Control Law for UNI-Axial Vehicle Using Lyapunov Analysis

Ahmed Farid, Tehseen Zia, Fahad Maqbool, Saad Razzaq, Fahad Shahbaz & Kashif Irfan

Abstract— **Lyapunov's direct method is useful technique especially for nonlinear systems. Lyapunov function is energy-like function. This function may draw conclusions about the stability of system without solving the set of non linear equations. Discovering a Lyapunov function for particular system is really a hard job which requires experience and physical insight. Stability problem is a real problem for any control system which requires appropriate control law for a given plant. This paper presents Lyapunov's local stability for uni-axial vehicle by designing appropriate control law. After the designing of control law candidate Lyapunov function becomes a real Lyapunov function.**

***Index Terms*— Nonlinear control, Lyapunov's direct method, , Local stability, State-space.**

I. INTRODUCTION

Nonlinear control is one of the biggest challenges in modern control theory. Nonlinear processes are difficult to control because there can be so many variations of the nonlinear behavior. The second method of Lyapunov also know as Lyapunov s direct method [1] , is becoming increasingly recognized as having great potentiality, both for resolving nonlinear stability and performance problems. Lyapunov's direct method is now being widely used for designing stable controllers for various fields like Power system stabilizing controllers [2], Adaptive Control of a MEMS Gyroscope [3], replicator system with entropy-like applications [4], Neuro stabilizing control for power systems [5], Suppression Control of Rotor Oscillation for Stepping Motor [6], Application to induction machines [7].
In order to find the stability characteristics of the system we have to solve the system equations. Sometimes especially for nonlinear systems it is hard to solve system equations. Lyapunov's theory purpose a way to analyze stability of the system without necessarily solving the system equations [1]. Lyapunov's direct method is a useful technique for determining the stability of non linear system [1]. An energy-like scalar function is generated through this method. Stability of the system can be examined, through variations of that function [8]. There is no effective approach to find the lyapunov function; it requires experience, physical insight and trial-error approach. Candidate lyapunov function has to fulfill certain conditions in order to become a valid lyapunov function i.e. it should be zero at equilibrium and positive elsewhere (positive definite function)[8][9].

In many control problems the task is to find an appropriate control law for a given plant [8]. Stable control systems can be designed by using lyapunov's direct method. In this paper we have designed a control law for uni-axial vehicle by hypothesizing a lyapunov fuction candidate. Later in the paper it is showed that this candidate function is a real lyapunov function.

Section 2 presents lyapunov's applications in various fields of stability in sense of lyapunov and conditions for real lyapunov function. In the section 3, dynamics of uni-axial vehicle are described. Section 4 contains the hypothetical lyapunov function, control law and lyapunov's local stability proof for uni-axial vehicle. In the fifth section simulations and results are shown and last section concludes our work.Procedure for Paper Submission.

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II. RELATED WORK

Lyapunov's direct method is applied to design various controllers, their brief description is provided in this section.
M. Januszewski, J. Machowski and J.W. Bialek presented in [2] an approach to improve damping of power swings. This is done by using the unified power flow controller (UPFC). Available signals of real and reactive power are used and then state-variable strategy has been derived. This state-variable control provides damping independent of operating conditions.
Robert P. Leland has explained [3] two adaptive controllers for a vibrational MEMS gyroscope. One based on low frequency model and other based on the full
gyroscope model. Good transient response is obtained force-to-rebalance and automatic gain control loops using
The Lyapunov function used is critical in obtaining a good transient response, especially for the force-to-rebalance and automatic gain control loops.
Yuri A. Pykh has shown in [4] that there exists entropy-like lyapunov function. For replicator systems known entropy measures may be obtain from entropy-like Lyapunov function.
Hirata, Atsushi and Nishigaito used [5] Particle swarm optimization to find set of connecting weights matrix of NN. Probability of agents entrapped into local optima by improving algorithm is reduced.
Senjyu, Nakahama and Uezato achieved [6] the suppression control of the rotor oscillation by excitation control using Lyapunov function. Because of this the rotor oscillation was effectively vanished by the simple excitation sequence and quick tracking control of rotor position is becomes possible.
Ludvigsen, Ortega, Albertos and Egeland studied in [A] that, due to technological or information transmission considerations, fast switching is not possible. Predictive

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feature minimizes the delay and average behavior of lyapunov function. The scheme is applied to the problem of direct torque and flux control of induction motors, for which a complete stability analysis is carried out.

