Quantity-Setting Competition Under Uncertain Demand

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Abstract— **We consider a quantity-setting duopoly model, and we study the decision to move first or second, by assuming that the firms produce homogeneous goods and that there is some demand uncertainty. The competitive phase consists of two periods, and in either period, the firms can make a production decision that is irreversible. As far as the firms are allowed to choose (non-cooperatively) the period they make the decision, we study the circumstances that favour sequential rather than simultaneous decisions.**

Keywords: Industrial Organization, Game Theory, Cournot model, uncertainty

1 Introduction

In a standard duopoly, firms choose either prices or quantities in a non-cooperative fashion. If the decisions are made simultaneously, these models are called, respectively, Bertrand model and Cournot model (see [2, 3]). Sometimes, one of the firm has the opportunity to make his decision before the other firm. In a quantity setting, the situation is called Stackelberg model (see [17]). Stackelberg leader-follower relations have most often been modeled in association with the chronological order of moves. Namely, there are a first mover (leader) and a second mover (follower). In spite of such a supposedly dynamic setting, it has been common to overlook what happens during the period between these two moves, by assuming a static market which clears only once, after the second mover's move. This builds certain biases into the analysis of firms' strategic incentives either to lead or to follow, which are the contributing forces to endogenous Stackelberg outcomes.

In the earlier literature, endogenous leader/follower has been imbedded most often in the context of a timing game played by oligopolists. Hamilton and Slutsky [7] construct an 'extended game' framework, in which each firm faces the choice of production timing. A fair number of theoretical explanations have been attempted with regard to firms' incentives for Stackelberg behaviour, especially a follower's incentive to wait. Robson [14] imposes costs associated with an early action. Albaek [1] takes into account cost uncertainty. The effect of *a priori* informational heterogeneity between firms, broadly defined, have been discussed in several studies, including Mailath [12] and Normann [13]. On the other hand, when the oligopolists are *a priori* equally uncertain about the market demand, as in Maggi [11], Sadanand and Sadanand [15] and Spencer and Brander [16], earlier production can utilize less information in exchange for the strategic advantage of commitment, whereas later production does the converse. Hirokawa and Sasaki [8] employed a similar framework to Hamilton and Slutsky's 'extended game', except that the static market is replaced with an explicitly two-period market. Lagerlöf [10] shows that if the distribution of the demand uncertainty has a monotone hazard rate and if another, rather weak, assumption is satisfied, then uniqueness of equilibrium is guaranteed. Ferreira et al. [4] study the effects of demand uncertainty in a Stackelberg duopoly.

In this paper, we follow closely the paper of Kultti and Niinimäki [9], by considering a more general demand function. We assume that the competitive phase consists of two periods. In either period, the firms can make a production decision that is irreversible. As far as the firms are allowed to choose (non-cooperatively) the period they make the decision, one can study the circumstances that favour sequential rather than simultaneous decisions. If this is the only change in the standard setting with perfect information there are now three pure strategy subgame perfect equilibria when the firms are symmetric (see [7]). Either of the firm is a leader and the other one a follower, or both of them make the same choices as in the standard setting in the first period. The firms prefer the equilibrium in which they move first. Even though no equilibrium selection is helpful here the symmetry of the situation makes the symmetric equilibrium appealing.

In our work the production period plays a non-trivial role since we assume that the demand is uncertain, and that the uncertainty is resolved once either firm makes its production decision. The enterprise bears a close relationship to the literature about endogenous timing of moves in oligopolies (see [1, 5, 6, 7, 12, 13]). Hamilton and Slutsky [7] study two different games: A game in which

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firms announce in which period they are going to produce, and are committed to this announcement, and a game in which the firms can choose in which period to produce only by actually producing. Our model corresponds to the second game, the difference being uncertainty about demand and differentiated goods. In our model, simultaneous moves is never an equilibrium, and depending on the variance of the demand, either the first mover or the second mover may be more profitable.

