The Study of DEA Evaluating Model on Road Traffic Safety

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Abstract—Data Envelopment Analysis is a kind of important analyzing tool and research method in evaluation. As a method, introducing secondary relative evaluation method, the Data Envelopment Analysis (DEA) is applied to evaluate road traffic safety in different areas based on optimizing evaluation index system. This paper mostly discusses the relatively safety of different areas.

Keywords—road traffic safety; data envelopment analysis; evaluation; evaluation index system

Ⅰ. Introduction

Data Envelopment Analysis, DEA is an efficiency evaluation method which was advanced by famous operational researcher A.Charnes and W.W.Cooper, etc[1][2]. DEA has spread the concept of single input-output project efficiency to the efficient evaluation of the multi input-output DMU, which has greatly enriched the theory of Production Function and its appliance.It has valuable superiority in avoiding subjective factors, simplizing the arithmetic, reducing the error, etc[3].DEA has received wide attention for its unique characteristic and advantages, which has a fast development both in academic research and practical appliance. DEA now has become an important and common analyzing tool as well as researching means in management, systemic project, decision-making, evaluation and many other fields[4][5][6].

Ⅱ. Data Envelopment Analysis Model

A. C^2R model

Suppose n production decision units $DMU_j(j=1,2,\cdots,n)$,each DMU has m items inputs $X_j=(x_{1j},x_{2j},\cdots,x_{mj})^T$, s items outputs $Y_j=(y_{1j},y_{2j},\cdots,y_{sj})^T$ .And the j_0 th efficiency assessment model of DMU is:

$$\left.\begin{array}{c} \max \mu^T Y \\ s.t.\omega^T X_j - \mu^T Y_j \ge 0 \\ \omega^T X_0 = 1 \\ \omega^T X_0 \geqq \varepsilon \bullet \hat{e}^T, \mu^T \geqq\in \varepsilon \bullet e^T \end{array}\right\} \quad (1)$$

where (X_0,Y_0) are the outputs and inputs of DMU_{j0}, ε is Non-Archimedes infinitesimal ,and $\hat{e}$ 、e is s -dimensional vector of element 1.And its Dual Problem is:

$$\left.\begin{array}{c} \min\left[\theta-\varepsilon(\hat{e}^T s^- + e^T s^+)\right] \\ s.t.\sum_{j=1}^{n} X_j\lambda_j + s^- = \theta X_0 \\ \sum_{j=1}^{n} Y_j\lambda_j - s^+ = Y_0 \\ \lambda_j \ge 0(j=1,2,\cdots,n), s^+ \geqq 0, s^- \geqq 0 \end{array}\right\} \quad (2)$$

The DMA model is the feasible production set $T=\left[(X,Y)\middle|\text{Ycan be producted from X}\right]$, meeting the following axiomatic assumptions: convexity, cone, invalidity, and minimum.

B. BBC model

Sometimes the cone assumption of production possible set is unrealistic or unreasonable ,therefore the assumption is removed .When the feasible production set only meet the convexity (add conditions $\sum\lambda_j=1$), invalidity and the minimum, we can gain the BBC model that meets variable returns to scale.

Manuscript received October 9, 2007
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$$\left.\begin{aligned} &\min\left[\theta-\varepsilon(e^T s^- + e^T s^+)\right] \\ &s.t\sum_{j=1}^{n}\lambda_j X_j + \overline{s} = \theta X_0 \\ &\sum_{j=1}^{n}\lambda_j Y_j - s_+ = Y_0 \\ &\sum_{j=1}^{n}\lambda_j = 1 \\ &s^- \geqq 0, s^+ \geqq 0, \lambda_j \geq 0; j=1,2,\cdots,n \end{aligned}\right\} \quad (3)$$