The control law which we have derived for uni-axial vehicle does not involve intensive mathematical computations as the above mentioned applications involves. Lyapunov local stability is guaranteed.

III. PRELIMINARIES

Some concepts of stability and characteristics of real lyapunov function are explained in light of few definitions and theorem [8].

Let B_R denote the spherical region (or ball) defined by $||x||< R$ in state-space, and S_R the sphere itself, defined by $||x||=R$.

Definition I:

"The equilibrium state x=0 is said to be stable if, for any R>0, there exists r>0, such that if $||x(0)||< r$, then $||x(t)||<R$ for all $t \geq 0$. Otherwise, the equilibrium point is unstable".

The definition states that the origin is stable, if, given that we don't want the state trajectory x(t) to get out of a ball of arbitrarily specified radius B_R, a value r (R) can be found such that starting the state from within the ball B_R at time 0 guarantees that the state will within the ball B_R thereafter.

The definition of real lyapunov is as fallows:

Definition II:

"If in a ball B_{Ro}, the function V(x) is positive definite and has continuous partial derivation and if its time derivative along any state trajectory of the system is negative semi-definite.

$V'(x) \leq 0$

Then V(x) is said to be lyapunov function".

After finding the valid lyapunov function, we have to discuss the lyapunov local stability of the system. The local versions are concerned with stability properties in the neighbourhood of equilibrium point and usually involve a locally positive definite function.

Theorem I

"If in a ball B_{Ro}, there exist a scalar function V(x) with continuous first partial derivate such that,

V(x) is positive definite (locally in B_{Ro});

V'(x) is negative semi definite (locally in B_{Ro}) then the equilibrium point '0' is stable. If actually, the derivative V'(x) is locally negative definite in $B_{Ro,}$ then the stability is asymptotic".

The discussion about the lyapunov function can be summarized as:

- V (0) =0
- V(x) is positive throughout some region of the sate-space 'S' outside the origin that is V(x) > 0; $x \epsilon S$; $x \neq 0$.
- V(x) is continuous throughout 'S'.
- V(x) has continuous first partial derivative with respect to x_i ; $\partial V(x)/\partial x_i$; i=1………n
- $V'(x) \leq 0$ (for locally stable)

In extension to the theorem 1; if V'(x) is negative definite in B_{Ro}, then the stability of the system is as asymptotic [8].

IV. CONTROL LAW FOR UNI AXIAL VEHICLE

In order to design a control law using lyapunov direct method, we have taken uni-axial vehicle [10] as an example. The state-space representation of a uni-axial vehicle is:

$$x'_1=1/2(u_1 + u_2)\cos x_3 \quad (1)$$
$$x'_2=1/2(u_1 + u_2)\sin x_3 \quad (2)$$
$$x'_3=1/2(u_1 - u_2) \quad (3)$$
$$y= [y_1, y_2]^T = [x_1,x_2]^T \quad (4)$$

u_1 and u_2 are inputs; left and right track velocities. y_1 and y_2 are the system's output. The state x_3 is angle between vehicle velocity and y_1-axis.

Control law is derived by hypothesizing the candidate lyapunov function that is:

$$V(x) = 1/2\ (x_1^2 + x_2^2 + x_3^2) \quad (5)$$

This function justifies all conditions which have been mentioned above for candidate lyapunov function.

After applying derivative.

$$V'(x) =x_1{*}x'_1 + x_2{*}x'_2 + x_3{*}x'_3 \quad (6)$$

Putting the values of x'_1 , x'2 and x'_3 . in (6), we get;

$$V'(x) = 1/2{*}x_1{*}\ (u_1+u_2){*}\cos x_3 + 1/2{*}x_2{*}\ (u_1+u_2){*}\sin x_3 + 1/2{*}x_3{*}\ (u_1-u_2) \quad (7)$$

$$V'(x) =1/2{*}\ (u_1+u_2){*}\ (x_1{*}\cos x_3 + x_2{*}\sin x_3) + 1/2{*}x_3{*}\ (u_1-u_2) \quad (8)$$

Our supposed candidate function will be acceptable, if it satisfies $V'(x) \leq 0$. The desired control law plays an important role to make this candidate a real lyapunov function.

