elements of an array . Since the inter- element phase, depends on both the AOA and frequency, for a linear array, a non- zero-band-width signal appears as an extended angular source, whereas a zero-band width signal appears to come form a discrete angle of arrival. This effect is known as dispersion. For the zero – bandwidth case, the exact covariance matrix for the data received by the array from *M* signals arriving from AOAs $\theta_1\ \theta_2\ \theta_3 . \ . \ . \ . \ \theta_M$, the (zero mean and spatially white) noise is typically modeled as:

$$R_x = E\{X(t)X^H(t)\} = A(\theta)SA^H(\theta) + \delta_n^2 I \quad (6)$$

where: $A(\theta) = [a(\theta_1),.....a(\theta_M)]$, $'I'$ is an identity matrix,

$$X(t) = A(\theta)s(t) + n(t) \quad (7)$$

and *n(t)* represents the noise vector .

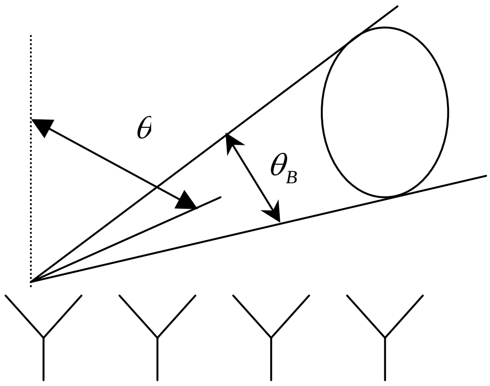

Fig 1 : The model of received signal by BS

3. MUSIC AND MVDR ALGORITHMS

In this section, the MUSIC algorithm is briefly described and applied to different angular separation of mobile users. CCI is subsequently reduced by placing nulls of the array pattern in the direction of undesired signals.

Using vector notation we can write the array output on the matrix form:

$$X(t) = As(t) + n(t) \quad (8)$$

Where ,$'A'$ is the $N \times L$ matrix and :

$$X(t) = [X_1(t), X_2(t),...., X_N(t)]^T \quad (9)$$

$$s(t) = [s_1(t), s_2(t),.....s_L(t)]^T \quad (10)$$

The different components of noise $\{n_i, i = 1,...., N\}$ are assumed to be uncorrelated and white with their variance “δ_n^2” . The correlation matrix will be of the form:

$$R_x = E\{X(t)X^H(t)\} = ASA^H + \delta_n^2 I$$
$$= \sum_{i+1}^{N} \lambda_i e_i e_i^H = Q\Lambda Q^H \quad (11)$$

Where $S = E\{ss^H\}$. Note that S is the signal covariance matrix and has dimension L*L, and *I* is the unit matrix. The superscript "H" denotes the conjugate transpose .

R_X has dimension N*N, and $[\lambda_1, \lambda_2,...., \lambda_L, \delta_n^2, \delta_n^2,....., \delta_n^2]$ are the eigenvalues of R_X. The eigenvectors corresponding to the first L largest eigenvalues are referred to as the signal eigenvectors, and those corresponding to the minimum eigenvalues are referred to as the noise eigenvectors. The subspace spanned by the signal eigenvectors is called the signal subspace, and its orthogonal complement spanned by the noise eigenvectors is called the noise subspace[4].

The diagonal matrix ' Λ ' consists of the eigenvalues which are arranged from large to small resulting:

$$\Lambda = diag[\lambda_1, \lambda_2,..\lambda_L,..\lambda_N] =$$
$$diag[\lambda_1, \lambda_2,....\lambda_L, \delta_n^2, \delta_n^2,.....\delta_n^2] \quad (12)$$

$$Q = [q_1, q_2,...., q_L, q_{L+1},...., q_N] \quad (13)$$

The matrix

$$R_X - \delta_n^2 I = (ASA^H) \quad (14)$$

has the same eigenvectors as R_X, with eigenvalues

$\lambda_i = \sigma^2$ for i=1 , 2 ,.,L and λ_i = 0 for i >L

It follows that :

$$(A(\theta)SA^H(\theta)) = \sum_{i=1}^{L} (\lambda_i - \sigma^2) e_i e_i^H \quad (15)$$

Therefore the signal direction vectors and the signal eigenvectors span the same subspace.