This model only evaluates the technology efficiency of DMU .Its dual form is as follows:

$$\left.\begin{aligned} &\max(\mu^T Y_0 - u_0) \\ &s.t\mu^T Y_j - \omega^T X_j - \mu_0 \leq 0 \\ &\omega^T X_0 = 1 \\ &\omega \geqq \varepsilon, \mu \geqq \varepsilon, u_0 \in E^1; j=1,2,\cdots,n \end{aligned}\right\} \quad (4)$$

Where, u_0 is the indicatrix of Return to Scale, in the optimal solutions $\mu=\mu_0^*$,then (1) $u_0^*<0$,return to scale increase ; (2) $u_0^*=0$ return to scale unchanged; (3) $u_0^*>0$,return to scale decrease.

Ⅲ. Establishing macro overall evaluation model of road safety

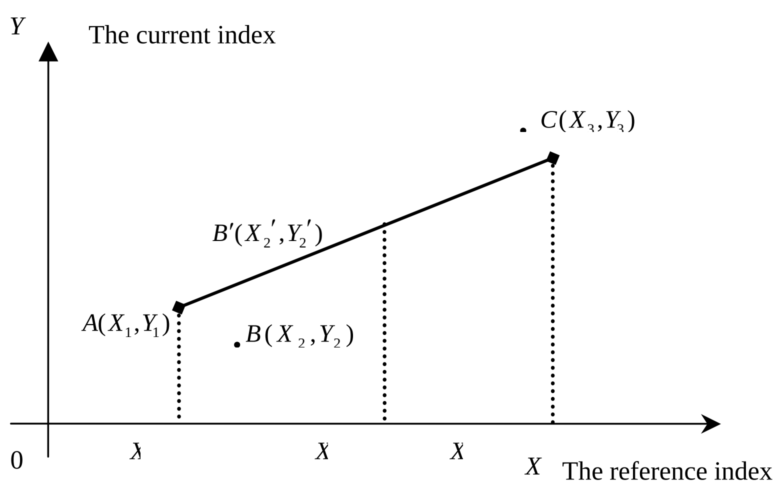
A. Quadratic relative evaluation methods

1. Reference Index, the current index, the state of index

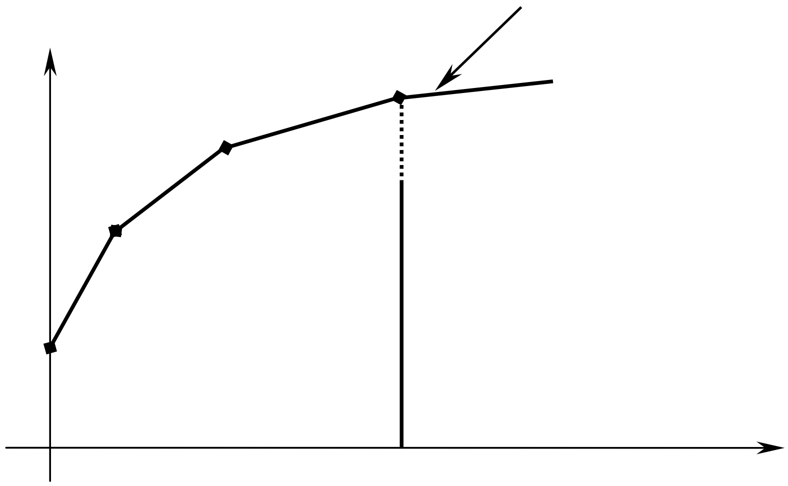
In order to eliminate the impact of the quality of the objective basic conditions, acquiring the index that can reflect the subjective efforts of the assessed areas, we should mine in the dynamic changes of the indicators of the assessed areas. After the establishment of the composite indicators, we choose a scientific and reasonable evaluation method and calculate the current situation, and the indicators obtained are called the current index. Of course, regional security situation in the past can also be calculated with the same index system and approach, and the indicators obtained which reflect the station of the objective condition are called reference index.

