The Scaling Model for Credit Limit Management

M.J. Konovalikhin, V.V. Kulik, D.O. Sergienko and O.V. Balaeva

***Abstract*—Proper credit limit evaluation plays significant role in credit risk management. This paper introduces a scaling model for optimal credit limit allocation, based on the cause add effect analysis of client's personal data**

***Index Terms*—credit limit, scaling model, scoring, probability of default.**

I. Introduction

One of the consequences of impressive growth of russian retail market is a very strong competition between the banks. This makes Basel Committee on Banking Supervision (Basel II) [1] requirement related to the necessity of development of the modern flexible bank systems for client reliability assessment [2] becomes particularly important. Increased competition and growing pressures for revenue generation have made financial institutions to look for more effective ways to attract new creditworthy customers and at the same time to control the losses. Aggressive marketing efforts have resulted in deeper penetration of the risk pool amongst potential customers. The need to process them rapidly and effectively has initiated growing automation of the credit and insurance application and adjudication processes. Banks need rapid credit decision making process in order to maintain high borrowing power, but on the other hand it should not result in deterioration of portfolio quality. The risk manager is now challenged to provide solutions that not only assess the creditworthiness, but also keep the per-unit processing cost low, while reducing turnaround time for the customers. In addition, customer service quality requires this automated process to be able to minimize the denial of credit to creditworthy customers, while keeping out as many potentially delinquent customers as possible.

In the recent years a particular attention has been paid to risk scoring, which along with other predictive models is a tool evaluating the risk level associated with applicants or customers. While it does not identify "good" (no negative behavior expected) or "bad" (negative behavior expected) applications on individual basis, it provides the statistical odds, or probability, that an applicant with any given score will be either "good" or "bad". These probabilities or scores along with other business considerations, such as expected approval rates, profits and losses are used as a basis for decision making.

When bank decides to provide the loan to a client, decision has to be made on the credit amount and the term. Credit limit is the approved level of a loan amount, which theoretically must represent client's financial rating. In other words, correct calculation of the optimal credit limit is a powerful tool for minimizing the credit losses.

At the same time it should be noted that most of the existing methodologies for credit limit calculation do not take into consideration the explicit assessment of the probability of default. Traditional formula for credit limit calculation does not deal with the credit risk. It is postulated that the limit, calculated by this formula, provides zero credit risk, or, in other words, the probability of default reduces to zero [3]

Some studies looked into possibility of taking into consideration the probability of default while choosing the optimal credit policy by introducing synthetic coefficient [4] or VaR technology [5]. These studies are based on the estimation of a hypothecation value for inter-bank credits, which is not always applicable to retail business.

This paper suggests the scaling methodology for calculation of coefficients, which are required for optimal credit limit estimation. While developing such scaling methodology, three principles have been considered:

First, scaling rationale is based on the goal-oriented parameters. In this case, scaling parameters are determined from credit limit quality perspective.

Second, while seeking the simplicity of the scaling equation, it is required to reduce the (potential) effects of characteristics, involved in calculations, on component phenomena.

Third, equations and correlations that were used in development of the scaling methodology are transparent and can be separately validated.

The method is built on comparison of two competing forces: (i) potential increase of the limit caused by client's positive characteristics such as income, good credit history, ownership of movable and immovable properties etc. and (ii) potential limit decrease, which is a result of the probability of default calculated from the client score. Such model can be related to the quantitative cause-effect model type, which is focused on the final result without consideration of the process' dynamics

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II. ANALYSIS

Existing credit limit calculation methodology, which is currently used at some banks is presented below. Here, the total client income *(D)* is defined as a sum of documented and undocumented incomes:

$$D = DCI + B3 \times UCI \,, \qquad (1)$$

where *B3* is a coefficient showing the presence of some attributes (movable and immovable properties for instance), that could indicate additional income.

