

A Fourier Extension Based Algorithm for Impulse Noise Removal

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Abstract—In this paper a novel Fourier extension based algorithm is introduced which is able to remove impulse noise from corrupted images faster than conventional algorithms. This algorithm can function faster while it preserves image details. This algorithm uses two phases, at first phase an impulse detector finds corrupted pixels and at the second phase a noise cancellation algorithm removes salt and pepper noise from the image. The results show that the proposed method is able to restore noisy pixels better than conventional algorithms and its restoration time is less than other methods making it a useful algorithm for real-time applications.

Index Terms — Image Enhancement, Impulse Noise, Nonlinear Filter.

I. INTRODUCTION

Images are usually contaminated by impulse noise while they are transmitted through communication channels or due to noisy sensors. The effect of an impulse noise on image pixels is so that the value of a pixel becomes a lot more or less than the pixels its neighborhood. It's important to remove impulse noise before subsequent processing, such as edge detection, image segmentation and object recognition. So far different techniques are proposed to eliminate impulse noise [1]-[6], [7]-[10].

Median-based filters were amongst the first methods which attracted the attention because of their capability of preserving image edges and also because of their simplicity [5], [6]. Of course, the typical median-based filters are implemented uniformly on the image and are inclined to modify both noisy and good pixels. One way to improve this situation is to use the weighted median filter [7]-[10], an extension of the conventional median filter, which gives more weight to some values within the window around noisy pixel. It emphasizes or de-emphasizes some pixels because in most applications, some of the pixels are more important than the others. Center-weighted median (CWM) filter is the special case of the median filter [11], which gives more weight to only the central pixel of the window. It is also reasonable to give importance to the central pixel, because it is the one that has the most correlation with the desired estimate. In some recently published works a better method is introduced to enhance the restoration process which includes

a switching algorithm [2], [12]. This switching algorithm is in a way that the noisy and good pixels are separated so that the filtering algorithm changes only noisy pixels, leaving good ones unchanged. This method of filtering is called “*impulse detector*”.

In [1], a median-based switching filter, called progressive switching median (PSM) filter, is proposed where both the impulse detector and the noise filter are applied progressively in iterative manners. But due to its iterative manner the execution time will be noticeable and it can not be used in real-time applications. In [3], an algorithm based on long-range correlation (LRC) is introduced. This method aims to increase the range seen by conventional methods to enhance image restoration rate. Due to the fact it looks for the most similar pixel in a long range around noisy pixel, it will be necessary that the image have the property like texture. So this method will not be able to restore all kinds of images specially where there is no similar pixel to the noise disturbed one. The simulations show that this method takes a long time to finish the filtering process together with producing a poor filtered image. In [13] an adaptive rank-ordered mean (AROM) filter is proposed that employs the switching scheme based on the two-stage impulse detection mechanism. The objective is to utilize the rank-conditioned median (RCM) filter [14, 15] and CWM filter [11] to define more general operators. In the first stage of impulse detection scheme, the RCM mechanism sees if the central sample lies outside the trimming range and how much small or big the central pixel is in comparison with other pixels that lie within the trimming range in the window. In the second stage of impulse detection scheme, the CWM mechanism with variable center weights is used to decide the values of local thresholds in the sliding window. The ultimate output is switched between the current pixel itself and the rank-ordered mean of two central ranks of the surrounding pixels in the window.

Reference [16] introduces a two-phase scheme for removing salt-and-pepper impulse noise. In the first phase, an adaptive median filter is used to identify pixels which are likely to be contaminated by noise and in the second phase, the image is restored using a specialized regularization method that applies only to those selected noise candidates.

In this paper a new filtering procedure is introduced which not only removes the impulse noise from images to a great extent but also reduces the filtering time. These features make this method to be applicable to real-time applications. This algorithm works in two phases: first an impulse detector finds and flags the noise disturbed pixels and then, considering the flagged pixels, a filtering algorithm restores the noisy pixels to their original values. This algorithm works in a way that image details are preserved while impulse noise is almost fully removed. The simulation results show the

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excellence of the proposed method in comparison with some conventional algorithms over different images.

Structure of rest of the paper is as follows: in section 2, impulse detection algorithm is described. Section 3 studies noise removal algorithm. Implementations and simulation are shown in section 4. Exhaustive conclusions and discussions are given in section 5.