Suppose x_j is the j th reference index of the assessed area, y_j is the current index of the assessed area, where $x_j \in E_1$, $y_j \in E_1$, then arrays (x_j, y_j) are called the j th state of index of the assessed area.

We make the reference index as abscissa X , and the current index as the vertical coordinate Y . Assuming there are three evaluation subjects whose exponent state are $A(X_1,Y_1)$, $B(X_2,Y_2)$, $C(X_3,Y_3)$ in the evaluation, and their positions in the Plane Coordinate are as figure 1.The reference index of evaluation subject B is between A and C , that is $X_1<X_2<X_3$ .if $B(X_2,Y_2)$, the exponent state of B is below the Joint Line AC between $A(X_1,Y_1)$ and $C(X_3,Y_3)$,we can consider that the Effective Effort Degree of B inferior to A and C .



We can line out the exponent state of the evaluation subject on the Plane Coordinate, and get the frontier of the exponent state possibility set with the DEA method. If an evaluation subject (X, Y) is between two subjects that on the frontier, its current index Y should be marked as Y' on the frontier. η the Ratio of Y and Y' can be a Measurement of Effective Effort Degree. as Fig. 2 shows.



2. Quadratic relative evaluation method

Suppose (x_j, y_j) is the exponent state of the observed area, $j = 1,2,3\cdots, n$ and

$$T = \left\{ (x,y) \middle| \sum_{j=0}^{n} \lambda_j x_j \le x, \sum_{j=0}^{n} \lambda_j y_j \ge y, \sum_{j=0}^{n} \lambda_j = 1, \lambda_j \ge 0, j = 0,1,2,\cdots n \right\}$$ is

the exponent state possibility set that is composed by (x_j, y_j), $j = 1,2,\cdots, n$,where $(x_0, y_0) = (0,0)$. Making the reference index as input and the current index as output, structuring frontier of the exponent state and establishing Data Envelopment Analysis Model as follows by the DEA method for structuring frontier which was advanced by Charnes.

$$\max Z$$

$$s.t \begin{cases} \sum_{j=0}^{n} \lambda_j x_j \leqq x_{j0} \\ \sum_{j=0}^{n} \lambda_j y_j \geqq Z y_{j0} \\ \sum_{j=0}^{n} \lambda_j = 1 \\ \lambda_j \ge 0, \; j = 0,1,2\cdots n \end{cases} \qquad (5)$$

If the Optimum Value of MODEL（5）exist and $Z^0 = 1$,then the exponent state of this region is considered on the frontier of the exponent state possibility set. Generally, if Z^0 is the Optimum Value of MODEL（5）,let $\bar{x}_{j0} = x_{j0}$, $\bar{y}_{j0} = Z^0 y_{j0}$,we can see $(\bar{x}_{j0}, \bar{y}_{j0})$ is on the frontier, and it is called the Projection on the frontier of the exponent state possibility set from (x_{j0}, y_{j0}) which is the j_0 th region's exponent state.

The frontier of the exponent state Envelopes all the exponent states (x_j, y_j) , $j = 1,2,\cdots, n$ and reflect the optimum relation between the inputs and out puts of the evaluation region. Suppose Z^0 is the Optimum Value of MODEL（5）,it can reflect the degree of Deviation between the exponent state of the evaluation region and the frontier, hence we can obtain Relative effective value of the evaluation region

Suppose $\eta = 1/Z^0 \times 100\%$, take η as the Quadratic relative evaluation value of the evaluation region, and it is a measurement that reflect the unit efficient work of the evaluation region that eliminate the objective basic situation.

B. adopting the traffic safety evaluation model of quadratic relative evaluation method

This paper has advanced a new method of evaluating the traffic safety based on deeply research into the theory of Quadratic relative evaluation method, and has adopted it to the evaluation of the traffic safety. The basic idea is to apply Quadratic relative evaluation method to evaluate the road traffic safety through the variation of the evaluated areas. It can be divided into two stages: the first stage is to survey the safe situation of the evaluated area with four steps concretely; the second stage, quadratic relative evaluation method. The new model is shown as fig3.

