Generalized Minimum Variance Controller as a Velocity Loop Controller of a Casting Drum Drive in a Polyester Manufacturing Line

S Donoghue, J.W. Finch, D. Giaouris, A. Jones

***Abstract*— The vast majority of controllers found in manufacturing plants use a combination of Proportional, Integral and Derivative (PID) control. These controllers are easily understood, and there are only three controller values to set up when attempting to achieve an acceptable system response. This paper looks at one industrial application of a PI controller, controlling the speed of a casting drum in a polyester manufacturing plant. The desired objective of the controller is to minimize the variance of the casting drum speed when in steady state. It therefore makes sense to compare a Generalized Minimum Variance Controller with the present PI to see if any improvements can be achieved.**

***Index Terms*—Generalized Minimum Variance Control, motor speed control, PI Control, D.C. Motor.**

NOMENCLATURE

V_s is the supply voltage,

IR is the voltage drop across the motor winding resistance

$L\frac{di}{dt}$ is the voltage drop across the winding inductance

E is the back emf generated by the motor rotation.

$K_b\omega$ = back emf.

K_b is the back emf constant

ω represents the angular velocity.

T_m equals the motor torque

K_m is the motor torque constant

I_a is the armature current.

J is the inertia in kgm^2

α is the angular acceleration $\frac{d\omega}{dt}$ in rad/s

ΣT is the sum of the torques acting on the drum.

T_E is the torque developed by the motor

T_L is the load torque

β is the coefficient of dynamic friction

L = length of web

E_m = Young's modulus

S = cross section area.

V_1 is the downstream roller velocity

V_2 is the upstream roller velocity

T_1 is the downstream web tension

T_1 is the upstream web tension

σ^2 is the variance of a white noise input signal

I. INTRODUCTION

Polyester film production involves the melting of polymer chip by extrusion. This process involves a blend of polyester chip and reclaim flake being transported through a heated barrel by a rotating screw. The force of friction between the chip and the barrel wall heats the chip up to its liquid state, forming molten polymer. A melt pump is used to deliver the molten polymer to a die at a constant metered rate. A curtain of molten polymer drops from the die onto a casting drum.

The molten curtain is electrostatically pinned to a chilled casting drum where it is rapidly cooled, forming a continuous sheet of amorphous film. The amorphous sheet of film is subsequently passed over a series of heated rollers to increase the film temperature to a point where the film can be stretched or drawn. This drawing process imparts mechanical strength into the film. The film is wound and taken to subsequent processing stages as required.

II. IMPORTANCE OF CASTING DRUM SPEED HOLDING

The casting drum rotation has to be controlled to ensure that as the polymer is pinned to the casting drum, the smooth rotation of the drum takes away the polymer at an even rate. A continuous sheet of brittle amorphous film is formed from the casting process, whose thickness is largely determined by the speed of the casting drum. One of the main specifications of a film type is the mean thickness of the film across the whole web. The film thickness variation from a mean value is one measure of the quality of the film. It is important both for the quality of the film delivered to customers and to the winding process, that the film thickness variation is minimized. The film thickness variation, or profile as it is termed, is measured by using a sensor located just prior to the point where the finished film is wound onto a central core.

The casting drum provides the speed reference for all the subsequent drives used in the remainder of the process, which can number between 30 and 40 drives on some manufacturing lines. The film web, which is usually under tension, physically connects the casting drum to the downstream process and any

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torque disturbances may be transmitted through the web.

It is very rare that modern manufacturing plants will be used to produce a single product at the same manufacturing conditions. Producing a range of products will usually involve an equal range of process conditions being applied in the product manufacture. The dynamics of a process may change as the operating conditions of the process change.

Tuning a controller whilst manufacturing one particular product may mean that the controller gains may not be optimum when the manufactured product changes. Changes in process dynamics may mean that new controller gains are needed to ensure optimum system control. It is impractical to tune a controller every time a new product is produced. Time and plant availability would prevent this being a practical option. It is therefore advantageous to have a controller, which is capable of quickly changing its parameters to maintain a desirable response at all times.