This implies that all signal direction vectors are orthogonal to the noise subspace. The MUSIC algorithm estimates the AOA of the L signals by finding the value of θ corresponding to the L maxima of the function:

$$P_{MUSIC} = \frac{1}{\sum_{i=L+1}^{N} \left| q_i^H a(\theta) \right|^2} \qquad (16)$$

The weights of the element outputs be represented in the N-dimensional vector :

$$W = [W_1, W_2, ...W_N]^T \qquad (17)$$

Then the array output can be written as :

$$y(t) = \sum_{i=1}^{N} W^{*}{}_i X_i = W^H X(t) \qquad (18)$$

This procedure is known as filter and sum beamforming [5]. The mean output power is thus given by

$$P(w) = E[y(t) y^*(t)] = W^H R W \qquad (19)$$

In order to derive the optimal weight vector the array output is minimised so as the desired signals are passed with specific gain while minimising the contributions due to noise and interference. I.e.

Minimise $W^H R_X W$ (20)

subject to : $W^H a = \rho$ (21)

where ρ is a constant and a is the steering vector associated with the look direction.

Again the method of Lagrange multiplier can be used to solve equation (20) resulting [6]:

$$W_{opt} = \rho \frac{R_X^{-1} a}{a^H R_X^{-1} a} \qquad (22)$$

When $\rho = 1$ known as MVDR beamformer [7,8].

4. Theories for the proposed method

Suppose there are ' L 'elements , M co-channel sources and the total power transmitted is Peq .
Let complex number $W = re^{j\xi}$ (23)
represents the weighting element in which r is the amplitude and ξ is the phase. Therefore for a multielement scenario the weighting become

$$W_i = r_i e^{j\xi_i} \qquad (24)$$

Furthermore matrix 'A' can be expressed as

$$a_{ij} = 1 e^{j\theta ij} \qquad (26)$$

So the original problem of the optimisation leads to formation of related problem of:

Minimise $P_{eq}\left(\sum_{i=1}^{L}\sum_{j=1}^{L} r_i r_j\right)$ (27)

subject to:

$$\begin{cases} \sum_{i=1}^{L} \{\, r_i \cos(\theta_{i1} - \xi_i) \,\} = 1 \\ \sum_{i=1}^{L} \{\, r \sin(\theta_{i1} - \xi_i) \,\} = 0 \end{cases} \qquad (28)$$

In order to prove this, let substitute the correlation matrix *'R'* into the coast function, we have:

$$P(W) = \sum_{i=1}^{L} \sum_{j=1}^{L} r_i r_j e^{j(\xi_i - \xi_j)} R_{ij} \qquad (29)$$

And: $R_{ij} = \sum_{k=1}^{M} P_K a_{ik} a_{jk}^{*}$ (30)

Let:

$$W = [W_i]_{i=1}^{i=l} \;\&\; W_i = r_i e^{j\xi_i} \qquad (31)$$

Then we have:

$$|P(W)| = \left| \sum_{i=1}^{L}\sum_{j=1}^{L} r_i r_j e^{j(\xi_i - \xi_j)} R_{ij} \right| \le$$

$$\sum_{i=1}^{L}\sum_{j=1}^{L} r_i r_j \left| \sum_{k=1}^{M} P_k a_{jr} a_{ir}^{*} \right| \le \qquad (18)$$

$$\sum_{i=1}^{L}\sum_{j=1}^{L} r_i r_j \left(\sum_{k=1}^{M} \left| P_k a_{jk} a_{ik}^{*} \right| \right) =$$

$$\sum_{i=1}^{L}\sum_{j=1}^{L} r_i r_j \left(\sum_{k=1}^{M} |P_k| \, |a_{jk}| \, |a_{ik}^{*}| \right) = \qquad (32)$$

