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Improving Sharpe Ratios and Stability of Portfolios by Using a Clustering Technique

Jin Zhang* Dietmar Maringer †

Abstract—**This paper proposes a method which combines a clustering technique with asset allocation methods, to improve portfolio Sharpe ratios and weights stability. The portfolio weights are computed based on cluster members and cluster portfolios, which are decided by an optimal cluster pattern. The optimized cluster pattern tells the belonging of assets to particular clusters, which is identified by using a population-based method, i.e. the Differential Evolution, subject to maximizing the Sharpe ratio of terminal portfolios. We employ two different asset allocation methodologies, i.e. the mean-variance Markowitz allocation and the parameter-free equal weights allocation, with the Financial Times and Stock Exchange market and Dow Jones Industrial Average market data, to study the clustering impact on Sharpe ratio and weights instability of the terminal portfolios. As experimental results suggest that, the terminal portfolios from the clustered markets have higher Sharpe ratios than that without clustering. Furthermore, as a side effect of the clustering, the terminal portfolio weights become more stable than that in the non-clustered markets. Portfolio managers may cluster their assets with the Sharpe ratio criterion before distributing asset weights to improve portfolio weights stability and risk-adjusted returns.**

Keywords: Clustering Optimization, Asset Allocation, Sharpe Ratio, Portfolio Stability, Differential Evolution

1 Introduction

At the Markowitz efficient frontier, an efficient portfolio yields higher return than other portfolios given a same risk level. However, two elements in the Markowitz analysis, i.e. the assets expected returns and covariance can hardly be predicted precisely using historical data, due to errors from estimation procedure and noise in financial data itself. Furthermore, the investors who manage large portfolios containing hundreds of assets, tends to face the 'information deficit' problem when historical data has limited observations leading insufficient degrees of freedom in estimating the two elements. The literature suggests several approaches to reduce or avoid the error and noise impact on portfolios. For example, to reduce the error impact on the output stability after the variance-covariance matrix inversion, Harris and Yilmaz [1] combine the return-based and range-based measures of volatility to improve the estimator of multivariate conditional variance-covariance matrix. Some investors just ignore the mean and variance measures and turn in favor of the equally weighted investment strategy, or the so-called 1/N rule. For example, Windcliff and Boyle [2] propose that 1/N rule should be optimal in a simple market where the assets are indistinguishable and uncorrelated. Benartzi and Thaler [3] discuses the 1/N puzzle in the context of asset allocation decision in a contribution saving plan. In the research by DeMiguel *et al.* [4], the 1/N strategy is found outperforming other thirteen allocation models in terms of the Sharpe ratio, certainty-equivalent return and turnover. This paper proposes a method that introduces clustering to asset allocation procedure, to reduce the negative impacts from the estimation errors and noise on the constructed portfolios.

Many researchers have applied traditional clustering techniques to portfolio management. Pattarin *et al.* [5] employ a clustering technique to analyze mutual fund investment styles. Lisi and Corazza [6] develop an active fund management strategy which selects stocks after clustering equity markets. However, the clustering techniques applied in finance area still follow the traditional clustering criteria, i.e. minimizing a measure of dissimilarity between the objects inside clusters, whereas maximizing the dissimilarity between clusters. We propose a different clustering criterion, which groups market assets to maximize Sharpe ratio of portfolios. The proposed asset allocation model first groups the market assets to a series of disjoint clusters according to the optimal clustering pattern, then uses an asset allocation and cluster members to construct cluster portfolios. After that, the model employs the same asset allocation to construct a terminal portfolio based on the cluster portfolios. In this paper, we refer the assets in same clusters as cluster members, the portfolios which are constructed using the cluster members as cluster portfolios, and the portfolio that is built up using the cluster portfolios as terminal

*Centre for Computational Finance and Economic Agents, University of Essex, United Kingdom. Email:jzhangf@essex.ac.uk. The author gratefully acknowledges the financial support from the EU Commission through MRTNCT2006034270 COMISEF for attending this conference.

†Faculty of Economics and Business Administration, University of Basel, Switzerland. Email:dietmar.maringer@unibas.ch

portfolio. The terminal asset weights are decided by two parts, the cluster portfolio weights and cluster member weights. Three asset allocation methods are employed for the clustering study: the so-called naive 1/N allocation, the Markowitz minimum variance portfolio (MVP) allocation and the modified Tobin tangency portfolio allocation. A population-based evolutionary method, the Differential Evolution (DE) is employed to tackle the clustering problem.