Then the *D* value is corrected according to the client's score:

$$BP = D \times B2 - OL \,, \qquad (2)$$

where *B2* is a coefficient, which represents client's rating, *OL* is a total amount of promissory notes. Client's expected income is influenced by a number of dependencies, which is reflected by *B1* coefficient.

$$SD = BP \times B1 \qquad (3)$$

The credit limit is calculated as following:

$$L = \frac{SD \times T}{1 + \frac{R \times T}{12}} \,, \qquad (4)$$

where *T* is the loan term and *R* is the bank interest rate (%))/100.

The weakness of this method is in the fact, that coefficients *B1*, *B2* and *B3* are defined by the expert judgment. Calculation of the limits, which is based on the expert judgement may result in ungrounded over- or understatement of credit limits. In the case of understatement of the credit limits the bank cannot use all the credit resources, that in turn leads to reduced profit. One of the main consequences of credit limit overstatement is the increase in credit risks, which leads to additional losses.

On the other hand, applying precise methods to the credit limit calculation allows to reach full bank credit potential and to maximize bank's profit. Also, one should take into account client's rating (scoring) in order to minimize the risk of default.

A. B3 coefficient calculation

B3 coefficient (equation (1)) can be estimated by examining the dependency between the ratio $\frac{UCI}{DCI + UCI}$, which indicates the share of client's undocumented income in the total income, and the default probability $P(X) = \frac{1}{1 + e^{-\frac{1}{X}}}$, where *X* is the client score. We assume, that this value can grow only with the improvement of client's rating or with the score increase and, respectively, with the decrease of the probability of default:

$$\frac{UCI}{DCI + UCI} = f(P(X)) = C(X) \times \frac{1}{P(X)} \,, \qquad (5)$$

where *C(X)* – a proportional coefficient, which may be independent of the score *X*. In order to assess this dependency a pool of 37890 clients (for one product within certain timeframe) was analyzed. Then a number of clients were selected that had declared *UCI* incomes. From this data two groups were selected - 9681 clients without delinquency and 1334 with delinquency over 30 days.

The first problem, which had to be investigated, was to check for a potential correlation between $\frac{UCI}{DCI + UCI}$ and the presence of delinquency. The comparison of average values in both cases (with and without delinquency) showed that the difference between these two values is about 6%, which can be explained by statistical uncertainty (Fig. 1 and 2). This fact confirms the random nature of $\frac{UCI}{DCI + UCI}$.

With the knowledge of relation (5) it is possible to find the level of undocumented income *UCI*, which statistically corresponds to the documented income *DCI*:

$$(UCI)_{calc} = DCI \times C(X) \times \frac{1}{P(X)} \Big/ \left(1 - C(X) \times \frac{1}{P(X)}\right) = \frac{DCI \times C(X)}{P(X) - C(X)} \,, \qquad (6)$$

First, it was necessary to estimate parameter *C(X)* by using statistical data. For this purpose two regions of possible risk levels (high and low) were selected. The line “good clients average ratio” was chosen as upper conservative boundary because the region, which is located above this line and correspondingly above the line “bad clients average ratio” was considered a high risk zone. The lower boundary was estimated by averaging (dashed line “low risk region”, Fig. 1 and 2) bad client data, which occurs under the line “good clients average ratio”. The region under the line “low risk region” was defined as a low risk region, which contains the values $(UCI)_{calc}$.

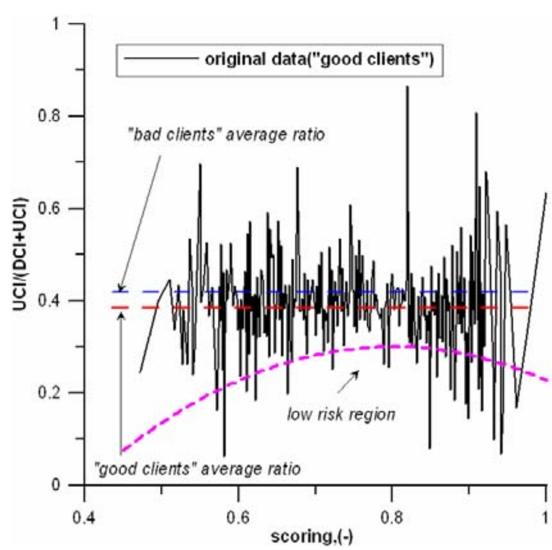

Fig. 1. Dependency of average $\frac{UCI}{DCI + UCI}$ value on the score of the “good” clients.