III. MODELING OF SYSTEM

A model was constructed to determine the effectiveness of the present system to maintain acceptable speed holding in the face of torque disturbances. The system comprises the casting drum, the direct drive motor and the analogue PI controller. In the model, a digital controller can replace the existing analogue velocity loop controller. This offers the advantage of flexibility in changing control law, by simply loading modified software, as opposed to changing resistor and capacitor values. The model can be used to give a degree of confidence that implementation of hardware would have a lower risk and the benefit that a new control law could be examined before being applied to the plant. A model of the motor, drive and plant was constructed in Simulink using a combination of data obtained from the manufacturers' data books and experimental results.

The model of the motor was obtained using Kirchhoff's voltage law:

$$V_s = IR + L\frac{di}{dt} + E \tag{1}$$

$$V_s(s) = I(s)R + L(s)I(s) + K_b\omega(s) \tag{2}$$

$$V_s(s) = I(s)(R + L(s)) + K_b\omega \tag{3}$$

$$\therefore I(s) = \frac{V_s(s) - K_b\omega(s)}{R + L(s)} \tag{4}$$

For a permanent magnet d.c. motor, the motor torque equals

$$T_m(s) = K_m I_a(s) \tag{5}$$

It can be shown by energy conservation that K_m is equal to K_b when a consistent unit system is used.

The input to the motor is a voltage given by

$$V_s(s) - K_b\omega(s) \tag{6}$$

and the output of the motor is torque expressed in Nm .

IV. MODEL OF CASTING DRUM ASSEMBLY.

The casting drum assembly was modeled by determining the sum of the torques.

$$J\alpha = \sum T \tag{7}$$

The net torque can be written as,

$$J\alpha = T_E - T_L - \beta\omega \tag{8}$$

For the casting drum assembly the load torque acts in the same direction as the motor rotation, but the motor torque acts in the opposite direction, effectively braking against the pulling force of the film. Therefore the load torque changes sign.

$$J\alpha = T_E + T_L - \beta\omega \tag{9}$$

Choosing the velocity ω, as the state variable gives:

$$\frac{d\omega}{dt} = \frac{1}{J}\{T_E + T_L - \beta\omega\} \tag{10}$$

The motor torque determined from (4) and (5) is therefore given as

$$T_m(s) = K_m\left(\frac{V_s - K_b\omega(s)}{R + L(s)}\right) \tag{11}$$

$$\therefore T_m(s)(R + L(s)) = K_m(V_s - K_b\omega(s)) \tag{12}$$

$$\therefore \frac{O/P}{I/P} \equiv \frac{T_m(s)}{(V_s - K_b\omega(s))} = \frac{K_m}{R + L(s)} \tag{13}$$

The back emf constant was obtained from the manufacturer's data book and the field flux is a constant since the field is provided by a permanent magnet.

V. MODEL OF FILM TENSION ON DRUM.

The model of a web transport system is built by modeling the web tension between two consecutive rollers and the dynamic velocity of each roller. This web tension model can be developed using three laws:

Hooke's law, giving the elasticity of the web,

Coulomb's law, giving the web tension, and Mass conservation, which defines the coupled relationship between web velocity and web strain.

Web Tension between two consecutive rolls can be derived from these 3 laws and given as [1],

$$L\frac{dT_2}{dt} \approx E_m S(V_2 - V_1) + T_1V_1 - T_2(2V_1 - V_2)$$

The output from two load cells measuring film tension on the real plant, were used to provide a realistic disturbance to the model. The variation in film tension translates as a load torque disturbance acting on the casting drum. The casting drum speed holding is affected by the disturbance.

VI. COMPARISON OF DIFFERENT CONTROLLERS

Using the model new controllers can be simulated to determine if the speed holding can be improved. Various methods of motor control exist, from the traditional cascade control using PI loop controllers, to more advanced controllers. The purpose of this section is to develop a Generalized Minimum Variance controller [2, 3]and compare its effectiveness in improving the speed holding of the casting drum, comparing it with both the existing narrow bandwidth controller and a controller with a wider bandwidth.