The paper is organized as follows. Section 2 introduces the clustering optimization problem, the data for empirical experiments, and the heuristic approach to solve the complex clustering optimization problem. Section 3 provides experimental results and discussions, and Section 4 draws the concluding remarks.

2 The Asset Allocation Model

2.1 The Optimization Problem

The optimization problem is to identify an optimal clustering pattern set $\mathcal{C}$, which is an union of optimal subsets $\mathcal{C}_1, \mathcal{C}_2, ..., \mathcal{C}_G$. The terminal portfolio computed based on the optimal pattern yields higher in-sample Sharpe ratio than that are computed by using other patterns with a same cluster number G. The optimization objective of the clustering problem can be expressed as follows:

$$\max_{\mathcal{C}} SR = \frac{\bar{r}_p - \bar{r}_f}{\sigma_p}, \tag{1}$$

where SR represents the Sharpe ratio, $\mathcal{C}$ is the optimal partition set, $\bar{r}_p$ is the average return of the portfolio, $\bar{r}_f$ is an estimate of the risk-free return, and σ_p is a measure of the portfolio risk over the evaluation period.

When we apply clustering techniques to segment equity markets, there are several clustering constraints must be satisfied. G is the number of subsets in cluster set $\mathcal{C}$ with a value range $1 \leq G \leq N$, and N is the number of market assets. When G is set at 1 or N, we have the non-clustered market. The union of segmented markets contains all market assets, and there is no intersection between two different clusters. If we denote $\mathcal{C}_g$ as the g-th subset of assets and $\mathcal{M}$ as market assets, the constraints can be expressed as:

$$\bigcup \mathcal{C} = \mathcal{M}, \tag{2}$$

$$\mathcal{C}_g \cap \mathcal{C}_j = \emptyset, \quad \forall g \neq j. \tag{3}$$

To avoid the cases that one single cluster contains too many assets or an empty cluster exists, we impose cardinality constraints to cluster size, which are related to the cluster number G; if we let $\tilde{N}^{\min}$ and $\tilde{N}^{\max}$ denote the minimum and maximum asset number in a cluster, the constrains are described as follows:

$$\tilde{N}^{\min} \leq \sum_{s=1}^{N} I_{s\in\mathcal{C}_g} \leq \tilde{N}^{\max} \qquad \forall g \in G, \tag{4}$$

$$\text{where} \quad I_{s\in\mathcal{C}_g} = \begin{cases} 1 & \text{if } s \in \mathcal{C}_g, \\ 0 & \text{otherwise,} \end{cases} \tag{5}$$

$$\text{with} \quad \begin{cases} \tilde{N}^{\min} = \lceil \frac{N}{2G} \rceil, \\ \tilde{N}^{\max} = \lceil \frac{3N}{2G} \rceil. \end{cases} \tag{6}$$

When we apply asset allocations to construct portfolios, we impose weight constraints to the cluster members and cluster portfolios: the sum of cluster member weights in a cluster, and the sum of cluster portfolio weights should be equal to 1, respectively. In addition to that, depending on the asset allocation allowing short sales or not, we have positive or negative weights constraints. If we let w_g denote the weight of gth cluster portfolio, and $w_{g,s}$ denote the weight of cluster member s in cluster $\mathcal{C}_g$, the constraints are described as follows:

$$\sum_{g=1}^{G} w_g = 1, \quad \text{and} \quad \sum_{s\in\mathcal{C}_g} w_{g,s} = 1, \tag{7}$$

with either

$$w_g \geq 0, \quad w_{g,s} \begin{cases} \geq 0 & \forall s, g : s \in \mathcal{C}_g \\ = 0 & \text{otherwise} \end{cases}, \tag{8}$$

or

$$-\infty < w_g < +\infty, \quad w_{g,s} \begin{cases} \in (-\infty, +\infty) & \forall s, g : s \in \mathcal{C}_g \\ = 0 & \text{otherwise} \end{cases}. \tag{9}$$

The terminal weight of an asset s is denoted by $\widetilde{w}_s$, which is the product of the cluster portfolio weight w_g and the cluster member weight $w_{g,s}$,

$$\widetilde{w}_s = w_g \cdot w_{g,s} \quad \forall g : s \in \mathcal{C}_g. \tag{10}$$

Thus the reward r_p and risk σ_p of the portfolio can be described as:

$$r_p = \sum_{g=1}^{G} w_g r_g = \sum_{g=1}^{G} w_g \sum_{s\in\mathcal{C}_g} w_{g,s} r_s = \sum_{s=1}^{N} \widetilde{w}_s r_s, \tag{11}$$

$$\sigma_p = \sqrt{\sum_{s=1}^{N} \sum_{k=1}^{N} \widetilde{w}_s \widetilde{w}_k \sigma_{s,k}}. \tag{12}$$

where r_g is the return of cluster portfolio,

$$r_g = \sum_{s\in\mathcal{C}_g} w_{g,s} r_s, \tag{13}$$

and $\sigma_{s,k}$ is the covariance between asset s and k.