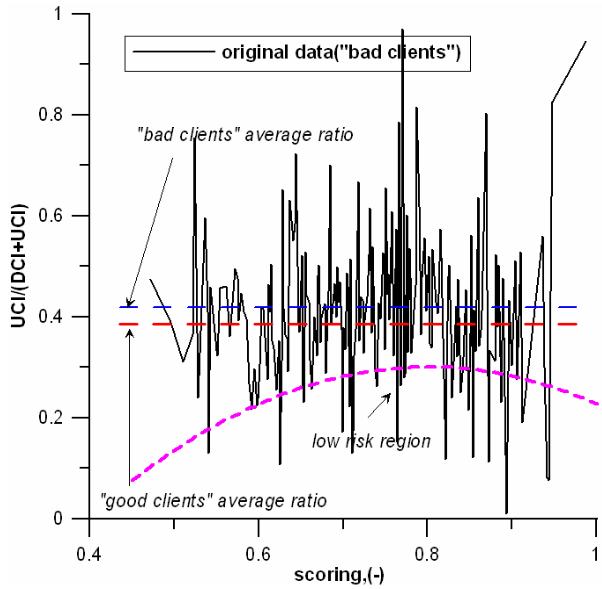

Fig. 2. Dependency of average $\frac{UCI}{DCI+UCI}$ value on the score of the "bad" clients.

Then two alternatives were examined, when (1) applied undocumented income $(UCI)_{appl}$ is less than $(UCI)_{calc}$ and (2) when it is greater than $(UCI)_{calc}$. In the first case the whole UCI amount can be taken into account and be employed for client income calculation or, in other words, $B3 = 1$ in Equation 1. In the second case the *UCI* value can be calculated as a sum of two items: $(UCI)_{calc}$ and some part of the value $[(UCI)_{appl} - (UCI)_{calc}]$:

$$UCI = (UCI)_{calc} + C_3 \times [(UCI)_{appl} - (UCI)_{calc}], \quad (7)$$

During the next step the coefficient C_3, which demonstrates the optimal degree of confidence in the client, should be estimated. Fig. 3 shows the difference between real client undocumented income *UCI* and calculated values $(UCI)_{calc}$.

One of the possible solutions is to use the cause-effect relations, which have been successfully applied to quantitative analysis of various technical problems [0]. Such equations provide solution to the problem without describing process' dynamics. The result in this case is the ratio between positive and negative characteristics, that are represented by client's financial condition, such as movable and immovable properties etc in the numerator and the default probability *P(X)* in the denominator:

$$C_3 = C_{3,corr} \times \frac{\sum_i A_i Z_i}{P(X)}, \quad (8)$$

where $C_{3,\,corr}$ is the coefficient, which correlates equation (8) to statistical data, Z_i - parameters of client financial condition, A_i – regressive coefficients, that are calculated by the *Interactive Grouping* methodology. In order to estimate C_3 the following parameters were used: cost of movable properties (car, yacht etc.), country house, apartment and business shareholding. Results of the calculation are presented in Fig. 4.