C. Optimizing the evaluation index system

The complexity of evaluation model and various data collection have brought the organizer, data collector and evaluation specialists great burden, therefore, to optimize the evaluation index system has its own practical significance. First, optimized evaluation model can greatly save financial and human resource; second, it can make the dispose of data much easier; third, improve the efficiency; forth, it is much easier to control the process of evaluation with optimized model; last, improve the implemental feasibility of evaluation. The evaluation index system is the core of evaluation model, and the optimization of the evaluation index is the most important part of evaluation index optimization. The evaluation index system has in some extent determined the evaluation method, data dispose and investigate method, as a result, the optimization of the evaluation index system shares more importance in optimizing the evaluation model.

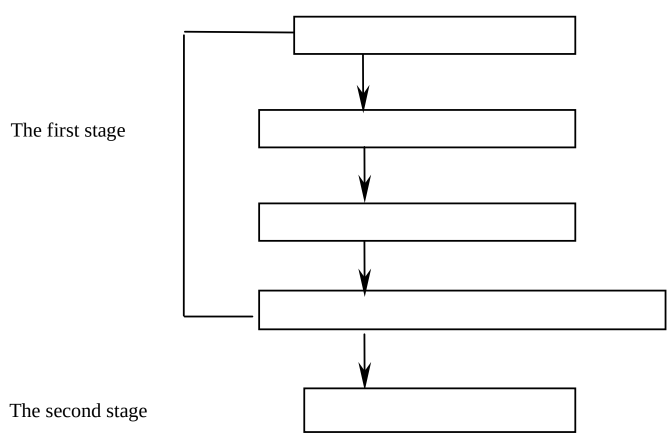
In the process of optimizing the index, it may come cross several situations as follows: combine the index, delete the index, append the index, and replace the index. The mathematic description of optimization is as follows；

Let index set $B = (B_1, B_2, \cdots, B_n)$

Suppose choosing any m evaluation subjects, index score matrix correspond to index set B gained by practical evaluation

$$X = \begin{bmatrix} x_{11} & x_{12} & \cdots & x_{1n} \\ x_{21} & x_{22} & \cdots & x_{2n} \\ \vdots & \vdots & \vdots & \vdots \\ x_{m1} & x_{m2} & \cdots & x_{mm} \end{bmatrix} \quad (6)$$

$X_j = (x_{1j}, x_{2j}, \cdots, x_{mj})$, $j = 1,2,\cdots,n,$ X_j is the evaluation point value index of every evaluation index correspond to index set B



1. Combine the index

New index set B' was obtained after the first degree index combination of index set B , where $B' = (B'_1, B'_2, \cdots B'_{n-1})$,the index B_p combine with B_q ,and $p < q$,then index set

$$B'_j = \begin{cases} B_j & j = 1,2,\cdots,p-1,p+1,\cdots,q-1 \\ B'_p & j = p \\ B_{j+1} & j = q,\cdots,n-1 \end{cases} \quad (7)$$

2. Delete the index

New index set B' was obtained after the first degree index deletion of index set B ,where $B' = (B'_1, B'_2, \cdots B'_{n-1})$ .The index B_p was appended, then the index set is

$$B'_j = \begin{cases} B_j & j = 1,,2,\cdots p-1 \\ B_{j+1} & j = p,\cdots n-1 \end{cases} \quad (8)$$

3. Add the index

New index set B' was obtained after the first degree index addition of index set B, where $B' = (B'_1, B'_2, \cdots B'_{n+1})$,the index B was replaced, the score value set is $X'_p = (x'_{1p}, x'_{2p}, \cdots x'_{mp})$,then the index set is

$$B'_j = \begin{cases} B_j & j = 1,2,\cdots, p-1 \\ B'_p & j = p \\ B_{j-1} & j = p+1,\cdots, n+1 \end{cases} \quad (9)$$

