Design of A Novel Sliding Mode Observer for Chaotic Systems

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Abstract - This paper describes the state reconstruction of nonlinear chaotic systems with uncertainty having unknown bounds. The new observer presents simple structures that contain a sliding mode term which is established from the available output measurements. To the best of authors' knowledge, in iteration such classes of observer have not been used in synchronization problem. The convergence of the proposed observer has been proved by Lyapunov inequality equation; LMI; while the performance of the system has been verified by famous chaotic systems such as Lorenz and Rossler . A design procedure for the proposed technique is described and simulation results are presented to show that the proposed chaos observer is vary efficient with regards to the class of chaotic systems.

Keywords: sliding mode Observer, chaotic system, nonlinear system1.

Introduction

Chaotic synchronization has received the attention of researches in many fields since the 1980[1-5] and is still an area of active research [6-10]. Recently information theories and concepts were applied to analyze and to quantify synchronization [11–15]. Mutual information measures have been introduced in the past for evaluating the degree of chaotic synchronization[12,13]. The methods of symbolic dynamics have also been used to relate synchronization precision to capacity of the information channel and to the entropy of the drive system[11-14]. It is well known that the study of the synchronization problem for nonlinear systems has been very important for sciences, in particular the applications to biology, medicine, cryptography, secure data transmission and so on. In general, the synchronization research has been focused onto two areas.

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The first one relates to the employment of state observers, where the main applications lie on the synchronization of nonlinear oscillators. The second one is the use of control laws, which allows achieving the synchronization with different structure/ order among nonlinear oscillators [16]. A particular interest is the connection between the observers for nonlinear systems and chaos synchronization, which is also known as master–slave configuration. Thus, chaos synchronization problem can be regarded as observer design procedure, where the coupling noise is viewed as an output and the slave system is the observer [17]. The main approaches, which are related to the construction of asymptotic observers for nonlinear processes, use geometric differential methods. The idea is to find a state transformation to represent the system as a linear equation plus a nonlinear term, which is a function of the system output. However, finding a nonlinear transformation that places a system of order *n* into the so-called observer canonical form requires the integration of *n* coupled partial differential equations. Furthermore, this approach needs an accurate knowledge of the nonlinear dynamics of the system.

The early works dealing with sliding mode observers which consider measurement noise were proposed by Utkin and Drakunov [18]. They discussed the state estimation using sliding mode technique. Anulova [19] treated an analysis of systems with sliding mode in the presence of noises. Slotine *et. al.* [20], successfully designed, so named, sliding-mode approach to construct observers which are highly robust with respect to noises in the input of the system. But, it turns out that the corresponding stability analysis can not be directly applied in the situations with the output noise (or, mixed uncertainty) presence. So, it is still a challenge to suggest a workable technique to analyze the stability of identification error generated by sliding-mode (discontinuous non-linearity) type observers [20-24].

In this paper we propose a new model-free observer: a sliding mode observer for the synchronization problem. The intention of choosing two examples as the Lorenz system, and Rössler system is to clarify the proposed methodology.

2. Problem formulation and assumption

Consider the nonlinear system and a measurement model of the form

$$\dot{x} = f(x), \quad y = Cx \qquad (1)$$

Where: $x \in \Re^n$ is the state to be estimated from, $y \in \Re^p$ is the measured output, f(x) is bounded unknown and C is the constant matrix. We can write Eq.(1) in the following form:

$$\dot{x} = Ax + F(x,u), \quad y = Cx \qquad (2)$$

Where: $A \in \Re^{n\times n}$ is a linear time inverting matrix, pair of (A,C) are observable and $F : \Re^n \to \Re$ is the nonlinear function that may have perturbations.

2.1 Theorem 1

Suppose a chaotic system having the form as Eq.(2) and for $\forall(x_1,u_1),(x_2,u_2) \in \Re^n$, the function F(x,u) satisfies the Lipschitz condition on $\Re^n$, that is

$$\|F(x_1,u_1) - F(x_2,u_2)\| \le L\|(x_1,u_1) - (x_2,u_2)\| \qquad (3)$$

Where: $L \rangle 0$ is Lipschitz constant and $\|\cdot\|$ is the standard Euclidean norm in $\Re^n$.