The velocity loop bandwidth of the present system is around 1Hz. Increasing the bandwidth of the velocity loop will improve the disturbance rejection capabilities of the system, and give the system a faster response time. High frequency disturbances are largely damped out by the high inertia of the casting drum. The frequency disturbances measured on the real plant occur at the low end of the frequency spectrum, typically less than 2Hz. Increasing the velocity loop bandwidth to 12Hz can be accomplished by changing the gains of the PI controller. There is a limit to the bandwidth that can be safely used as noise in the feedback signal will not be attenuated and cause increased steady state variance in the casting drum speed. There is also a well-established rule of thumb when using cascade control: ensure the bandwidth of the current loop is at least 10 times the bandwidth of the velocity loop.

The disturbance measured in the plant film tension is applied to the Simulink model of the casting drum assembly. A comparison is made between the present low frequency velocity loop and the wide bandwidth velocity loop. The simulation is conducted with and without the film tension disturbance. A step response to an increase in casting drum speed is also compared.

VII. THE EFFECT OF WIDENING THE VELOCITY LOOP BANDWIDTH.

Figure 1 shows the simulated response of the casting drum system when subjected to a step of amplitude 2m/min. Clearly the rise time of the wide bandwidth velocity loop is much faster then the present low bandwidth loop.

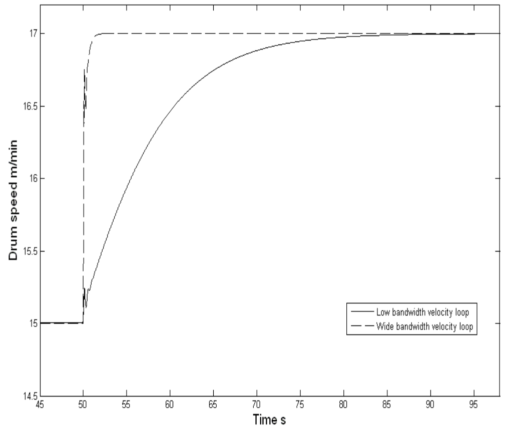

Figure 1: Response of casting drum to step of amplitude 2m/min.

A simulated disturbance, which was originally measured on the film line was added to the film tension and is shown in Figure 2.

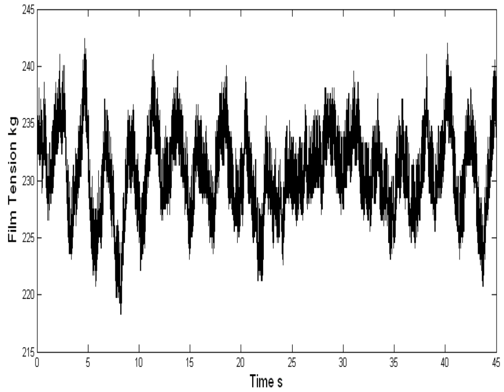

Figure 2: Simulated Load torque in the form of film tension applied to casting drum.

The drum speed output with the two applied loop bandwidths is shown in Figure 1. The wider loop bandwidth reduces the amplitude of the disturbance as seen on the casting drum speed output. One downside of having a wider bandwidth can be seen in Figure 4, with the wider bandwidth allowing more measurement noise through and this affects the casting drum speed.

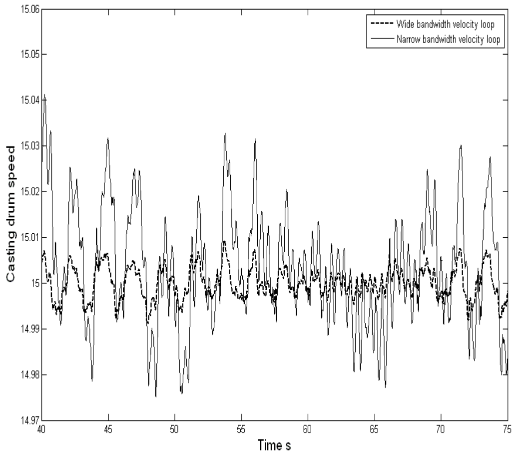

Figure 3: Comparison of loop bandwidths with no added measurement noise.