4. Replace the index.

new index set B' was obtained after the first degree index replacement of index set B ,where $B' = (B'_1, B'_2, \cdots B'_{n+1})$,and be replaced by the index B'_p ,the score value set is $X'_p = (x'_{1p}, x'_{2p}, \cdots x'_{mp})$,the index set is

$$B'_j = \begin{cases} B_j & j = 1,,2,\cdots p-1, p+1,\cdots n \\ B'_p & j = p \end{cases} \quad (10)$$

Ⅳ. The practical analysis of traffic safety evaluation model

A. Traffic safety evaluation index system and weight

1. Traffic safety evaluation index system

Based on the characteristic of traffic safety and management in china, the traffic safety evaluation index has been optimized to the evaluation index system shown as fig4 from the combined angle of qualitative analysis and quantitative analysis. This evaluation index system includes four aspects with eighteen evaluation indexes in total (with eight quantitative indexes shown as indexes 5-12 in fig4 and ten qualitative indexes)

2. Establishment of index system weight

This paper analyses and mark weight composition of the index system when established it. 1 represents the road traffic safety overall level and meanwhile the final weight of index layer correspondence to target layer .The final weight composition which is calculated in AHP based on the summary of the mark results was shown as fig4.

B. Obtaining the original data

This paper has gained the data of a city with five counties by data collection and processing. These data conclude the following: GDP(RMB Billion), population of the city, total road mileage of the city, the people died in traffic accident on road, the mileage of the county, the mileage of countryside, the people died in traffic accident on country and countryside road, the mileage of national road, the people died in traffic accident on national road, the mileage of provincial road, the people died in traffic accident on provincial road, the injured people in traffic accident and the left motor vehicles in traffic accidents. These data are not shown in the present paper considering the secrecy of the data, in addition, the five cities are shown as City One-City Five.

C. Calculation of quantitative index and qualitative index

1. Calculation of Quantitative Index data

The original data is calculated and collected to obtain quantitative index data for evaluating, shown as Table.1 and Table.2.

2. Data pretreatment

This paper select the standardized method, the formula is as follows

$$x'_{ij} = \frac{x_{ij} - \bar{x}_{ij}}{s} \quad (11)$$

Where:

$$\bar{x}_{ij} = \frac{1}{n}\sum_{j=1}^{n} x_{ij}$$

$$s = \sqrt{\frac{1}{n-1}\sum_{j=1}^{n}(x_{ij} - \bar{x}_{ij})^2}$$

And the correction formula is:

$$x^*_{ij} = \frac{x_{ij} + 1}{\max\{x_{ij} + 1\}} \quad (12)$$

x_{ij} 、 x^*_{ij} in formulas（11）、（12） are the i th evaluation index data and the i th evaluation index dimensionless value through pretreatment of city j ,and the dimensionless results of quantitative index data are shown as Table.3 and Table 4.

The qualitative index can only be established by the expert’s commentaries and be made as the evaluation data, for it can’t be expressed quantitatively.

Comprehensive Level of road traffic safety

- Perfect the mechanism and policy of road traffic safety（A）（0.52）
  - 2 comprehensive planning of road traffic safety（0.28）
  - 3 responsibility system of road traffic safety（0.15）
  - 4 publicity and Education of road traffic safety (0.16)
- Scientific analysis of road traffic safety status and trend（B）（0.08）
  - 5 road traffic accident death rate per hundred million GDP（0.12）
  - 6 the equivalent accident death rate per ten thousand vehicles（0.35）
  - 7 road traffic accident death rate per 100 000 resident population（0.14）
  - 8 national and provincial highway mortality rate per hundred million vehicle kilometer (0.13)
  - 9 city road traffic accident death rate per hundred miles
  - 10 country road traffic accident death rate per hundred miles (0.03)
  - 11 risk analysis of road traffic（0.06）
  - 12 decreased percentage of death number of road traffic accident（0.09）
- Strengthening road traffic safety supervision and management（C）（0.15）
  - 13 inspection and management force of road traffic safety（0.19）
  - 14 inspection and management level of road traffic safety（0.15）
  - 15 conditions of road traffic safety（0.06）
- Strengthening road traffic safety supervision and management（D）（0.25）
  - 16 road traffic accident analysis（0.12）
  - 17 the Regulation of frequently-occuring Spots of Traffic Accident（0.63）
  - 18 Emergency Rescue of road traffic accident（0.25）

