Fuzzy Iterative Learning Control in 2-D Systems

A. Argha, P. Karimaghaee, M. Roopaei, M. Zolghadri Jahromi

Abstract- **In this paper, we discuss the application of Iterative Learning Control (ILC) algorithms in two dimensional problems. We propose five new algorithms for control of 2-D systems. In all of these algorithms, the control signal is calculated based on control signal at last iteration plus a corrective term. The difference between various algorithms presented is the way the mentioned corrective term is calculated. The performance of various algorithms was assessed in simulation environment. The results of simulations show that our proposed Fuzzy Iterative Learning Control (FILC) can reduce the trajectory error in far less number of iterations. This can be due to the fact that FILC is providing a nonlinear mapping while all others are linear**.

I. INTRODUCTION

Most industrial systems and machines perform their tasks repetitive in nature. Such industrial systems include robots, motors, steel mills, hard-disk drives, and many more. These systems are required to carry out tasks with precision and high speeds subject to modeling variations, disturbances and repeatability imperfections. Higher quality of their performance in accuracy and efficiency can enhance product quality, increase productivity, improve efficiency and reduce costs.

Iterative learning control (ILC) and repetitive control (RC) aim to improve control system performance against the lack in knowledge of system modeling, disturbances and initial offsets. They have capabilities in learning through iterations and are feed-forward control techniques based on tracking errors. They compensate the limitations of feedback control design in the deficient knowledge of modeling errors, disturbances and initial offsets.

The subject of controlling the iterative process has taken into consideration a lot by investigators since these two recent decades, it has achieved good improvements in both theoretical field, and utilization field as it is became one of the specialty majors of control science. Interested readers can refer to the book [1] and the articles [2,3] to achieve more information about formation and improvement of this subject.

Many researches and studies have been done at the field of iterative learning control up to now, are specified to the one-dimensional system. Many cases in the nature are function of two independent variables, therefore for modeling these phenomena, two-dimensional (2-D) signals, and systems are used.

Some of the basis usage of two-dimensional systems is large-scale systems, heat transfer, image processing, biologic systems, processing earthquake signals, sonar etc. The usage of industrial systems contain of some subjects like web-forming process, metal forming equipments, exploited system of mines etc. Suitable usage index of iterative learning control algorithm causes this motive, which is also use in controlling the two-dimensional systems. The two-dimensional systems and their representation forms are discussed more in articles [4-8].

As a model free design method, Fuzzy Logic Control (FLC) has been successfully applied to control complex or ill-defined processes whose mathematical models are difficult to obtain. The ability of converting linguistic descriptions into an automatic control strategy makes it a practical and promising alternative to classical control schemes.

In this paper, we propose an FILC strategy that makes use of FLC and ILC schemes for control of 2-D systems. In this scheme, a fuzzy system is used to update the controller in each iteration of ILC.

The rest of this paper is organized as follow. In section II, the problem formulation for 2-D systems is introduced. In section III, various ILC schemes for control of 2-D systems are presented. In section IV, the effectiveness of various schemes is examined by simulation. Section V concludes this paper.

II. CONTROL OF 2-D ITERATIVE PROCESSES

In ILC, update of the controller is based on the performance of the controller in previous iterations. The update law, which is usually denoted as "learning" will cause the performance of the system to improve over several iterations. This will also result in stability of the system.

In control of 2-D iterative processes, we are dealing with systems that their dynamic is constructed using three independent variables. Two of these represent dimension (*i,j*) and the third represents the iteration number (*k*). A linear iterative 2-D system with large-scale time can be formally described as [4]:

$$\begin{aligned} &x_k(i+1,j+1)=A_1x_k(i,j)+A_2x_k(i+1,j)+A_3x_k(i,j+1)+Bu_k(i,j) \\ &y_k(i,j)=Cx_k(i,j) \\ &i=0,1,\ldots,M\ ,\ j=0,1,\ldots,N\ ,\ k=0,1,\ldots, \\ &x_k(i,j)=x_0\quad for\ (i\ or\ j\ )=0 \end{aligned} \tag{1}$$

In which i and j show the independent 2-D variables and k shows the number of iteration, $x\in R^n$, $y\in R^q$, $u\in R^p$ are

Mehdi Roopaei is with the Dept. of Computer Science and Eng., Shiraz University, (corresponding author, Post Box: 71955-177 Shiraz-Iran, phone: +98-711-6309641 mobile: +98-9177137538, Mehdi Roopaei, e-mail: mehdi.roopaei@gmail .com).