We consider

$$f(x) - Ax = P^{-1}C^T h(x) \qquad (4)$$

Where: $h(x_t,u_t)$ is bounded as $\bar{h} \rangle \|h(x,u)\|$

3. Sliding mode observer design

For the above system, the following sliding mode observer is designed:

$$\dot{\hat{x}} = A\hat{x} + H(e_y) + S(\hat{x},e), \quad \hat{y} = C\hat{x} \qquad (5)$$

Where e is the state reconstruction error, defined as $e = x - \hat{x}$ and e_y is output error defined as

$$e_y = y - \hat{y} = C\Delta = C(x - \hat{x}) = Ce$$

$S(\hat{x},e)$ is selected as

$$S(\hat{x},e_y) = -\rho \frac{\Gamma C \Delta}{\|C\Delta\|} = -\rho\Gamma sign(e_y) \qquad (6)$$

Now we can reconstruct the observer in the following form:

$$\dot{\hat{x}} = A\hat{x} + H(e_y) + \rho\Gamma sign(e_y), \quad \hat{y} = C\hat{x} \qquad (7)$$

Where the gain matrix H and the switching gain ρ are chosen such that the stability of the observer system is preserved. The discontinuous feedback input is defined as:

$$sign(e_y) = \begin{cases} +1, & e_y \rangle 0 \\ -1, & e_y \langle 0 \end{cases} \qquad (8)$$

Since the sliding mode observer is not dependent on nonlinear plant, only the output y is needed. The derivative of the observer error is:

$$\dot{e} = Ae - HCe + F(x,u) - \rho\Gamma sgn(Ce) \qquad (9)$$
$$= (A - HC)e + F(x,u) - \rho\Gamma sgn(Ce)$$

The feedforward gain matrix H can be obtained in two ways; pole assignment method and LQ method. For the latter method, it is easier to derive the gain matrix H using Riccati equation as:

$$AP + PA^T - PC^T R^{-1} CP = -Q \qquad (10)$$

Where Q and R are arbitrary semi-positive definite and positive definite matrices, respectively. Equation (10) has a positive definite solution for P. Then $A^T - C^T H^T$ is stable assuming:

$$H^T = R^{-1}CP \qquad (11)$$

Which is equivalent to the stability of $\bar{A}_0 = A-HC$. In fact, H is the observer gain matrix for the system Eq. (2).

By using appropriate Lyapunov equation, we select a Γ such that the reconstruction error system is asymptotically stable. Let P_f be the positive definite solution of the Lyapunov equation as:

$$A_0 P_f + P_f A_0^T = -Q_f \qquad (12)$$

Where Q_f is an arbitrary positive definite matrix.
Let set:

$$\Gamma^T P_f = C \qquad (13)$$

3.1 Theorem 2

If f is bounded, the observer gain satisfies the following Equation:

$$\rho > \bar{h} \qquad (14)$$

Then the error between the sliding mode observer and the nonlinear system is asymptotically stable means that: $\lim_{t\to\infty} e = 0$.

Proof Let consider the a Lyapunov function candidate for Eq.(9) as:

$$V(e) = e^T P_f e \qquad (15)$$

Then;

$$\begin{aligned} \dot{V}(e) &= \dot{e}^T P_f + e^T P_f \dot{e} = e^T (A_0 P_f + P_f A_0^T) e \\ &+ (F - \rho\Gamma^T sign(ce)) P_f e \\ &+ e^T P_f (F - \rho\Gamma^T sign(ce)) \\ &= -e^T Q_f e + 2(P_f e F - \rho\Gamma^T P_f e\, sign(e_y)) \\ &= -e^T Q_f e + 2e^T P_f [S(\hat{x}, e_y) + F(x,u)] \end{aligned}$$

By using (5) we have:

$$F(x,u) = P_f^{-1} C^T h(x,u), \qquad \|h(x,u)\| < \rho$$

If we select $S(\hat{x}, e_y)$ and Γ as (6), (13); we have

$$\dot{V} = -e^T Q_f e + 2e^T C^T h(x,u) - 2\rho \frac{\Gamma C \Delta}{\|C\Delta\|}$$

$$= -e^T Q_f e + 2e^T C^T h(x,u) - 2\rho\|C\Delta\|$$

$$\leq -e^T Q_f e + 2\|C\Delta\|(\|h(x,u)\| - \rho) \leq 0 \qquad (16)$$

Since it is assumed that $\dot{V} \leq 0$, $e \in L_\infty$, from the error equation (9) it is also $\dot{e} \in L_\infty$ and V is bounded process, e is quadratically inerrable and bounded. Using Barbalat's Lemma we obtain that the observer error e is asymptotically stable, then $\lim_{t\to\infty} e = 0$.