The above optimization problem can hardly be solved by using traditional numerical methods, since Brucker [7] points out that the problem turns out to be NP-hard while the cluster number G turns higher.

2.2 Asset Allocation Methods

This subsection introduces three asset allocation methods which include the naive 1/N equal weights approach and two methods from the Markowitz mean-variance framework, to compute the cluster members weights $w_{g,s}$ and cluster portfolio weights w_g, respectively.

2.2.1 The $1/\tilde{N}$ Allocation

We denote the equal weights allocation as $1/\tilde{N}$ in this paper, to distinguish it from the traditional 1/N strategy. In the $1/\tilde{N}$ allocating procedure, the cluster portfolio weights are decided by the cluster number G, and cluster member weights in a cluster are decided by the cluster size, i.e. the number of assets in the cluster. One should note that the cluster number G is manually assigned, whereas the cluster size depends on the optimized pattern. Therefore, the cluster portfolio weights w_g are given by 1 over the cluster number G, and cluster member weights $w_{g,s}$ in the subset $\mathcal{C}_g$ are computed by taking 1 over the number of cluster members $\sharp\{\mathcal{C}_g\}$ respectively,

$$w_g = \frac{1}{G}, \tag{14}$$

$$w_{g,s} = \frac{1}{\sharp\{\mathcal{C}_g\}} = \frac{1}{\sum_{s=1}^{N} I_{s\in\mathcal{C}_g}}. \tag{15}$$

2.2.2 The Markowitz MVP Allocation

The MVP portfolio is the safest portfolio yielding the minimum variance at the Markowitz efficient frontier. We use quadratic programming to compute the cluster member weights, as well as the cluster portfolio weights:

$$\max_{\boldsymbol{w}} \lambda \boldsymbol{r'w} - (1-\lambda)\boldsymbol{w'\Sigma w}, \tag{16}$$

where $\boldsymbol{w}$ is either the cluster portfolio weights vector or cluster member weights vector, depending on whether the $\boldsymbol{r}$ represents the expected return vector of cluster portfolios or cluster members, and $\boldsymbol{\Sigma}$ is the variance-covariance matrix describing the correlation of cluster portfolios or cluster members. The λ is set at 0 for a minimum variance portfolio. The MVP allocation is a special portfolio at the Markowitz efficient frontier, which can be used as a proxy of other efficient portfolios, the cluster effect on the MVP is same as on the portfolios which locate on the efficient frontier. By setting the λ at a value range $0 < \lambda < 1$, we have other efficient portfolios at the frontier.

2.2.3 The Modified Tobin Tangency Allocation

The third asset allocation is an extension from the Tobin's original framework, which has an analytical solution when short sales is allowed and a market safe rate is available. In this study, the terminal portfolio is a tangency portfolio based on the cluster tangency portfolio returns, which are constructed using the assets returns in each cluster as inputs of the tangency allocation. The tangency allocation is also employed to construct the terminal portfolio based on the cluster portfolios return. The weights distribution from the tangency portfolio allocation is described as follows:

$$\boldsymbol{A} = \begin{bmatrix} \boldsymbol{r'} \\ \boldsymbol{I'} \end{bmatrix} \boldsymbol{\Sigma^{-1}} \begin{bmatrix} \boldsymbol{r} & \boldsymbol{I} \end{bmatrix} \equiv \begin{bmatrix} a & b \\ b & c \end{bmatrix}, \tag{17}$$

$$\boldsymbol{w} = \boldsymbol{\Sigma^{-1}} \begin{bmatrix} \boldsymbol{r} & \boldsymbol{I} \end{bmatrix} \begin{bmatrix} \frac{1}{b-r_f\cdot c} \\ \frac{-r_s}{b-r_f\cdot c} \end{bmatrix}, \tag{18}$$

where $\boldsymbol{I}$ is the unity vector, r_f is the risk-free rate. $\boldsymbol{r}$ represents a vector of the expected returns of either cluster portfolios or cluster members, $\boldsymbol{\Sigma}$ is a variance-covariance matrix describing the correlation of cluster portfolios or cluster members, and correspondingly the $\boldsymbol{w}$ is the vector representing either cluster portfolio weights w_g or cluster member weights $w_{g,s}$.