Ahmadreza Argha is with the Dept. of Electrical Eng., Shiraz University, Zand street, Shiraz, Iran (e-mail: rezaargha@yahoo.com).

Paknoosh Karimaghaee is with the Dept. of Electrical Eng., Shiraz University, Zand street, Shiraz, Iran(e-mail: kaghaee@shirazu.ac.ir).

Mansoor zolghadri is with the Dept. of Computer Science and Eng., Shiraz University, Zand street, Shiraz, Iran, (e-mail: zjahromi@shirazu.ac.ir).

ISBN:978-988-98671-9-5

state vector, input and output vector, respectively. The matrices A_1, A_2, A_3, B, and C are real and have suitable dimensions. M and N represent bound of each dimension. and x_0 is the boundary condition in iterations, which is supposed to be unknown. The ultimate goal of ILC in controlling 2-D systems is to determine $u_K(i, j)$ such that the following relation is satisfied.

$$\lim_{k\to\infty}\{y_d(i,j)-y_k(i,j)\}=0 \;, i=0,1,\ldots M,\; j=0,1,\ldots N \tag{2}$$

III. ILC SCHEMES FOR 2-D SYSTEMS

In the following, we present five ILC algorithms for control of 2-D systems

a. A 1-D ILC

The controller is updated based on extending the classical ILC (for 1-D systems) to the case of 2-D systems. For process (1), the controller is updated using the following relation.

$$u_{k+1}(i,j)=u_k(i,j)+\Delta u_k(i,j) \tag{3}$$

Where $\Delta u_k(i,j)$ is constructed by:

$$\Delta u_k(i,j)=q\times e_k(i+1,j+1) \tag{4}$$

In which:

$$e_k(i,j)=y_d(i,j)-y_k(i,j) \tag{5}$$

where $i=0,1,\ldots, M$, $j=0,1,\ldots, N$.

As it observed in the above relations, tracking error at point $(i+1,j+1)$ is used to generate $\Delta u_k(i,j)$. The learning rate q, which is held constant in this paper, can have an impact on convergence speed. In general, q is chosen by try and error such that the tracking error converges to zero.

b. A 2-D ALGORITHM

The corrective term in this scheme uses error at $(i+1,j)$, $(i,j+1)$ and $(i+1,j+1)$. The corrective term is defined as:

$$\Delta u_k(i,j)=q_1 e_k(i+1,j+1)+q_2 e_k(i+1,j)+q_3 e_k(i,j+1) \tag{6}$$

This algorithm has some advantages over the scheme presented in (4). Using three learning rates (q_1, q_2 and q_3) in (6) gives more degrees of freedom and suitable choice of these parameters can improve the convergence speed.

c. A 2-D PID OVER DIMENSIONS

The traditional *P*, *PD*, *PI* and *PID* algorithms have been used as ILC strategies to control 1-D systems. In the following, a *PID* algorithm for 2-D systems is defined as:

$$\Delta u_k(i,j)=Kpe_k(i+1,j+1)+K_I\sum_{m=1}^{i+1}\sum_{n=1}^{i+1}e_{k-1}(m,n) \tag{7}$$
$$+K_D\{e_k(i+1,j+1)-e_k(i,j)\}$$

As seen, the corrective term is constructed using error at the point $(i+1,j+1)$, summation of error at all previous points and also the difference in error between the points $(i+1,j+1)$ and (i,j).

d. A 2-D PID OVER ITERATIONS

In this scheme, the corrective term is defined as the sum of following three terms:

a) Tracking error at k^{th} iteration.
b) Summation of tracking error at all previous iterations.
c) The difference in tracking error at point $(i+1,j+1)$between k^{th} and $(k-1)^{th}$ iterations.

The corrective term is formally defined as:

$$\Delta u_k(i,j)=Kpe_k(i+1,j+1)+k_I\sum_{m=1}^{K}e_m(i+1,j+1) \tag{8}$$
$$+k_D(e_k(i+1,j+1)-e_{k-1}(i+1,j+1))$$

e. TWO-DIMENSIONAL FUZZY ITERATIVE LEARNING CONTROL

In this scheme, a fuzzy system is used to generate the corrective term. In general, fuzzy systems can provide nonlinear mapping between its inputs and outputs.