The crucial assumption in our model is the way demand is revealed. If either firm produces in the first period, demand is known in the second period. In case neither firm produces in the first period, demand remains unknown in the second period. This is clearly a very specific assumption that applies only to some markets. Perhaps, the most important is the case of new products. Demand for new products is highly uncertain, and many times the only way to find it is to enter the markets by producing the product. By the assumption made, the demand is revealed since some products are sold in the first period, and then we can ask why the firm cannot produce more in the second period if demand turns out to be strong. The answer is the same as in the standard Stackelberg model: it is assumed that the firms are committed to the levels of production which they choose. The standard static case is an approximation of a dynamic real life situation that is compressed into two stages. Our model can be regarded as an approximation of a real life situation in which a producer brings a new product to the market. First, he has to expend his time in production, and only after this he sells the product which is time consuming as well. The competitor produces while the first producer sells his products. This model allows us to analyse the trade-off between producing early, and being well informed about demand.

2 The model

There are two firms and two time periods. Both firms produce a differentiated good. The demand, for simplicity, is linear, namely

$$p = \alpha - \beta(q_1 + q_2),$$

with $\alpha > 0$ and $\beta \geq 1$, where p is the price and q_i the amount produced of good i, for $i \in \{1, 2\}$. Firms have the same constant marginal cost c. We consider that the demand intercept is a random variable which is assumed to have a continuous density. The density of α is, however, common knowledge. The expected value of α is $E(\alpha)$. We assume that the variance $V(\alpha)$ of α is not too large in a sense that in no case the firms produce so much that price drops to zero.

Our aim is to study the effect of information revelation on the timing of the firms' production decisions. To this end we model the firms playing the following extensive game. The firms make their decisions non-cooperatively, and they may choose the quantity to be produced in either period. If a firm produces already in period 1 the choice is common knowledge in period 2, and the true demand is revealed. If neither firm produces in the first period no information about demand is revealed in the second period. Notice that the game is not a signalling game, and that unlike in many models only actions speak; firms commit to a production decision by producing, not making announcements about when they intend to produce and how much (see [1, 7]). Next we determine the profits when the firms move sequentially and simultaneously, and then we compare the profits in the two cases.

2.1 Sequential decisions

Without loss of generality, let firm 1 be the first mover and make its decision in the first period. Firm 2 is the follower which delays its production decisions until the second period. Firm 1 believes (correctly) that its production decision in period 1 will influence firm 2's decision a period later. That is, the follower will select q_2 to maximize its profit

$$\pi_2 = (\alpha - \beta q_1 - \beta q_2(\alpha) - c)q_2(\alpha).$$

Thus,

$$q_2(\alpha) = \frac{\alpha - \beta q_1 - c}{2\beta}. \tag{1}$$

First mover's decision problem is to maximize his expected profit

$$\begin{aligned} E(\pi_1) &= E((\alpha - \beta q_1 - \beta q_2 - c)q_1) \\ &= \frac{E(\alpha) - \beta q_1 - c}{2} q_1. \end{aligned}$$

Thus,

$$q_1 = \frac{E(\alpha) - c}{2\beta}. \tag{2}$$

Using equation (2), the follower's choice (1) can be rewritten

$$q_2(\alpha) = \frac{2\alpha - E(\alpha) - c}{4\beta},$$

and the equilibrium price turns out

$$p(\alpha) = \frac{2\alpha - E(\alpha) + 3c}{4}.$$

First mover's expected profit is

$$E(\pi_1) = \frac{(E(\alpha) - c)^2}{8\beta}, \tag{3}$$

while the follower's expected profit is

$$E(\pi_2) = \frac{(E(\alpha) - c)^2}{16\beta} + \frac{V(\alpha)}{4}. \tag{4}$$

First mover has an advantage, if

$$\frac{(E(\alpha) - c)^2}{8\beta} > \frac{(E(\alpha) - c)^2}{16\beta} + \frac{V(\alpha)}{4},$$

which is equivalent to

$$(E(\alpha) - c)^2 > 4V(\alpha). \tag{5}$$

If the variance in α is small, the usual Stackelberg case where the first mover has always an advantage prevails. Only if the variance is large, the first mover may fare worse than the second mover. Note that the first mover's profit does not depend upon the variance. This comes from the linear demand and the fact that the variance is assumed small enough so that realized prices are always positive. The second mover's profit depends upon the variance since variance indicates the pay-off from waiting as the second mover knows the realized demand.

So, we have proved the following result.

Theorem 1. The second mover earns higher profits than the first mover, if the variance is large enough (i.e., if $V(\alpha) > (E(\alpha) - c)^2/4$). Otherwise, the first mover earns higher profits.