$$\sum_{i=1}^{L}\sum_{j=1}^{L} r_i r_j \left(\sum_{k=1}^{M} P_k \right) =$$

$$Peq\left(\sum_{i=1}^{L}\sum_{j=1}^{L} r_i r_j \right)$$

Which is a relationship for the coast function. Next, we consider an appropriate subject function. For $\rho = 1$,

$$W^H a_0 = 1 \qquad (33)$$

and for the first desired user we have:

$$W^H a_0 = 1 \Rightarrow \begin{bmatrix} W_1^* W_2^* \cdots W_L^* \end{bmatrix} \begin{bmatrix} a_{1i} \\ a_{2i} \\ \vdots \\ a_{Li} \end{bmatrix} = 1$$

$$\Rightarrow \quad r_1 e^{j(\theta_{11}-\xi_1)} + r_2 e^{j(\theta_{21}-\xi_2)} + \cdots + r_L e^{j(\theta_{L1}-\xi_L)} = \sum_{i=1}^{L} r_i e^{j(\theta_{i1}-\xi_i)}$$

$$= \sum_{i=1}^{L} \{ r_i \cos(\theta_{i1}-\xi_i) + j r_i \sin(\theta_{i1}-\xi_i) \} = 1$$

$$\Rightarrow \begin{cases} \sum_{i=1}^{L} \{ \ r_i \cos(\theta_{i1}-\xi_i) \ \} = 1 \\ \sum_{i=1}^{K} \{ \ r \sin(\theta_{i1}-\xi_i) \ \} = 0 \end{cases} \qquad (34)$$

This is very practical for computing purposes.

5. SIMULATION AND INTERPRETATION

Non deterministic nature of incident angle of transmitted signals in equation (28) is modified into a fuzzy optimization as follow:

Minimise $P_{eq}\left(\sum_{i=1}^{L}\sum_{j=1}^{L} r_i r_j\right)$

subject to:

$$\begin{cases} \sum_{i=1}^{L} \{ \ r_i \cos(\theta_{i1}-\xi_i) \ \} = 1 \\ \sum_{i=1}^{L} \{ \ r \sin(\theta_{i1}-\xi_i) \ \} = 0 \end{cases} \qquad (35)$$

θ_{i1}: Fuzzy Set for i=1,2, . . . L

$\mu_{\theta_{i1}}\left(\theta_{i1}, \theta_{i1}^{-}, \theta_{i1}^{0}, \theta_{i1}^{+}\right)$: Membership function of the θ_{i1} .

With reference to equation (35) let assume we are considering ' 3 ' elements i.e., L=3 and only ' 1 ' co-channel sources (M=1). The proposed model is thus simplified as:

$$X = [x_1 = r_1, x_2 = r_2, x_3 = r_3, x_4 = \xi_1, x_5 = \xi_2, x_6 = \xi_3] \quad (36)$$

θ_{i1}^{-}, θ_{i1}^{+} can be expressed in terms of θ_{i1}^{0} using nonlinear relationship:

$$\theta_{i1}^{-} = 0.9 * \left|\theta_{i1}^{0} - \theta_c \ \right| * \theta_{i1}^{0} \qquad (37)$$

$$\theta_{i1}^{+} = 0.2 * \left\{ \left|\theta_{i1}^{0} \theta_c \ \right| / 3 + \log\left(3.5 + \theta_{i1}^{0}\right) \right\} * \left|\theta_{i1}^{0} - \theta_c \ \right| \theta_{i1}^{0} \qquad (38)$$

for: i=1,2,3

θ_c in above equations identifies the co-channel angle of arrival.

The carves shown in figure 2 and 3 clearly demonstrate that firstly the proposed model is more accurate in that the minimum null occurs at precisely 5^{O} (Fig 2) and 7^{O} (Fig 3) whereas for the MVDR algorithm it occurs at about 5.3^{O} and 7.5^{O}. Secondly in both Figs 2 and 3, the proposed model gives a CCI loss (expressed in dB_s) far less compared to the MVDR algorithm.