Fig.4 weight composition of road traffic safety evaluation index system

Table.1 Quantitative evaluation index data collecting table of the evaluated cites in 2003

The sequence Number of index	Evaluation Index	CITY 1	CITY 2	CITY 3	CITY 4	CITY 5
5	death rate per hundred million GDP	0.73	1.25	0.58	0.82	0.67
6	the equivalent death rate per ten thousand vehicles	21.10	29.19	13.41	30.12	24.93
7	death rate per 100 000 resident population	7.94	8.93	4.84	9.04	12.47
8	national and provincial highway Mortality rate per hundred million vehicle kilometer	12.31	14.12	12.30	4.01	13.02
9	city road traffic accident death rate per hundred miles	31.53	24.04	8.28	26.87	4.58
10	country road traffic accident death rate per hundred miles	5.10	4.49	1.52	7.49	9.84
11	Risk Analysis of road traffic	16.74%	15.10%	26.53%	17.87%	17.26%
12	decreased percentage of death number	-13.34%	-4.92%	-11.28%	-1.71%	3.34%

Table.2 Quantitative evaluation index data collecting table of the evaluated cites in 2004

The sequence number of index	Evaluation Index	City1	City2	City3	City4	City5
5	death rate per hundred million GDP	0.67	1.04	0.54	0.79	0.63
6	the equivalent death rate per ten thousand vehicles	20.03	27.04	12.31	28.79	22.57
7	death rate per 100 000 resident population	7.58	8.24	4.57	8.61	12.16
8	national and provincial highway mortality rate per hundred million vehicle kilometer	11.89	13.46	11.76	3.73	12.09
9	city road traffic accident death rate per hundred miles	30.67	23.48	8.09	24.34	4.26
10	country road traffic accident death rate per hundred miles	4.97	4.13	1.37	6.98	8.78
11	Risk Analysis of road traffic	15.45%	15.4%	22.75%	17.34%	15.79%
12	decreased percentage of death number	-9.65%	-3.78%	-9.12%	-0.89%	-0.59%

D. Road traffic safety evaluation

Traffic safety evaluation is the comprehensive evaluation of the regional road safety situation based on the index value and the corresponding weights of the evaluation index system, and it has lots of methods such as value analysis, comparison analysis, fuzzy evaluation method and so on. This paper adopted the value analysis method to make comprehensive evaluation of the five cities' road traffic safety situation in 2003 and 2004, which was called the first degree evaluation, and the evaluation which were shown as Table.5 were made as the reference index and current index of the binary evaluation.

Table.3 The dimensionless result of quantitative index data of the evaluated cites in 2004

The sequence number of index	Evaluation Index	City1	City2	City3	City4	City5
5	death rate per hundred million GDP	0.4594	1	0.3035	0.5530	0.3971
6	the equivalent death rate per ten thousand vehicles	0.3164	0.9294	-0.2676	1	0.6062
7	death rate per 100 000 resident population	0.3041	0.4287	-0.9351	0.4426	1
8	national and provincial highway mortality rate per hundred million vehicle kilometer	0.7425	1	0.7411	-0.4143	0.8435
9	city road traffic accident death rate per hundred miles	1	0.6928	0.0464	0.8089	-0.1054
10	country road traffic accident death rate per hundred miles	0.3511	0.2676	-0.139	0.6783	1
11	Risk Analysis of road traffic	0.197	0.1203	1	0.2901	0.240
12	decreased percentage of death number	-0.04315	0.4834	0.023	0.6836	1