II. IMPULSE DETECTION

Similar to other impulse detection algorithms [1-4], this impulse detector is developed by some prior information on natural images, *i.e.*, a noise-free image should be locally smoothly varying and be separated by edges. The noise considered in this paper is only salt-and-pepper impulsive noise which means:

- 1) Only a proportion of all the image pixels are corrupted while other pixels are noise-free and
- 2) A noise pixel takes either a very large value as a positive impulse or a very small value as a negative impulse.

Let x_{ij} and $x_{ij}^{(new)}$ represent the pixel values at position (i,j) in the corrupted image and the restored image, respectively. The impulse detector generates a binary flag map where each pixel (i,j) is given a binary flag value, f_{ij} , indicating whether it is considered as an impulse; *i.e.*, $f_{ij} = 1$ means the pixel in position (i,j) is a corrupted pixel and $f_{ij} = 0$ means the pixel in position (i,j) is noise free. In *Sun and Neuvo's switching I scheme* they used the difference between the pixel value itself and the median value of a local window centered about it as the measurement to detect impulses. The impulse detection algorithm used here is a modified version of Sun and Neuvo's method.

Two image sequences are generated during the impulse detection procedure [1]. The first is a sequence of gray scale images, $\{\{x_{ij}^{(0)}\}, \dots, \{x_{ij}^{(n)}\}, \dots\}$, where the initial image $\{x_{ij}^{(0)}\}$ is the noisy image to be detected, $x_{ij}^{(0)}$ denotes the pixel value at position (i,j) in the initial noisy image and $x_{ij}^{(n)}$ is the same pixel value after n^{th} iteration. The second is a binary flag image sequence, $\{\{f_{ij}^{(0)}\}, \dots, \{f_{ij}^{(n)}\}, \dots\}$, where the binary value $\{f_{ij}^{(n)}\}$ is used to indicate whether the pixel at position (i,j) has been detected as an impulse, *i.e.*, $f_{ij}^{(n)} = 0$ means the pixel at position (i,j) is good pixel and $f_{ij}^{(n)} = 1$ means it has been found to be an impulse. Before the first iteration, it is assumed that all the image pixels are good, *i.e.*, $f_{ij}^{(0)} = 0$. In the n^{th} iteration ($n = 1, 2, \dots$) for each pixel $x_{ij}^{(n-1)}$, the median value of the samples is found in a $W_D \times W_D$ (W_D is an odd integer not smaller than 3) window centered about it ($m_{ij}^{(n-1)}$). The difference between $m_{ij}^{(n-1)}$ and $x_{ij}^{(n-1)}$ provides a simple measurement to detect impulses. At first it is supposed that the image is noise free. So before the impulse detection starts $f_{ij}^{(0)}$ will be equal to zero.

$$f_{ij}^{(n)} = \begin{cases} f_{ij}^{(n-1)} & \text{if } |x_{ij}^{(n-1)} - m_{ij}^{(n-1)}| < T_D \\ 1 & \text{else} \end{cases}$$

where T_D is a threshold value.

When a pixel is found to be impulse noise disturbed, the value of $x_{ij}(n)$ is modified as follows:

$$x_{ij}^{(n)} = \begin{cases} m_{ij}^{(n-1)} & \text{if } f_{ij}^{(n)} \neq f_{ij}^{(n-1)} \\ x_{ij}^{(n-1)} & \text{else} \end{cases}$$

If impulse detection algorithm stops after N_D^{th} iteration there will be two output images, $x_{ij}^{(N_D)}$ and $f_{ij}^{(N_D)}$.

It should be mentioned that the impulse detection measurement used here is first introduced by *Sun and Neuvo* in their *switch I scheme* [2].

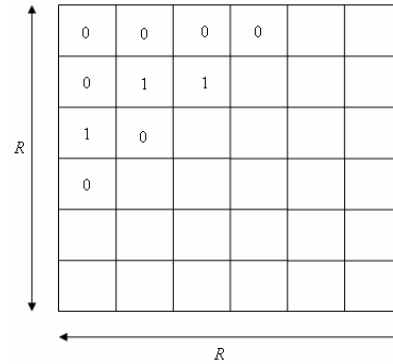


Fig.1 A binary flag matrix, the same size as the original image which is $R \times R$ pixels, used for impulse detection