The tracking error $e(i+1,j+1)$ and its variation $\Delta e(i+1,j+1)$ were used as input to fuzzy system giving u_{IFLC} as its output. Variation of tracking error was defined using the following scanning methods:

1- Row Scanning:

$$\Delta e(i+1,j+1)=e(i+1,j+1)-e(i,j+1)$$

2- Column Scanning:

$$\Delta e(i+1,j+1)=e(i+1,j+1)-e(i+1,j)$$

The fuzzy system uses zero order *Tackagi-Sugeno* rules. Seven fuzzy sets were used to partition each input using triangular fuzzy sets expressed as Negative Big (*NB*), Negative Medium (*NM*), Negative Small (*NS*), Zero (*ZE*), Positive Small (*PS*), Positive Medium (*PM*) and Positive Big (*PB*). Similarly, seven singleton fuzzy sets were assigned to output. The fuzzy rule table was designed as in Table 1. The membership functions of input and output fuzzy sets are shown in Fig. 2. The fuzzy iterative learning controller (FILC) can be represented as:

$$\Delta u_{FILC}(i,j)=k_{FILC}FILC(e(i+1,j+1),\Delta e(i+1,j+1)) \tag{9}$$

TABLE 1.The rule-base of FILC

u_{FILC}		*e*						
		NB	NM	NS	ZE	PS	PM	PB
Δe	PB	NB	NB	NB	NB	NM	NS	ZE
	PM	NB	NB	NB	NM	NS	ZE	PS
	PS	NB	NB	NM	NS	ZE	PS	PM
	ZE	NB	NM	NS	ZE	PS	PM	PB
	NS	NM	NS	ZE	PS	PM	PB	PB
	NM	NS	ZE	PS	PM	PB	PB	PB
	NB	ZE	PS	PM	PB	PB	PB	PB

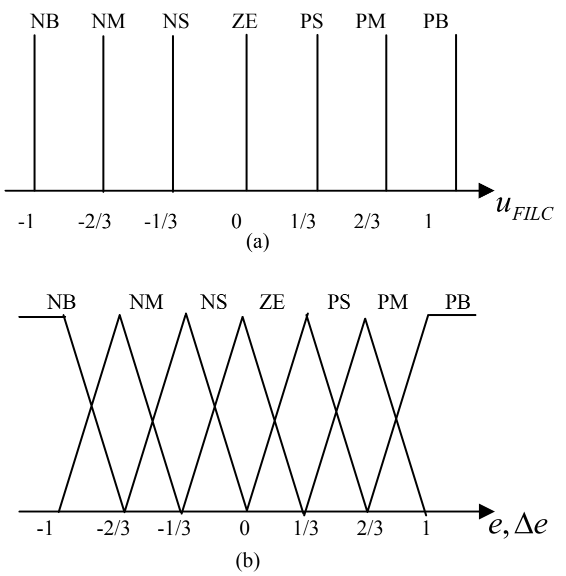

Fig. 2. Membership functions of (a) output fuzzy sets, (b) input fuzzy sets

IV. Simulation Results

We applied the discussed methods for controlling a 2-D system. The system used in simulation is an F-M two-dimensional model with parameters $A_1=-0.2$, $A_2=-0.1$, $A_3=-0.1$, $B=1$ and $C=1$. The desired output of system $y_d\,(i, j)$ was selected as:

$$y_d\,(i, j) = \sin(i+0.2j),\quad 1 \le i \le 10, \quad 1 \le j \le 10$$

The desired trajectory $y_d\,(i, j)$ is shown in Fig. 3.

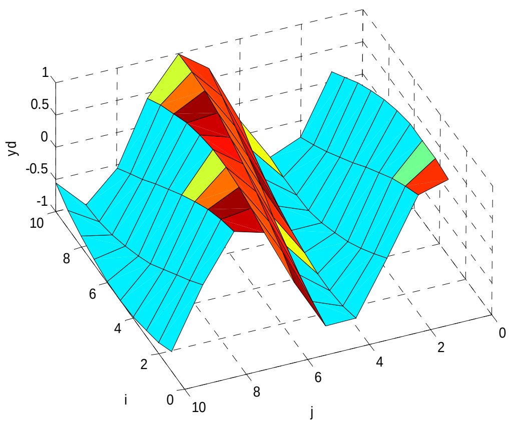

Fig.3. The desired trajectory of 2-D system.