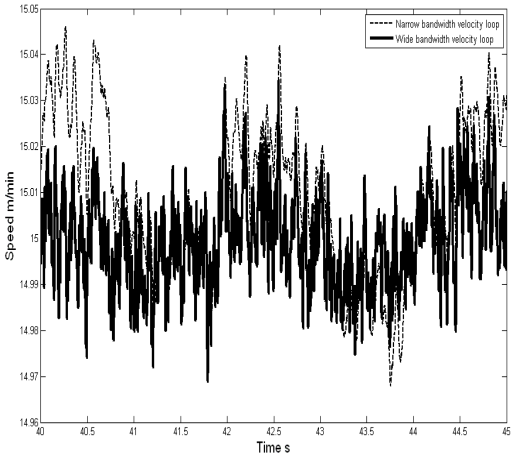

Figure 4: Comparison of wide (12Hz) bandwidth and narrow (2Hz) velocity loop bandwidth, with added torque disturbance and measurement noise.

Using the integral of time multiplied with the absolute error (ITAE) can be used as a measure of the two different loop bandwidths. The resulting 'figure of merit' can be used to compare the performance of the two controllers. The results in Table I show that the narrow loop bandwidth has a figure of merit higher than the wider bandwidth, and does not perform as well in reducing the error between the reference speed and the output drum speed when there is a disturbance present. However, with no disturbance present the narrow bandwidth controller performs better than the wide bandwidth controller. This is because of the wider bandwidth allowing more noise to pass through.

The figure of merit of the wide bandwidth controller does not alter significantly with or without the added disturbance. The narrow bandwidth controller however, performs poorly when the disturbance is added.

Table II. Variance of casting drum speed in steady state.

	Added noise and disturbance	No added noise or disturbance
Wide bandwidth ITAE	2.458	2.313
Narrow bandwidth ITAE	27.31	1.025

Figure 5 shows the casting drum speed when there is noise added to the feedback signal, but the film tension and disturbance are removed.

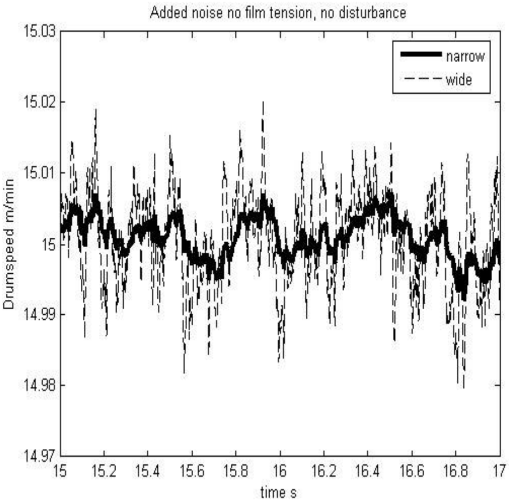

Figure 5: Casting drum speed output with no added disturbance or load.

As can be seen in Figure 5, the narrow bandwidth controller has less variance in drum speed than the wider bandwidth controller, due to the increased noise amplified by the wide bandwidth controller.

Figure 6, shows that with no disturbance and a steady state load present, the narrow bandwidth controller performs better than the wider bandwidth controller. It is only when the disturbance is added to the load in the form of a film tension disturbance, that the wider bandwidth controller outperforms the narrow band controller.

The continuous time plant model developed earlier is given as a transfer function using the Laplace operator s.

$$\frac{Y(s)}{U(s)} = \frac{20868s + 6.524e^{6}}{61.44s^{3} + 5.098e^{4}s^{2} + 1.507e^{7}s + 7.185e^{6}} \qquad (14)$$

To convert the continuous time equation of (14) to an equation containing the backward shift operator, the equation is first converted into a discrete transfer function using a sampling time of 0.01s and a zero order hold function [4]. The resulting discrete equivalent using the forward shift operator, q is given in (15).