III. NOISE FILTERING

To develop the algorithm proposed in this paper, following two points need to be considered: (a) as mentioned before it is assumed that the noise free image varies locally smooth and (b) a low frequency sinusoidal signal varies locally smooth too. The similarity of these two points leads to a new modeling of the image from signal processing point of view. In this paper, it is assumed that each $W_F \times W_F$ window of noise free image is a sinusoidal signal which has W_F components and varies smoothly. That is, the pixel values in a $W_F \times W_F$ window are assumed to be a sinusoidal signal like

$$f(t) = A_0 + A_1 \cos(\omega t) + A_2 \sin(\omega t) + A_3 \cos(2\omega t) + \dots$$

where the coefficients A_i ($i=0, 1, \dots, W_F$) are obtained from the pixel values in the window. This looks like the Fourier extension of a signal. The properties of the Fourier extension will be used to restore the noisy pixel values. For example in a noise free 3×3 window there will be maximum 9 coefficients, named A_i ($i=1, 2, 3$). Fig.2 gives a better sense of these coefficients.

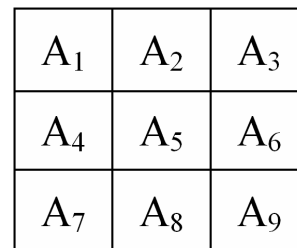


Fig.2 A typical 3×3 window

Fig. 3a shows a sample 3×3 window around a noisy pixel which is produced by impulse detection algorithm (flag

0	1	0
0	1	0
0	0	0

a

A_1		A_3
A_4		A_6
A_7	A_8	A_9

b

Fig. 3 Coefficients in a sample window around a noisy pixel used for restoration: (a) a sample flag matrix (b) the corresponding image pixels

matrix). The pixels corresponding to this flag matrix is shown in Fig. 3b. These pixels are noise free pixels which will be used to restore noise disturbed pixels.

As is known the Fourier extension of a signal has the *RMS* value of the following form:

$$RMS(f(t)) = \sqrt{A_0^2 + \frac{A_1^2 + A_2^2 + \dots + A_n^2}{2}}$$

where A_0 is the DC term and n is the number of coefficients.

Due to the fact that the information of the window around noisy pixel is modeled in a sinusoidal polynomial form, the *RMS* of the pixel values in the window is calculated and is then restored with the value of noisy pixel. Because only the noise free pixels are used as a source to estimate corrupted pixels, A_0 will be the DC term of only good pixels that can be calculated as:

$$A_0 = \frac{1}{n+1} \sum_{i=1}^n A_i$$

where n is the number of good pixels in the window.

So in the algorithm proposed in this paper a $W_F \times W_F$ sized window is assumed around a noisy pixel of position (i,j) and then the *RMS* value of the assumed sinusoidal polynomial is calculated using only good pixels. Afterwards the value of impulse pixel is changed to the calculated *RMS* value:

$$x_{ij}^{(new)} = RMS(f(t))$$

IV. IMPLEMENTATION AND SIMULATIONS

In our experiments the original test images were corrupted with fixed valued salt and pepper impulse noise, where the corrupted pixels take on the values of either 0 or 255 with equal probability. *Mean Square Error (MSE)* and *Peak Signal to Noise Ratio (PSNR)* are used to evaluate the performance of restoration algorithm. *MSE* is defined as:

$$MSE = \frac{1}{N} \sum_{i=1}^3 \sum_{j=1}^3 (O_{ij} - t_{ij})^2$$

Where N is the total number of pixels in the image while O_{ij} and T_{ij} are the pixel values at position (i,j) in the original image and test images, respectively. *PSNR* is defined as:

$$PSNR = 10 \times \log_{10} \frac{255^2}{\left(\frac{1}{r^2}\right) \sum_{i=1}^r \sum_{j=1}^r (O_{ij} - T_{ij})^2}$$

where r is the size of the image and O_{ij} and T_{ij} are the pixel values at position (i,j) in the original image and test images, respectively. For instance in a 512×512 image r will be 512. To implement the algorithm four parameters must be

predefined. These parameters are the filtering window size, W_F , the impulse detection window size, W_D , the impulse detection iteration number, N_D , and the impulse detection threshold, T_D .

Due to experiments in [1], the best restoration results are obtained at values $W_F=3$, $W_D=3$, $N_D=3$ and $T_D=40$.