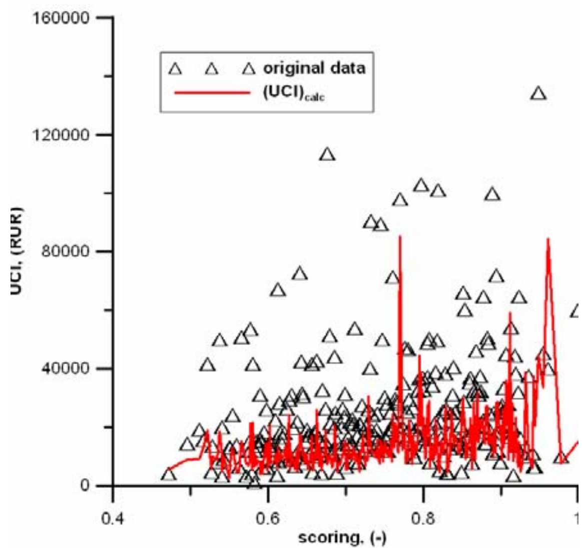

Fig. 3. Comparison of calculated UCI values and original data

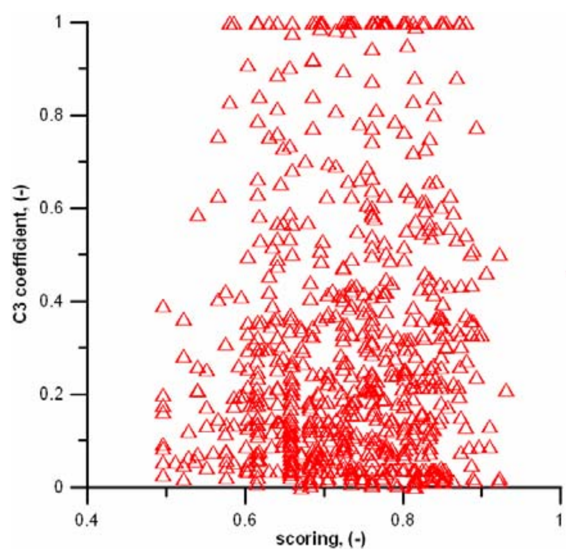

Fig. 4. Distribution of C3 coefficient.

B. B2 coefficient calculation

Similar approach could be applied for calculation of *B2* coefficient in equation (2). Here again we compare two competing parameters through their ratio. The upper parameter, which is responsible for potential limit increase, is represented by a regressive sum of characteristics such as documented income, occupation, employment position etc. The lower parameter, which is pushing down the credit limit, is the probability of default *P(X)*.

$$B2 = C_{2,corr} \times \frac{E'}{P(X)}, \quad (9)$$

where

$$E' = \sum_i D_i E_i \;, \quad (10)$$

where E_i are values of the positive characteristics, D_i are regressive coefficients, and $C_{2,corr}$ is a correlation coefficient between calculated values and statistical data range.

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In order to estimate *B2* coefficient two samples were chosen - 33664 clients with good credit histories and 4226 clients with delinquency over 30 days. The *B2* coefficient distribution, which is based on an expert judgment, is shown on the Fig. 5. Then specific weights of "good" and "bad" clients in each score range were determined. The comparison of these values indicates that they are almost equal and this fact attests to independency of *B2* distribution on the client quality. Also Fig. 5 shows that this expert distribution covers a small part of possible *B2* values, which may occur between 0.6 and 1 ordinate values and 0.47 and 1 abscissa values.

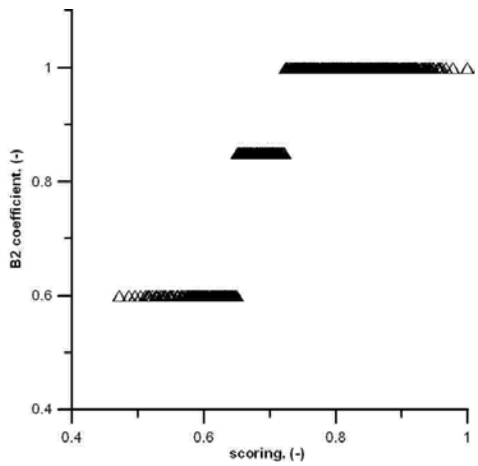

Fig. 5. Expert B2 coefficient distribution.