If we define the values of u_1+u_2 and u_1-u_2 in the following way then required results can be achieved.

$$u_1+u_2 = -(x_1{*}\cos x_3 + x_2{*}\sin x_3) \quad (9)$$
$$u_1-u_2 = -x_3 \quad (10)$$

Solving simultaneously (9) and (10) we get,

$$u_1=-1/2{*}\ (x_1{*}\cos x_3 + x_2{*}\sin x_3) - 1/2{*}x_3 \quad (11)$$
$$u_2=u_1+x_3 \quad (12)$$

So

$$u_2= -1/2{*}\ (x_1{*}\cos x_3 + x_2{*}\sin x_3) + 1/2{*}x_3 \quad (13)$$

u_1 and u_2 is desired control law for this particular plant. By putting the values of u_1 and u_2 in (8)

$$V'(x) = -1/2{*}\ (x_1{*}\cos x_3 + x_2{*}\sin x_3)^2 - 1/2{*}x_3^2 \quad (14)$$

$$V'(x) < =0 \quad (15)$$

Equation (15) may be considered as Semi Negative Definite, if ($x_1 = x_3 = 0$).

The desired results indicate that candidate is a real lyapunov function and derived control provides the lyapunov local stability, which is in accordance with the above mentioned theorem.

V. SIMULATION AND RESULTS

The presented control law design is simulated in MATLAB 7.0/SIMULINK using S-Function. u_1 and u_2 seems to be converging under derived control law for uni-axial vehicle as it can be seen from Fig,1. Initial values of 2.5 and -2.5

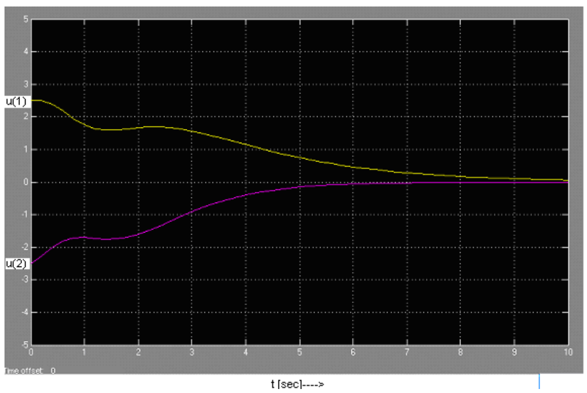

Fig. 1. Plant Behavior after implementing Control Law

Under same conditions, the behaviour of the system can be viewed over longer duration in Fig.2.

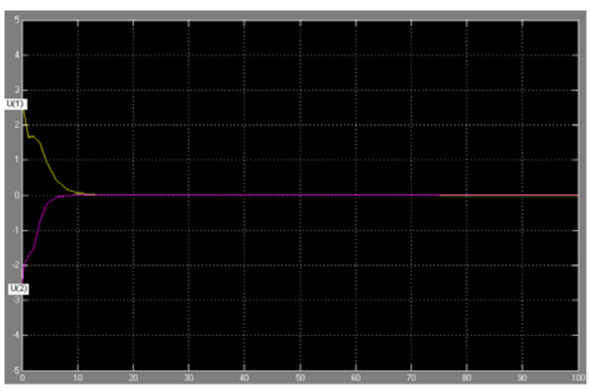

Fig.2. Plant Behavior over 100sec

If V(x) is positive definite and non-increasing and its derivative is negative semi-definite the state x(t) is bounded[9]. Under same conditions, the behaviour of the system can be viewed over longer duration in Fig.2. Both the figures reflect that the states are bounded and approaching towards equilibrium. In Fig.2, the performance of control law is obvious between the time interval around 13 to 75 sec.

VI. CONCLUSION

In this paper lyapunov local stability for uni-axial vehicle is proposed. Our supposed lyapunov function proved to be the real lyapunov function. Results are simulated after applying the control law to the plant and stability is guaranteed. Although lyapunov direct method is widely used in designing stable controllers for nonlinear systems but the difficulty is to find an appropriate lyapunov function. Lyapunov direct method is applicable to all dynamic systems weather it is linear or nonlinear. There are two ways to design control law using lyapunov analysis, one is hypothesising the candidate function, derive a control law and then justify the candidate as real lyapunov function. In other way control law is hypothesized and then lyapunov function is found. We have fallowed the first approach. In Fig 1. u_1 and u_2 are seem to be converging but the clear results can be seen in Fig.2. The region of complete overlapping is from 13 to 75 sec, which indicates the stability of uni-axial vehicle under derived control law.

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