4. Simulation results

4.1 Example 1

In this section we consider *Lorenz* system, which is known to exhibit a chaotic behavior to verify the effectiveness of the proposed methods in this paper [29]. The dimensionless version of Lorenz system is:

$$\begin{aligned} \dot{x}_1 &= \sigma(x_2 - x_1) \\ \dot{x}_2 &= -x_1 x_3 + r x_1 - x_2 \\ \dot{x}_3 &= x_1 x_2 - b x_3 \end{aligned}$$

With the measurement of equation $y=x_1$, and choosing $\sigma = 10, r = 28, b = 8/3$ then the system exhibits chaotic observer. Initial conditions are chosen as:

$$\begin{aligned} x_1(0) = 5, x_2(0) = 4, x_2(0) = 3 \\ \hat{x}_1(0) = -5, \hat{x}_2(0) = -4, \hat{x}_3(0) = -3 \end{aligned}$$

In this paper we assume that $Q = 50eye(3)$, $Q_f = 100eye(3)$, $\rho = 0.2$ and $R=1$. Simulation results are given in Figures 1,2 and 3. Fig. 2 shows the state error that rapidly converges to zero. Note that we assumed that the sampling period is T = 9×10^{-3} s and we defined the sum squared error (SSE) as:

$$Sse = \sqrt{(x_1 - \hat{x}_1)^2 + (x_2 - \hat{x}_2)^2 + (x_3 - \hat{x}_3)^2}$$

The SSE is illustrated in fig 3.

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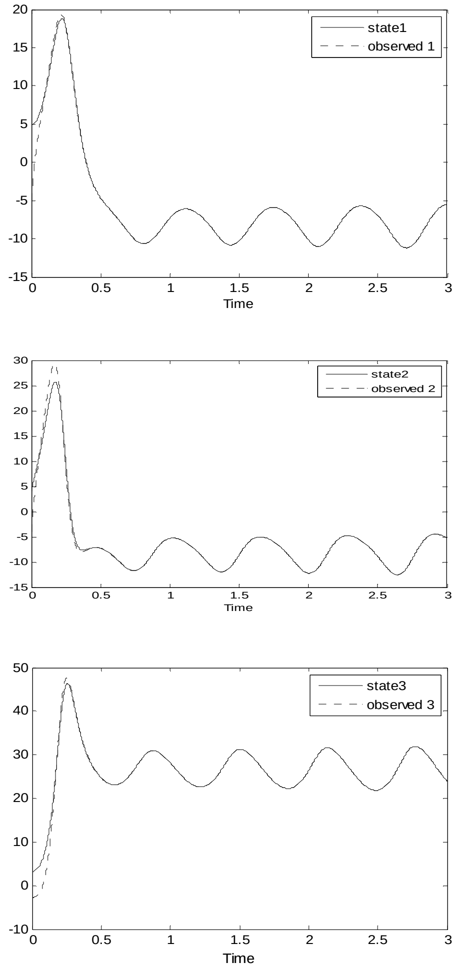

Fig .1 Drive state for Lorenz system

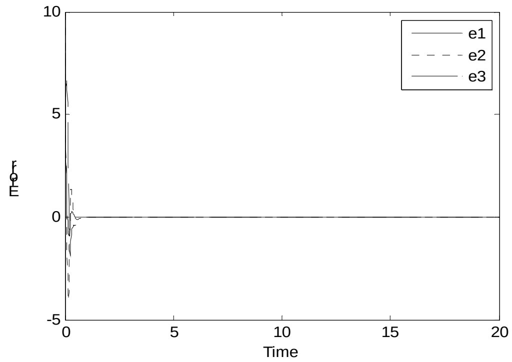

Fig.2 Synchronization state errors of Lorenz system

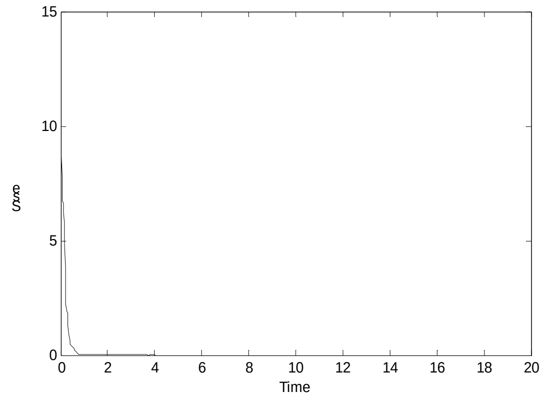

Fig.3 SSE propagation for Lorenz system

4.2 Example 2

In this example we consider *Rössler* system. Such a system can be implemented by a chemical reaction scheme[25-27]. It contains only one nonlinear term but produces a high-dimensional chaos phenomenon with two directions of hyper chaotic instability on the attractor. A four-variable Rössler system can be described by the following differential equations:

$$\dot{x}_1 = -\beta x_2$$

$$\dot{x}_1 = x_1 - x_2 + x_3$$

$$\dot{x}_3 = \alpha x_2 - \alpha x_3 - \frac{\alpha}{G} f(x_3)$$

$$y = x_1$$

Where: $\alpha = (C_2 / C_1)$ and $\beta = (C_2 / LG^2)$. The following parameters are assumed $G_1 = -0.8, G_2 = -.05, \alpha = 8, \beta = 11$ & *G=0.7[30]*. With these parameters, the equations given above exhibit a double scroll type chaotic behavior.
Initial conditions are chosen as:

$$x_1(0) = 5, x_2(0) = -5, x_2(0) = -4$$

$$\hat{x}_1(0) = -5, \hat{x}_2(0) = 5, \hat{x}_3(0) = 4$$

Simulation results are given in Figures 4,5 and 6.

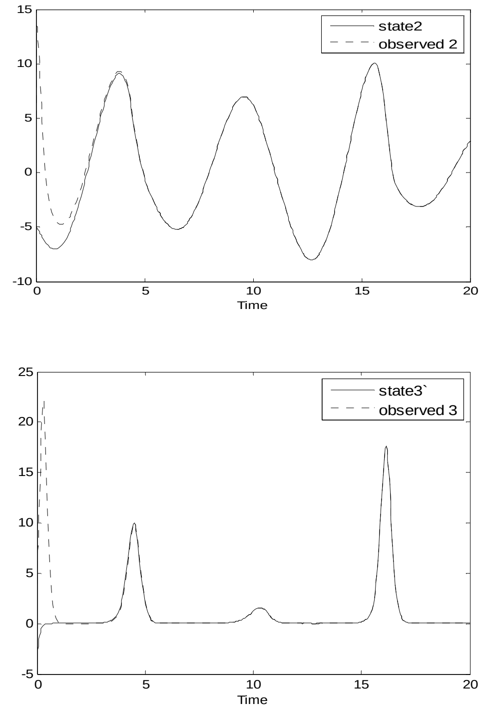

Fig.4 Drive state for Rössler system

From Fig.5 it can be seen that the error of the state converges to zero.

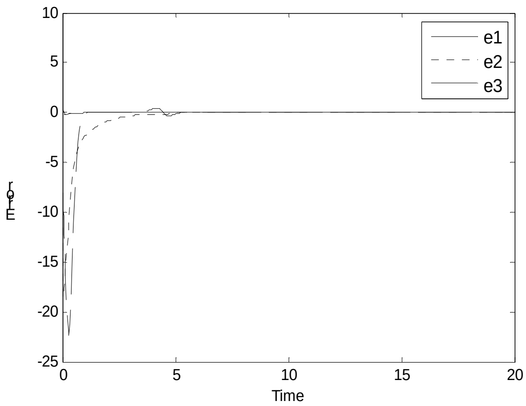

Fig.5 Synchronization state errors of Rössler system

Figure 6 shows the sum squared error (SSE).

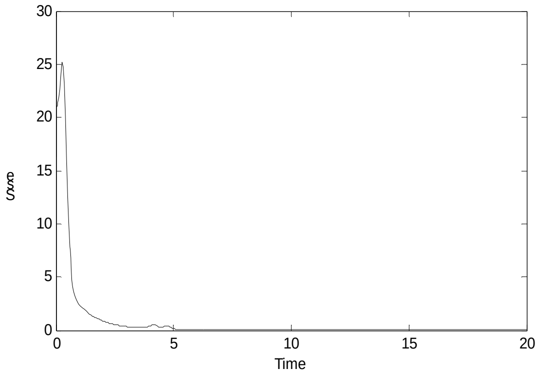

Fig.6 SSE propagation for Rössler system

The results achieved in this paper show that *e* converges to zero more rapidly and the observer performance is very efficient.

5. Conclusions

In this paper a sliding mode observer is proposed to reconstruct the states of uncertain nonlinear systems from available output measurements. The proposed approach is robust for synchronization despite the difference between transmitter and receiver parameters and their initial conditions. We have applied this approach on three systems, Lorenz, Rössler and Chua's circuit. With reference to the simulation results, it is shown that the correct estimation of real system can be obtained. The effectiveness of the proposed method is investigated through some examples that show a significant performance improvement.

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