2.3 Optimization method

Most of the heuristic algorithms provide a way to construct possible solutions and find a best solution based on an evolutionary concept. The algorithms generate new solutions by recombining or modifying existing solutions, then select better solutions comparing with their predecessors given a function that measures how good each solution is. Heuristic methods have been applied by Gilli *et al.* [8], Gilli and Winker [9] to solve optimization problems in finance and econometrics. Maringer [10] discusses constrained index tracking problems under investor loss aversion behavior.

Storn and Price [11] propose the Differential Evolution algorithm which is originally designed for the problems with continuous solution space. Here we propose an application of the algorithm to the clustering problem by using a variant of the original DE, which takes the advantage of diversity from noise to escape from local optima convergence and avoid premature convergence. Let the row vectors $\boldsymbol{v_p}$, $p = 1...P$ denoted as solutions, for each current solution p, a new solution $\boldsymbol{v_c}$ is generated by randomly choosing three different members from the current population ($p_1 \neq p_2 \neq p_3 \neq p$) with linear combination their corresponding solution vectors in probability (π_1), otherwise the new solution inherits the original pth solution with probability $1-\pi_1$. In the standard DE, only the population size P, the scaling factor F and the cross-over probability π_1 need to be considered. The extra noise is generated by adding normally distributed random number vectors with the mean value being zero, to F value and the difference of two solution vectors[1], respectively.

[1]Details of the standard DE algorithm please refer to Storn and Price's paper [11].

The noise vectors $\boldsymbol{z_1}$ and $\boldsymbol{z_2}$ have the property: they are random variables being zero in two probability $\boldsymbol{\pi_2}$ and $\boldsymbol{\pi_3}$, otherwise following normally distribution $N(0, \sigma_1^2)$ and $N(0, \sigma_2^2)$ respectively. Thus the modified linear combination of the cross-over procedure is decried as follows:

$$v_c[i] := \begin{cases} v_p[i] & \text{with probabiltiy } 1 - \pi_1, \text{ or} \\ v_{p1}[i] + (F + z_1[i]) \cdot (v_{p2}[i] - v_{p3}[i] + z_2[i]), & \end{cases} \tag{19}$$

where π_1 is the cross-over probability. We translate the solutions to cluster sets by rounding them to the nearest integers, which tell the belonging of each asset. After the linear combination, the algorithm updates the elitist by using the solution with higher fitness, which is defined as the Sharpe ratio value in this paper. Whether a replacement of the solution of $\boldsymbol{v_p}$ with $\boldsymbol{v_c}$ will process is decided from running a comparison, i.e. if the fitness value of $\boldsymbol{v_c}$ is better than the one of $\boldsymbol{v_p}$, the solution $\boldsymbol{v_p}$ is replaced by $\boldsymbol{v_c}$, and the updated $\boldsymbol{v_p}$ exists in the current population. The process is repeated until a halting criterion met. The DE optimization procedure is described using the following pseudo code.

Algorithm 2.1: *SR* MAXIMIZATION($\boldsymbol{v_p}$)

randomly initialize population of vectors $\boldsymbol{v_p}$, p= 1...P;
while do
  {*generate new solutions* $\boldsymbol{v'_p}$*:*}
  for all current solutions $\boldsymbol{v_p}$, p=1...P **do**
    randomly pick $p_1 \neq p_2 \neq p_3 \neq p$;
    $v_c[i] \leftarrow v_{p1}[i] + (F + z_1[i]) \cdot$ $(v_{p2}[i] - v_{p3}[i] + z_2[i])$ at probability π_1;
    or $v_c[i] \leftarrow v_p[i]$ at probability $1 - \pi_1$;
  interpret $\boldsymbol{v_c}$ into clustering partition;
  apply asset allocations to compute portfolio return and SR;
  end for;
  {*select new population:*}
  for all current solutions $\boldsymbol{v_p}$, $p = 1...P$ **do**
    if Fitness($\boldsymbol{v_c}$) > Fitness($\boldsymbol{v_p}$) **then**;
      $\boldsymbol{v_p} \leftarrow \boldsymbol{v_c}$;
    end if;
  end for;
until halting criterion met.