2.2 Simultaneous decisions

As long as both firms make their production decisions simultaneously the profits are the same regardless of the period, since the assumptions about the revelation of information guarantee that the demand is unknown. Firm 1 maximizes its expected profit

$$E(\pi_1) = E((\alpha - \beta(q_1 + q_2) - c)q_1).$$

Thus,

$$q_1 = \frac{E(\alpha) - \beta q_2 - c}{2\beta}.$$

Similarly, we get

$$q_2 = \frac{E(\alpha) - \beta q_1 - c}{2\beta}.$$

Thus, in equilibrium, output decisions are given by

$$q_1 = q_2 = \frac{E(\alpha) - c}{3\beta},$$

and the price given by

$$p(\alpha) = \frac{3\alpha - 2E(\alpha) + 2c}{3}.$$

So, the expected profits of both firms are equal, given by

$$E(\pi_1) = E(\pi_2) = \frac{(E(\alpha) - c)^2}{9\beta}. \tag{6}$$

It is easy to establish that, in equilibrium, both firms do not produce in the first period; a revealed preference argument is sufficient to establish this. Assume that there is an equilibrium in which both firms produce in the first period. Denote the firms' equilibrium outputs by q_1^* and q_2^*. Consider, say, firm 1. Suppose that it deviates and waits until the next period when it gets to know the realized demand. It can still produce q_1^*, but with full knowledge of the demand this output level is not the optimal choice. Firm 2 produces $(E(\alpha) - c)/(3\beta)$ and the upcoming production of deviating firm 1 will be

$$\frac{3\alpha - E(\alpha) - 2c}{6\beta}.$$

Thus, the expected profit is

$$\frac{(E(\alpha) - c)^2}{9\beta} + \frac{V(\alpha)}{4\beta},$$

which is $V(\alpha)/(4\beta)$ higher than if the firm would not deviate. This shows that there is no equilibrium with both firms producing in the first period. Thus, there are three possible equilibria: (i) Firm 1 produces in the first period and firm 2 in the second period; (ii) Firm 2 produces in the first period and firm 1 in the second; and (iii) both firms produce in the second period.

Next, we compare the profits in the sequential and simultaneous moves cases to determine whether or when sequential moves are more profitable than simultaneous moves. Whenever the first mover's expected profit is larger than his expected profit in the simultaneous move case, simultaneous moves is not an equilibrium. But, from equations (3) and (6), we see that this is always the case. We must still show that firm 2 does not deviate and produce in the first period, when firm 1 is already producing

$$\frac{E(\alpha) - c}{2\beta}$$

in the first period. Firm 2's optimal output choice in the first period is

$$\frac{E(\alpha) - c}{4\beta}.$$

Thus, firm 2's expected profit is

$$\frac{(E(\alpha) - c)^2}{16\beta},$$

$V(\alpha)/(4\beta)$ less than if the firm waited to the next period. Thus, firm 2 does not deviate and there does not exist an equilibrium in which the firms move simultaneously.

Now, we are going to compare the expected profits of the two moving alternatives. Firm F_2 prefers sequential solution to simultaneous moves, if (4) is larger than (6), which is equivalent to

$$36V(\alpha) > 7(E(\alpha) - c)^2. \tag{7}$$

Combining conditions (5) and (7), we get that

$$7(E(\alpha) - c)^2/9 < 4V(\alpha) < (E(\alpha) - c)^2, \tag{8}$$

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and we can say that if condition (8) holds the first mover earns more than the follower and both firms prefer sequential moves to simultaneous moves.

So, we have proved the following result.

Theorem 2. The game presented in this paper has exactly two equilibria in both of which the firms move sequentially.

This result implies that, in cases where demand uncertainty is revealed only after at least one firm produces, there are no simultaneous equilibria. The case in which both firms move simultaneously in the first period is not an equilibrium, since either firm can wait till the next period when it has the same choice set as in the first period, and additionally it knows the realized demand. The case in which both firms move simultaneously in the second period is not an equilibrium roughly because a deviating firm gains a first mover advantage. Generally, this is an advantage only with respect to the simultaneous moves case since it is possible that the second mover's profits are greater than the first mover's profits.

3 Conclusions

We have shown that in cases where demand uncertainty is revealed only after at least one firm produces, there are no simultaneous equilibria in a quantity-setting duopoly. We also proved that the second mover earns higher profits than the first mover, if the uncertainty is high; Otherwise, the first mover earns higher profits.

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