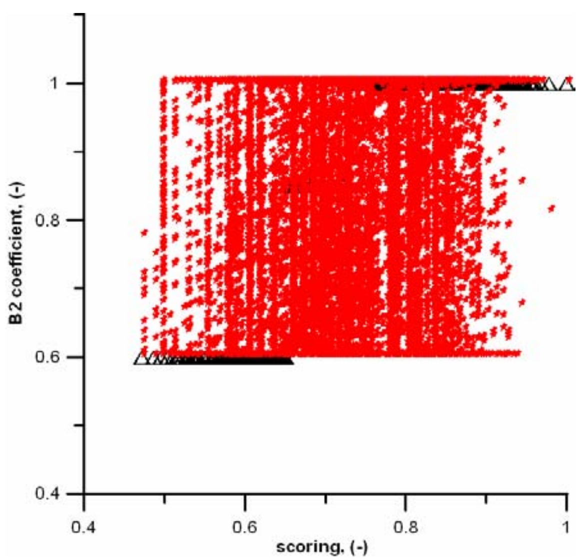

Fig. 6. Calculated B2 coefficient distribution.

Fig. 6 demonstrates the results of (9), where red points show *B2* values calculated for clients from the "good" sample. Each score corresponds to a number of *B2* values, which are determined by the personal characteristics of each client.

C. B1 coefficient calculation

In order to find *B1* coefficient in equation (3) it would be natural to use *P(X)* value, which reflects client's reliability:

$$B1 = C_{1,corr} \times \frac{BP}{FM \times P(X)} , \tag{11}$$

where *FM* is a number of dependencies, *BP* is the income, calculated from Equation (2), $C_{1,corr}$ is the coefficient, which represents the correlation between this ratio and statistical data.

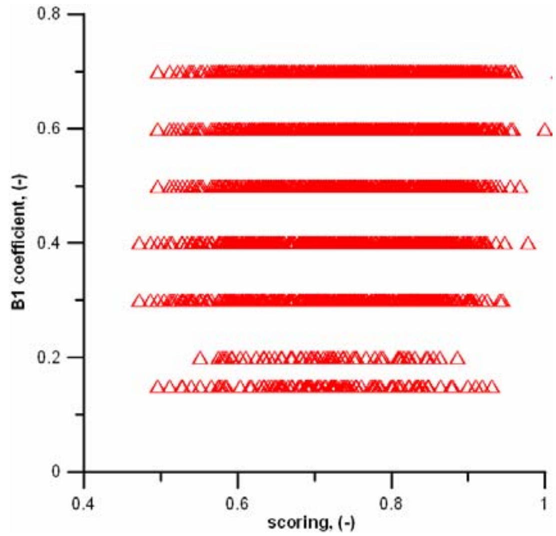

Fig. 7. Expert B1 coefficient distribution.

As shown on Figures 7 and 8 *B1* calculation through equation (11) provides mapping of discrete values (Fig. 7) to the value field (Fig. 8), demonstrating individual approach to each client.

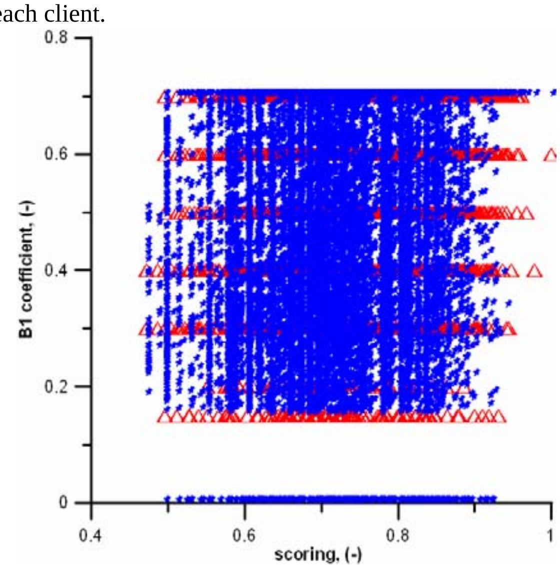

Fig. 8. Calculated B1 coefficient distribution.

D. Analysis results

Based on the described approach the credit limits for all clients from the samples were recalculated. The results are summarized in Table 1.