2.4 Data and Implementation

We downloaded the adjusted daily prices of FTSE 100 stocks in the period from January 2005 to December 2006, and the prices of DJIA 65 stocks in the period from January 2003 to December 2004 from Yahoo.com. We computed the asset returns by taking log return of the daily price series. The in-sample experiments and out-of-sample experiments employ the first year and second year data respectively. All experiments were performed on Matlab version 2007b and a Pentium 4 machine. Based on preliminary tests, the technical parameters of DE algorithm are set as follows. Population size is set at 100, although a general rule of thumb advises that the population size should be at least three times the number of variables, i.e. the size should be over 300. Instead of using such a large population, we set the iteration number at a higher level, says 100 thousands times. The weighting factor F is set at 0.7, the crossover probability π_1 is 50 percent. The parameters for generating the artificial noise $\boldsymbol{z_1}$ and $\boldsymbol{z_2}$ are listed as follows: $\pi_2 = 70\%$, $\pi_3 = 30\%$, $\sigma_1^2 = 0.1$ and $\sigma_2^2 = 0.03$.

Portfolio stability issue has been widely discussed in the literature. The portfolio stability issue in this paper referring how assets weights in a portfolio are sensitive to the errors in parameters inputs of an allocation. We use an instability measure which is proposed by Farrelly [12] to study weights changes due to estimation errors and noise subject to portfolio rebalance and transactions, which is defined as follows:

$$\mathcal{I} = \frac{\sum_{i=1}^{N} |\widetilde{w}_{i,o} - \hat{w}_{i,e}|}{2}, \tag{20}$$

where $\widetilde{w}_{i,o}$ stands for the asset weights in a portfolio that is constructed based on the 'true' asset information, while $\hat{w}_{i,e}$ are the asset weights of the portfolio that are computed by using the estimates containing errors and noise. In the instability experiments, we use the in-sample asset returns as 'the true' asset information, and artificially generate the asset returns containing errors and noise by adding noise disturbances to assets which are randomly selected each time. We randomly choose 20% of the market assets each time, and set the noise expected value at 0, standard deviation at 0.5% in the experiment. We compute the instability $\mathcal{I}$ each time and take the average as the final instability after 10,000 times, while setting the cluster number G at five different values, to tell the clustering impact on weights instability.

Since Sharpe ratio is widely used as a measure of portfolio performance, we use it to evaluate risk-adjusted return performance of the terminal portfolios over the both in-sample and out-of-sample period. The risk-free return r_f is set at 0 because the experiments employ daily returns. Furthermore, we employ the two sample Kolmogorov-Smirnov (KS) test to statistically judge whether the terminal portfolio return distributions over the in-sample and out-of-sample period will be affected by the clustering.

3 Computational Results

The section discusses the clustering impact on the weights instability, Sharpe ratios, and return distribution of the terminal portfolios over the in-sample and out-of-sample period.

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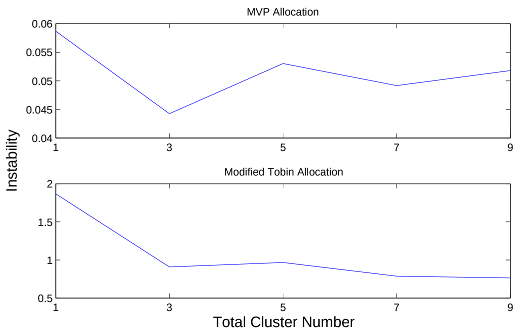

Figure 1: Clustering Impact on Portfolio Weights Instability, FTSE Market

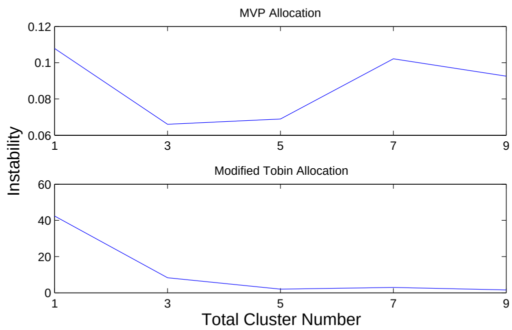

Figure 2: Clustering Impact on Portfolio Weights Instability, DJIA Market

3.1 Portfolio Instability

Figure 1 and Figure 2 show the instability $\mathcal{I}$ which is computed from the simulated FTSE and DJIA returns containing the errors and noise in five cluster number cases, i.e. the G value being 1, 3, 5, 7 and 9. We studied the weights instability of portfolios from the MVP allocation and Tobin tangency allocation using the FTSE and DJIA data respectively. We did not include the $1/\tilde{N}$ allocation in this experiment since the allocation distributes weights independently of return means and variances. As the figure shows that, the instability $\mathcal{I}$ turns lower while we set the cluster number G at larger values in the Tobin allocation case, and the weights instability from the MVP allocation is decreased further while the cluster number G turns higher. The above evidences support that the portfolio weights from the clustered markets are less sensitive to the changes, or to the estimation errors and noise in the input parameters than that from the non-clustered markets.