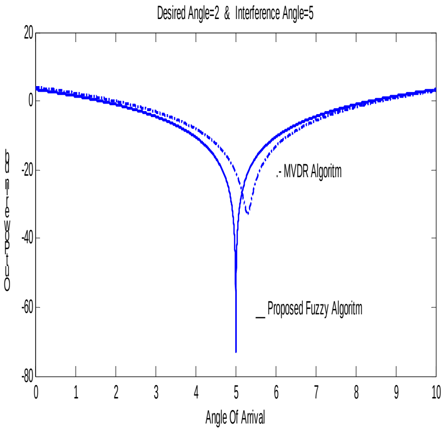

Fig 2: The out put power of the array antenna for common MVDR and the proposed model with interference Degree of 5^{o}

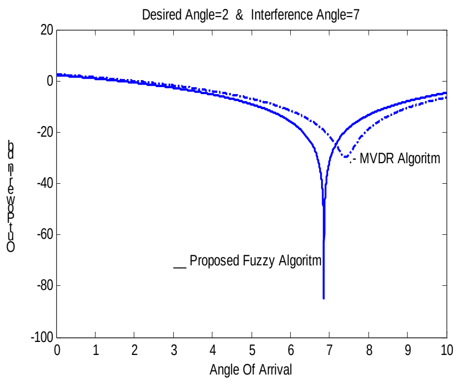

Fig 3: The out put power of the array antenna for common MVDR and the proposed model with interference degree of 7^{O}.

The analysis was repeated with ' 5 ' elements and ' 3 ' co-channel sources in order to examine the improvement of the proposed model . In this case the corresponding equations become :

$$error = \min\left\{ \left|\theta_{i1}^{\circ} - \theta_{c1}\right|, \left|\theta_{i1}^{\circ} - \theta_{c2}\right| \ \right\}$$

$$\theta_{i1}^{-} = 0.8 \times error \times \theta_{i1}^{\circ} \qquad (25)$$

$$\theta_{i1}^{+} = 0.5 \times \left\{ (error/3) + \log(3.5 + \theta_{il}^{\circ}) \ \right\} \times error \times \theta_{i1}^{\circ}$$

(39)

where θ_{c1}, θ_{c2} identify the co-channel angles of arrival.

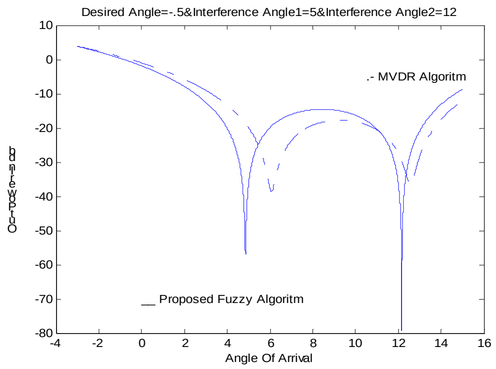

Fig 4: The out put power of the array antenna for common MVDR and the proposed model with interference degrees of 5^{o} and 12^{o}

The curve shown in figure (4) clearly shows that the proposed model is more accurate in that the minimum nulls occurs at precisely 5^{O} and 12^{O} compared to values of 5.3^{O} and 7.5^{O} for MVDR algorithm..

6. CONCLUSIONS

In developing the model of signals and receivers in section 2, we considered a uniform linear antenna array but our analysis can be easily extended to any arbitrary array geometry.

The proposed new algorithm based on the concept of fuzzy optimization using smart parameters improves the performance of the array antenna. The desired signal in the presence of co-channel interference signals can be extracted using a fuzzy algorithm. Due to the non deterministic nature of the users' location, by solving the proposed fuzzy optimisation problem, the weights of array can be determined. The cluster size can be decreased by suppressing the CCI from the neighbouring co-channel cells, and thus improve the spectral efficiency without decreasing the cell radius.

MATLAB Simulation curves presented in this paper demonstrate that the proposed new algorithm provides a superior result in comparison with the widely known MVDR algorithm.

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