Table.4 The dimensionless result of original quantitative index data of the evaluated cites in 2004

The sequence number of index	Evaluation Index	City1	City2	City3	City4	City5
5	death rate per hundred million GDP	0.2588	1	-0.0016	0.4992	0.1787
6	the equivalent death rate per ten thousand vehicles	0.3341	0.8668	-0.2530	1	0.5272
7	death rate per 100 000 resident population	0.3109	0.4117	-0.149	0.4684	1
8	national and provincial highway mortality rate per hundred million vehicle kilometer	0.7726	1	0.7546	-0.4559	0.8023
9	city road traffic accident death rate per hundred miles	1	0.6989	0.0543	0.7350	-0.1062
10	country road traffic accident death rate per hundred miles	0.4005	-0.1660	-0.2683	0.7168	1
11	Risk Analysis of road traffic	0.1436	0.1378	1	0.3667	0.1835
12	decreased percentage of death number	-0.5474	0.6286	0.0073	0.9650	1

E. The calculating of Quadratic relative evaluation method

$$\max Z$$
$$s.t\begin{cases}\sum_{j=0}^{n}\lambda_j x_j \leqq x_{j0}\\ \sum_{j=0}^{n}\lambda_j y_j \geqq Zy_{j0}\\ \sum_{j=0}^{n}\lambda_j = 1\\ \lambda_j \geq 0,\ j=0,1,2\cdots n\end{cases} \quad (13)$$

Where x_j、y_j are the first degree evaluation index value of city j in 2003 and 2004. Let $\eta = 1/Z^0 \times 100\%$ be the binary relative evaluation value of the j_0 th evaluated city then solving model（13）to get the optimal solution Z^0 and the calculating result was shown as Table.5. It's a measurement which reflect the road traffic safety performance of the evaluated city eliminating the objective basic conditions.

Table 5 The road traffic safety evaluation index and sorting of the five cities

city	The first degree evaluation result in 2003	Rank in 2003	The first degree evaluation result in 2004	Rank in 2004	Binary relative evaluation result	Performance ran
City1	0.3538	4	0.3433	4	0.7305	3
City2	0.7495	1	0.7268	1	0.6106	4
City3	-0.0302	5	-0.0527	5	1	1
City4	0.5885	3	0.6054	2	0.2504	5
City5	0.6354	2	0.5728	3	0.8766	2

Ⅴ Analysis of the evaluation results

It has been found from the evaluated results that, the five different cities in the evaluation of the year 2003 and 2004, the higher of their position in the evaluation are the lower level of their traffic safe condition are; although ,the fact in the binary relative performance rank is vice versa. The fact is that one evaluation is just based on various evaluating data of the traffic safe condition, therefore, the bigger the index value of a city, the worse of its traffic safe condition, but a comprehensive evaluation is the opposite. The binary relative evaluation value synthetically reflects every index value and their changes during the two years as well as the city traffic safe performance and changes. Therefore, the more the road traffic evaluation indexes decreased and the bigger the binary relative evaluation value are, the more the traffic change, that is the higher of their position in the binary relative performance rank are the higher level of their traffic safe level are. The calculating result show that the safety performance evaluated through observing the overall level of the cities' road traffic safety and worked out by the binary relative evaluation method can eliminate the performance error caused by the difference of the objective basic conditions. On the other side, the traffic safety evaluation of the corresponding region which adopted the first and binary evaluation method can not only reflect the current traffic safety level well but also evaluate the effect caused by the subjective effort on the traffic safety exactly in each region, which can improve the traffic safety level and the traffic safety management level, and the new traffic safety performance evaluation model achieve the unification of rational evaluation and non-rational evaluation.

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