3.2 Sharpe Ratios and Return Distribution

The second and third column in Table 1 provide the in-sample and out-of-sample Sharpe ratios, which are computed from the terminal portfolios returns based on the FTSE data using the three asset allocations. In the cases of using the $1/\tilde{N}$ and MVP allocation, we find that the out-of-sample Sharpe ratios are increased by 0.25% when the cluster number is set at 3 respectively. In the modified Tobin allocation case, the out-of-sample Sharpe ratio is increased by 20% when the cluster number G is set at 7. The fourth column in Table 1 provides the p-values from the K-S test. According to the p-values, the test rejects the hypothesis that the in-sample returns and out-of-sample returns follow a same distribution at a 10% confidence level. Surprisingly, the non-clustered cases from the three allocations have the p-values less than 10%, which indicates a structural break happened in the FTSE market between 2005 and 2006. To reconcile that the clustering improves the Sharpe ratio of terminal portfolios, we shall provide the experimental results using a different market data in different periods.

Table 1: Sharpe Ratios and p-values, FTSE Market

	SR(I)	SR(O)	p-values
$1/\tilde{N}$			
G=1	0.159	0.077	0.095
G=3	0.207	0.079	0.053
G=5	0.213	0.078	0.041
G=7	0.214	0.075	0.041
G=9	0.220	0.079	0.032
MVP			
G=1	0.155	0.083	0.008
G=3	0.207	0.085	0.008
G=5	0.230	0.082	0.006
G=7	0.239	0.081	0.012
G=9	0.245	0.076	0.012
Tobin			
G=1	0.622	0.073	0
G=3	0.610	0.064	0.001
G=5	0.610	0.082	0.002
G=7	0.611	0.088	0.001
G=9	0.612	0.066	0.001

Table 2 reports the experimental results using the DJIA market data in 2003 and 2004. Again we find the evidence that clustering the DJIA market improves the Sharpe ratios of the terminal portfolios in the out-of-sample period. Particularly, in the best scenarios, the out-of-sample Sharpe ratios are increased by 5.7%, 78% and 28% while using the $1/\tilde{N}$, MVP and the modified Tobin allocation respectively. Therefore, we are confident that the clustering can improve Sharpe ratio of the terminal portfolios. Now turning to results from the K-S test, in the cases of using the first two allocations, it is hardly to reject that the terminal portfolio returns over the in-sample and out-of-sample period are different since the p-values are close 1. However, the p-values in the modified Tobin allocation case are all close to zero, implying the difference in return distribution comes from the modified Tobin allocation, rather than the clustering.

Table 2: Sharpe Ratios and p-values, DJIA Market

	SR(I)	SR(O)	p-values
$1/\tilde{N}$			
G=1	0.069	0.087	1.000
G=3	0.102	0.092	0.936
G=5	0.104	0.092	0.967
G=7	0.103	0.088	0.893
G=9	0.102	0.091	0.967
MVP			
G=1	0.073	0.038	0.329
G=3	0.118	0.060	0.238
G=5	0.129	0.068	0.167
G=7	0.129	0.058	0.093
G=9	0.128	0.064	0.167
Tobin			
G=1	0.494	0.063	0
G=3	0.477	0.064	0
G=5	0.470	0.074	0
G=7	0.467	0.081	0
G=9	0.453	0.050	0

4 Conclusion

This paper presents a new asset allocation model that combines a clustering technique with asset allocation methods to improve portfolio Sharpe ratios and portfolio weights stability. Using the Differential Evolution algorithm, we identify optimal clustering patterns in different cluster number cases, which contribute to higher Sharpe ratios and lower portfolio instability than that of portfolios from the non-clustered markets over the both in-sample and out-of-sample period. Market practitioners may cluster assets before they distribute portfolio weights when they have desires to improve portfolio weights stability and risk-adjusted returns. In future research, we shall discuss the Differential Evolution stability and efficiency, in the comparison with that of other evolutionary methods, such as Threshold Accepting and Simulated Annealing while tackling the clustering problem, since the Differential Evolution is originally designed for problems with continuous solution space rather than the discrete solution space.

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