V. CONCLUSIONS AND DISCUSSIONS

In this paper a new algorithm for impulse noise removal was introduced. This algorithm has two phases: in the first phase an impulse detector detects the noisy pixels and then, at the second phase, the noise cancellation algorithm removes salt-and-pepper impulse noise from the images. The impulse detection algorithm is a modified version of *Sun* and *Neuvo's Switch I scheme*. Considering the properties of real noise free images and also the properties of the Fourier extension, the image is partly modeled in a sinusoidal polynomial manner. Then, using the properties of Fourier extension, the *RMS* value of noise free pixels are calculated and is replaced with the values of noisy pixels in each filtering window. The proposed algorithm is very fast and easy to implement. This is because there no need to any differentiation as used in [4] and also unlike some of conventional algorithms proposed in [3] does not need a large number of pixels to approximate the real value of noise disturbed pixels. Fig. 4 shows the results obtained by applying four different algorithms on "Lena" image with 25% impulse noise added to it. To demonstrate the efficiency of the proposed method in this paper, it is compared with three of the best algorithms introduced in [1], [3], [4]. Considering their results, compared with different methods, it can be concluded that these are among the best methods developed until now. So the comparison of the proposed method in this paper with these three algorithms will be meaningful.

The mentioned three algorithms are as follows: 1. Polynomial Approximation (PA) Method 2. Long Range Correlation (LRC) Method and 3. Progressive Switching Method (PSM)

Fig. 5 and Fig.6 show the *MSE* and *PSNR* values for different impulse noise values from 5% to 50%.

According to Fig. 5 and Fig. 6 it can be seen that the new algorithm proposed here in this paper has acceptable *MSE* and *PSNR* when compared with other conventional method. Fig. 7 shows the execution times of these four algorithms in compare with each other. According to Fig. 7 it can be seen that the introduced algorithm has the less execution time which together with acceptable *MSE* and *PSNR* values make it suitable for real-time applications.

In order to have a closer look on how good the proposed algorithm works, *MSE* and *PSNR* values for different noise values are gathered in table 1 and 2. Regarding these tables it is seen that the proposed algorithm produces acceptable *MSE* and *PSNR* values when compared with other methods. But the most efficient aspect of the proposed method is its minimum execution time shown in table 3. These values are for "Lena" image with size of 512×512 in gray scale. The computer used for these evaluations is a PC with 1.6GHz CPU, 40GB H.D.D. and 760MB of RAM. Considering table 3 it can be seen that there is a noticeable difference between the proposed method and other ones. The only algorithm that could produce better *MSE* and *PSNR* values than the

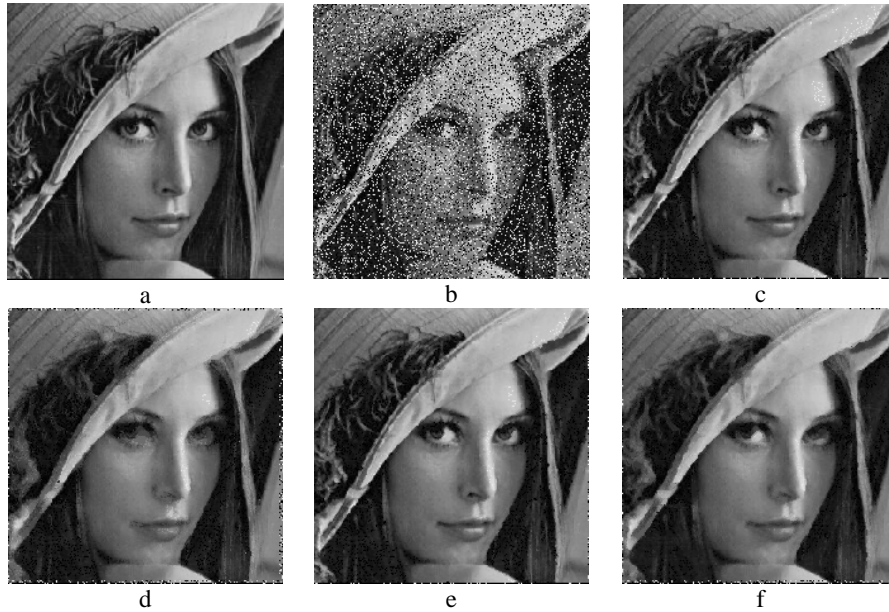


Fig. 4 – (a) Original image (b) 25% fixed value noise added image (c) Proposed Algorithm (d) Long Range Correlation (LRC) Method (e) Polynomial Approximation (PA) Method (f) Progressive Switching Method