By setting the initial value of the controller to one, we used various algorithms presented in this paper for the mentioned task. The parameters of various schemes were selected as: $q=1.2$ $q_1=1$, $q_2=0.17$ and $q_3=0.17$. In the *two-dimensional PID algorithm* the coefficients are considered as $K_D= 0.005$, $K_I= 0.007$ and $K_P=1$ also at the algorithm *PID on the iterations algorithm* $K_D = 0.2$, $K_I = 0.2$ and $K_P= 1$.
The second norm of tracking error for various shemes is given in Fig. 4. As seen, all schemes are capable of reducing the norm of tracking error to zero.

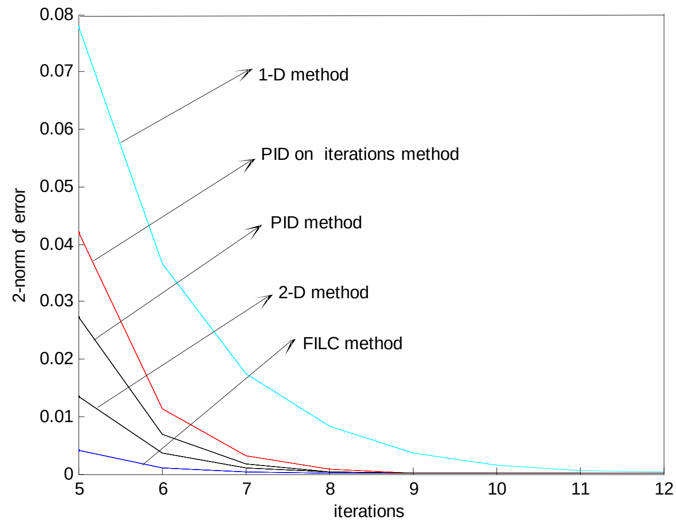

Fig 4. The 2-norm of the tracking error for presented algorithms (k= 5 - 12)

It can also be seen that the performance of FILC is much better than other schemes. This is mainly because the FILC is providing a nonlinear mapping.

V. Conclusion

In this paper we proposed five different ILC methods to control *2-D* systems. Simulation results showed that the convergence speed of *FILC* method is much better than all other schemes. This can be due to the fact that FILC is providing a nonlinear mapping while all others are linear

References

[1] Y. Q. Chen and C. Wen, Iterarive Learning Control: Convergence, Robustness and Applications, Berlin: Springer-Verlag, 1999.
[2] A. Tayebi, " Adaptive iterative learning control for robot Manipulators ", in Proceeding of the 2003 American Control Conference , May 2003.
[3] D. H. Owens, and K. Feng, " Parameter optimization in iterative learning control", International Journal of Control, vol. 76, no. 11, pp. 1059-1069, 2003.
[4] W. Fornasini & Marchesini, "State space realization theory of 2-D filters " IEEE Trans. Aut. Contr., Vol. 21, Aug. 1976.
[5] W. A. Porter & J. L. Aravena, "1-D model for M-D processes " IEEE, Trans. Ckt. & Systems, Vol. 31, pp. 232-239, Aug. 1984.
[6] P. E. Wellstead & W. P. Heat, " Two-dimensional control systems:Application to the CD and MD control problems" Pulp & Paper Canada 95, No. 2, pp. T75-T78, 1995.
[7] W. A. Porter and J. L. Aravena, " State estimation in discrete M-D systems" IEEE Trans. Automat. Contr., Vol. 31, pp. 833-841, Mar. 1986.
[8] J. L. Aravena & M. Shafiee & W. A. Porter, " State models and stability for 2-D filters " IEEE Trans. Circuit & Systems, Vol. 37, No. 12, pp. 1509-1519, Dec. 1990.
[9] Driankov D, Hellendoorn H, Reinfrank M. An introduction to fuzzy control. New York: Springer-Verlag; 1993.

ISBN:978-988-98671-9-5