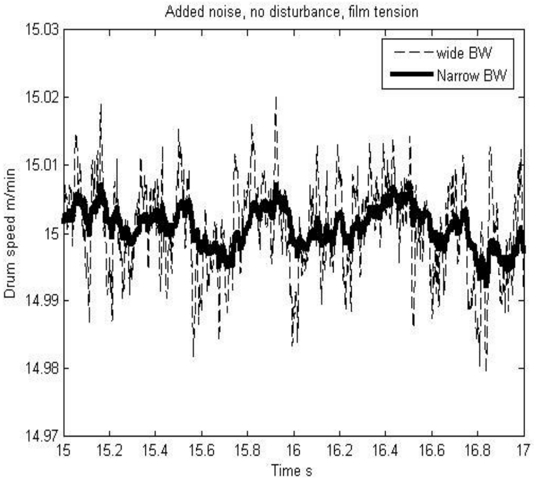

Figure 6: Casting drum speed with added noise and load, with no disturbance.

$$\frac{Y(z)}{U(z)} = \frac{0.004234z^2 + 0.0002241z - 7.765e^{-6}}{z^3 - 0.9666z^2 - 0.028242z - 0.0002491} \quad (15)$$

Dividing (15) by z^3 results in an equation in the backward shift operator. To distinguish between the operators, (16) uses q as the operator and the transfer function is given as a ratio of polynomials.

The transfer function is expressed in the form

$$A(q^{-1}) = 1 + a_1 q^{-1} + a_2 q^{-2} ... + a_n q^{-n}$$

$$B(q^{-1}) = b_0 + b_1 q^{-1} + b_2 q^{-2} ... + b_m q^{-m}$$

giving

$$\frac{B(q^{-1})}{A(q^{-1})} = \frac{0.004234q^{-1} + 0.0002241q^{-2} - 7.695e^{-6} q^{-3}}{1 - 0.9666q^{-1} - 0.028242q^{-2} - 0.0002491q^{-3}} \quad (16)$$

where $\frac{y(t)}{u(t)} = \frac{B(q^{-1})}{A(q^{-1})}$

and *y(t)* is the output from the plant, the casting drum velocity and *u(t)* is the input to the plant, the current demand reference. $B(q^{-1})$ and $A(q^{-1})$ are polynomials using the backward shift operator q, where $q^{-1}.x(t) = x(t-1)$

A. *Generalized Minimum Variance Control.*

The Generalized Minimum Variance controller (GMVC) has been used in a number of applications with good results [5-7], although it appears that there are few applications to speed control of electric drives. The objective of the GMVC is the minimization of a pseudo output Φ(t), defined as

$$\Phi(t+k) = Py(t+k) + Qu(t) - Rr(t) \quad (17)$$

so the cost function to be minimized is,

$$J = E\left|\Phi^2(t+k)\right|$$

Similar to the procedure exercised in determining the Minimum Variance controller, the GMVC involves separating past and future terms, in order to predict the output k steps ahead using known past values. Separating $\Phi(t+k)$ into two parts, one part is set to zero by the control action and the other part, whose values lie in the future and are not known. The Diophantine identity is used to determine the unknown polynomials required in the prediction of future values.

$$PC = EA + q^{-d} G \quad (18)$$

where C is a backward shift polynomial associated with the disturbance transfer function with an white noise input of mean 0 and variance σ^2 .