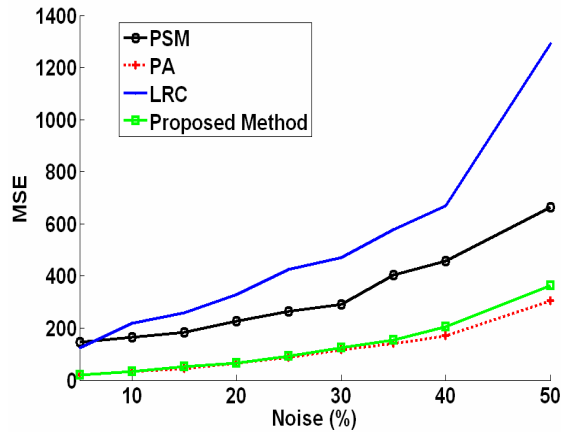


Fig. 5 A comparison of different median based filters' MSE for the restoration of corrupted "Lena" image under a range of impulse noise ratio

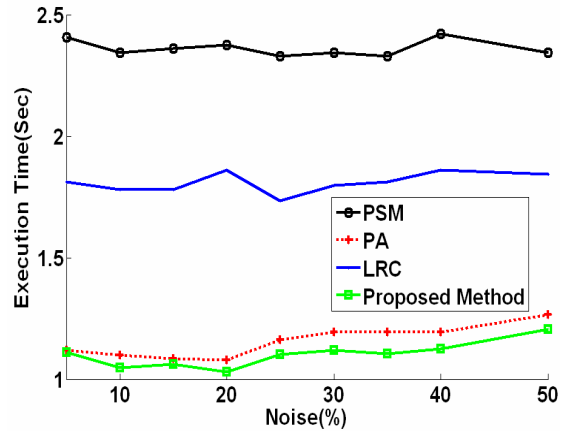


Fig. 7 – Execution times of four algorithms in compare with each other

proposed method, *i.e.* PA, takes 0.1307 sec. longer (in average) to finish filtration of the noisy image. Although it might seem that this is small value but it must be considered that these values are only for gray scale images of size 512×512. When a video with large number of frames needs

to be filtered this small value becomes a big trouble. These considerations and summaries mean that the proposed algorithm not only is able to produce acceptable results but also is able to be used in real-time applications.

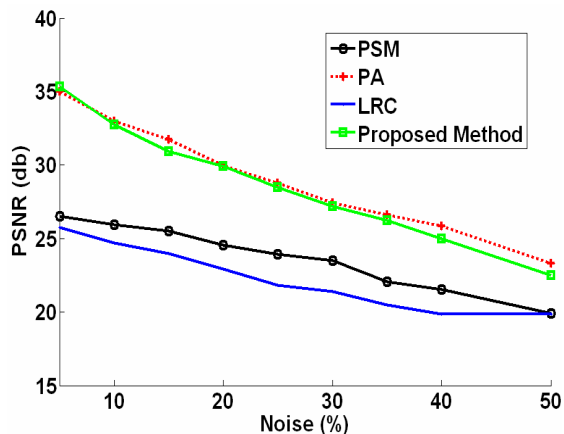


Fig. 6 A comparison of different median based filters' PSNR for the restoration of corrupted "Lena" image under a range of impulse noise ratio

TABLE 1 – MSE VALUES OF FOUR FILTERING METHODS FOR WIDE RANGE OF NOISE VALUES FROM 5% - 10%

Noise Value	PSM	LRC	PA	Proposed Method
5%	145.51	124.15	20.580	18.940
10%	165.28	219.15	32.860	34.320
15%	183.04	259.03	43.420	51.960
20%	227.63	327.27	65.160	65.430
25%	264.43	424.62	86.190	91.790
30%	289.94	470.35	117.35	123.80
35%	404.16	577.96	141.99	153.93
40%	457.56	668.92	169.62	205.19
50%	663.77	1290.4	303.97	362.45

TABLE 2 – PSNR VALUES OF FOUR FILTERING METHODS FOR WIDE RANGE OF NOISE VALUES FROM 5% - 10%

Noise Value	PSM	LRC	PA	Proposed Method
5%	26.500	25.730	34.990	35.320
10%	25.950	24.690	32.960	32.740
15%	25.500	23.960	31.750	30.940
20%	24.550	22.950	29.990	29.940
25%	23.910	21.820	28.770	28.470
30%	23.510	21.370	27.430	27.170
35%	22.060	20.480	26.610	26.220
40%	21.530	19.840	25.830	24.970
50%	19.910	19.870	23.300	22.500

TABLE 3 - MEAN EXECUTION TIMES FOR DIFFERENT NOISE VALUES IN DIFFERENT RUNS

PSM	LRC	PA	Proposed Method
2.3551 Sec.	1.8026 Sec.	1.2431 Sec.	1.1124 Sec.

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