Table 1 Average limit change

The score, X	**X < 0.6**	**0.6 ≤ X < 0.8**	**0.8 ≤ X ≤ 1**
Average limit change, %	+38.6	+14.3	+7.9

From the analysis results the following observations can be made:

- Total increase of the sample portfolio was +14.4% while maintaining the initial risk level. Calculations were performed in relative values.
- Number of clients, that had undocumented income*(UCI)* in the low risk region $(UCI)_{calc}$), was 32.3 % of the total number of clients, that had undocumented income.
- Table 1 shows that the maximum average increase was obtained in the low score range . This can be explained by the fact that clients in the range were undervalued.
- It should be noted that all *B* coefficients can be adjusted by the corresponding coefficients *C*, that depend on the product statistical data.
- Because of the normal score distribution most clients have scores in the range $0.6 \le X < 0.8$. This explains the fact that the average changes in this range and in the total portfolio are very close.
- The model enabled to substantially increase (Fig. 9) the credit portfolio (about + 17%) while maintaining or improvement (Fig. 10) its quality.

III. Conclusions

`This paper presented the scaling model that can be applied to the calculation of the optimal credit limit. Particular attention has been paid to this problem due to growing competition between the banks in the retail market, that leads to the increase in credit risk. Correct credit limit calculation plays one of the key roles in risk reduction.

The model described in the paper is based on comparison of two opposite client characteristics, one leading to potential credit limit increase, and the second leading to decrease of the credit limit. Credit limit calculation involved client 's personal data, which allows to approach each client on the individual basis during credit amount allocation. The scaling method was applied in order to analyze the data obtained from the real client database. The scaling ratio provides reasonable predictive capability from the risk point of view and therefore has been proposed to serve as a model for credit limit allocation. The model's flexibility allows coefficients' adjustments according to new statistical data. Although the present work is quite preliminary, it does indicate that presented solution allows to substantially increase the credit portfolio while maintaining its quality.

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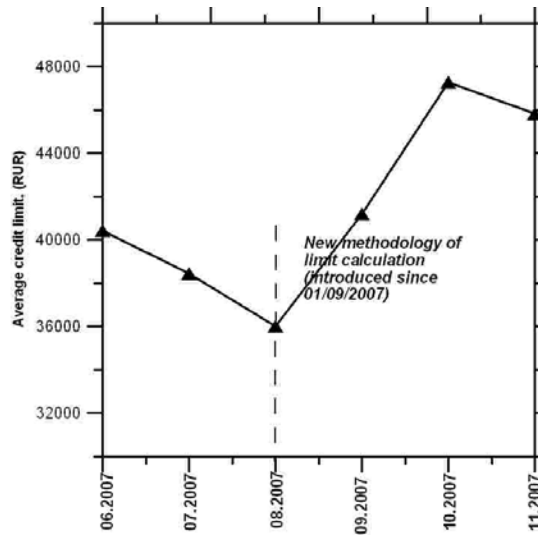

Fig. 9. The average credit limit dynamic of one product (the results are calculated at the end of each month).

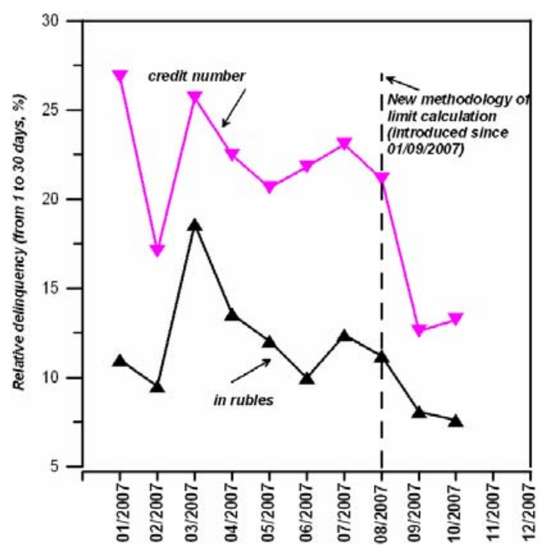

Fig. 10. The relative delinquency (from 1 to 30 days) level dynamic of one product (the results are calculated at the end of each month).

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