The polynomial P is chosen and will influence the dynamic response of the system as can be seen by the closed loop process equation written as

$$y(t) = \frac{q^{-d} BR}{PB + QA} r(t) + \frac{(BE + QC)}{(PB + QA)} e(t)$$

Polynomial E is monic and given by,

$$E = 1 + e_1 q^{-1} + ... + e_{d-1} q^{-(d-1)}$$

The order of polynomial G is given by,

$$G = g_0 + g_1 q^{-1} + ... + g_{ng} q^{-ng} \quad \text{where}$$

$$n_g = \max(n_a - 1, n_p + n_c - k)$$

For the plant model of casting drum, motor and current loop, given in the previous section, and with a delay of 1 sampling time, the polynomials can be calculated as,

$$E = 1 \quad (19)$$

$$G = g_0 + g_1 q^{-1} + g_2 q^{-2} + g_3 q^{-3}$$

giving from (18)

$$(1 - 0.8q^{-1})(1 - 0.226q^{-1} - 0.811q^{-2} - 0.414_q^{-3})$$

$$= (1)(1 - 0.9666q^{-1} - 0.028242q^{-2} - 0.0002491q^{-3})$$

$$+ q^{-1}(g_0 + g_1 q^{-1} + g_2 q^{-2} + g_3 q^{-3}) \quad (20)$$

Equating coefficients produces the unknown polynomials as

$$E = 1$$

and

$$G = -0.392 - 0.963q^{-1} + 0.234q^{-2} + 0.332q^{-3}$$

Multiply the system equation given by,

$$Ay(t+k) = Bu(t) + Ce(t+k)$$

by the polynomial E in order to obtain an expression EA, and substituting using (18). This gives the following expression,

$$PCy(t+k) = EBu(t) + Gy(t) + ECe(t+k)$$

Substituting for $y(t+k)$ leads to,

$$C[Py(t+k) + Qu(t) - Rr(t)] =$$

$$(BE + QC)u(t) + Gy(t) - CRr(t) + CEeP(t+k)$$

which on substituting into equation (17) gives

$$\Phi(t+k) = \frac{1}{C}[(BE + QC)u(t) + Gy(t) - CRr(t))] + Ee(t+k)$$

Similar to the Minimum Variance controller [8], the cost function is minimized by setting the first term on the right hand side to zero.

Therefore,

$$(BE + QC)u(t) + Gy(t) - CRr(t) = 0$$

This can alternatively be expressed as,

$$Fu(t) = -Gy(t) + Hr(t) \quad (21)$$

giving the GMV control law as

$$u(t) = -\frac{G}{F} y(t) + \frac{H}{F} r(t) \quad (22)$$

where $F = (BE + QC)$, and $H = CR$

This gives a controller in the form shown in Figure 6.

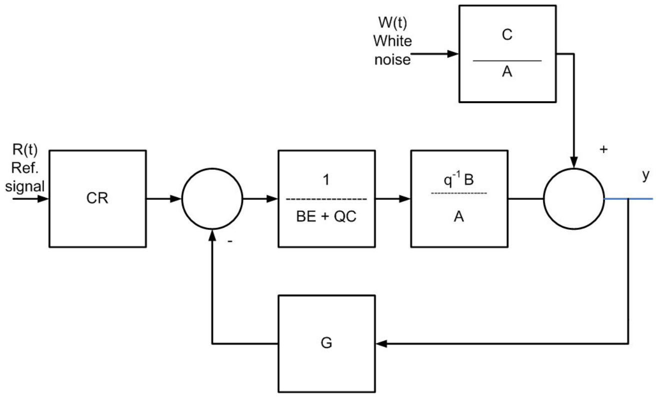

Figure 6: Overview of GMVC applied to the plant.

Table II shows the results of a step response applied to the simulated plant with three different controllers. Two PI controllers are simulated, one with a narrow 1Hz bandwidth representative of the existing controller. A PI controller of a wide bandwidth 12Hz controller is also simulated with a step response and both are compared to the GMVC, by determining the variance of the casting drum speed, y.

Table II. Variance of casting drum speed in steady state.

Controller	Bandwidth	Variance y
PI	1Hz	3.5982
PI	12Hz	3.3338
GMVC		2.4115

VIII. CONCLUSION.

The velocity loop controller of a casting drum drive, used in a polyester film manufacturing process was simulated using different controller options. A comparison was made between the present low bandwidth controller, a wider bandwidth controller and a GMVC. The variance of the casting drum speed after being subjected to a step in current demand was determined, using the three controllers. The GMVC produced the lowest variance in steady state casting drum speed.

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