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# Engineering Mathematical Study for the Visco-Elastic Impact Problem

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*Abstract*— **The research for the complex properties of visco-elastic material in various impact conditions have been interesting and difficult problems, especially in the auto-crash safety engineering process. In this paper an application of boundary-layer computing for the non-Newtonian rate type impact hardening and share thinning phenomenon are discussed. The numerical scheme yields the convergent finite element analysis (FEA) solution and stable semi-discrete Galerkin-Runge-Kutta(G-RK) iteration. To develope the passive protection model based on the non-recover solid metal (phase1) impact, we are deepening our knowledge of soft material recover protection (phase2) concept. The high performance computing (HPC) tool enabled us to carry out the research on the passive safety analysis with engineering computing.**

*Keywords*—**convergence, FEA, semi-discrete, transient, Garlerkin-RK**

## I. Introduction

Several articles are useful sources of information of how to create a link between the crucial life saving virtual test and the celebrated dry honeycomb structure (phase1-P1) [1,7] with the visco-elastic plastic (phase2-P2) theory [5,8,9]. for example, [2,3,4,11] are standard texts giving mathematical and engineering perspectives upon the subject. Further more [7] gave specific computational method, in which the rate control of honeycomb strength is based on the non-recoverable crush densification. In this paper we introduce the development of recoverable controlled fluid-structure interaction soft solid concept（P2）from 2 dimension [8] to 3 dimension [9,18] in positive definite FEA schemes.

Shear thinning flow and yield stresses are common effects of “visco elasticity”. Weissenberg effects [11] include die swell are non-Newtonian. Wherein fluid emerges from a pipe and then undergoes a subsequent and sudden radial expansion downstream. Visco-elastic flow past a bubble leads to a distinct cusp at the rear stagnation point due to a long filament of highly stretched polymers in the bubble wake. An important point that one should take from this discussion is that non-Newtonian fluid effects can be varied and unusual. As a result, the literature on non-Newtonian fluid mechanics contains many models of suspensions and polymeric fluids, each adding or encapsulating some observed effect. The first stable choice for the non-Newtonian viscosity has been made in the work by [11]. After Marchal and Crochet [2,3,16] the research had been focused on the reformulation of the momentum equation [4,19,20]. So that it includes the contribution of the micro-scopic effects towards the FEA solution of the P-T/T equation in calculation of the macro-scopic non-Newtonian stress flow [15,22,23]. In our paper, a specific numerical treatment can introduce the contribution of the micro-scopic shell element model [7] towards the currently validated macro-scopic non-Newtonian cubic element models.

As one can see in the framework of the general 3 dimension transient, positive, semi-discrete scheme [18,21], the objective derivative or the decoupled solution approach shall hide the expression of the term involving the convective derivative. On the basis of this argument it is important to carry out a numeric test for P2 in transient 3 dimension positive definite condition in the practical interests. The numerical results of non-Newtonian flow and stress are obtained with the F90 programming on the platform of the HPC centre in the mathematics department.

## II. The mathematical model

By use of standard rheologic term in material physics, we may analyze the special feature of the non-Newtonian P-T/T equation, for the resistance to the extensional and simple shear

$$\lambda \dot{\underline{\underline{\tau}}} = [2\eta \underline{\underline{D}} - \exp\{\frac{\varepsilon\lambda}{\eta_0}(\tau_{xx}+\tau_{yy})\}\underline{\underline{\tau}}] - \lambda[\underline{u}\bullet\nabla\underline{\underline{\tau}} - \nabla\underline{u}\bullet\underline{\underline{\tau}} - (\nabla\underline{u}\bullet\underline{\underline{\tau}})^T$$
$$+\xi(\underline{\underline{D}}\bullet\underline{\underline{\tau}} + (\underline{\underline{D}}\bullet\underline{\underline{\tau}})^T], \quad in \quad \Omega$$

which gives best estimate of the stress over-shoot for the elongating element over-stretch known as non-slip impact hardening besides the simple shear. Here $\underline{\underline{\tau}}$ is the stress field，$\underline{\underline{D}}$ is strain，$\underline{u}$ is the deformation velocity field in e/p material，$\eta$ is the viscosity，$\lambda$ the relaxation constant，$\varepsilon$ is the elongation rate，$\xi$ is the shear rate,

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$\rho$ is the density. The FEA calculation of the moving Maxwell type equation is at least 2nd-order of convergence by use of the Adini-type elements.

On the other hand, we calculated the large e/p deformation resulting from stress rate $\dot{\tau}$ known as flow shear thinning. That is the Cauchy conservation equation subject to the P-T/T stress effects,

$$\rho\, \dot{\underline{u}} = [\nabla \bullet \underline{\underline{\tau}} - \rho\, \underline{u} \bullet \nabla\, \underline{u}] \text{ in } \Omega - \Gamma_0,$$

including the velocity field $\underline{u}$ in region $\Omega - \Gamma_0$. The initial boundary condition of stress is decided by static test $\underline{\underline{\tau}}(0)$ in the impact experiment and the impact speed $\underline{u}(0)$， that is: $\underline{\underline{\tau}}(0) = \underline{\underline{\tau}}(static)$ ； $\underline{u}(0) = u_{x0}$ on $\Gamma_0 \subset \Gamma$； where $\Omega$ is the material volume， $\Gamma$ is the surface， $\Gamma_0$ is the contacted surface (along x moving boundary direction). We treat the boundary (usually singular) between the contacting and non-contacting surfaces as the free surface or flow from the over-stretched perturbation. Extensional dominated flow of visco-elastic fluids are encountered in the laboratory tests (e.g. fiber spinning, spray, foam deformation and extensional rheometry).

### 2.1 The discrete form

The positive definite semi-discrete form of the Euler-Galerkin method is the simplified step of the coupled Cauchy， P-T/T equations. The further higher order computation is based on the following analysis

$$\frac{\partial \underline{u}}{\partial t} = \frac{1}{\rho}\nabla \cdot \underline{\underline{\tau}} - \underline{u} \cdot \nabla\, \underline{u}$$

With discrete component form：

$$\frac{\partial u_x}{\partial t} = \frac{1}{\rho}\left(\frac{\partial \tau_{xx}}{\partial x} + \frac{\partial \tau_{xy}}{\partial y}\right) - \left(u_x^n \frac{\partial u_x^n}{\partial x} + u_y^n \frac{\partial u_x^n}{\partial y}\right),$$

$$\frac{\partial u_y}{\partial t} = \frac{1}{\rho}\left(\frac{\partial \tau_{xy}}{\partial x} + \frac{\partial \tau_{yy}}{\partial y}\right) - \left(u_x^n \frac{\partial u_y^n}{\partial x} + u_y^n \frac{\partial u_y^n}{\partial y}\right),$$

By use of the residuals method，the weak form of the Cauchy equation is：

$$\langle \frac{\partial u_x}{\partial t}\ \psi_i \rangle = \langle \frac{1}{\rho}\left(\frac{\partial \tau_{xx}}{\partial x} + \frac{\partial \tau_{xy}}{\partial y}\right) - \left(u_x^n \frac{\partial u_x^n}{\partial x} + u_y^n \frac{\partial u_x^n}{\partial y}\right), \psi_i \rangle, \quad (i = 1, \cdots, N),$$

$$\langle \frac{\partial u_y}{\partial t}\ \psi_i \rangle = \langle \frac{1}{\rho}\left(\frac{\partial \tau_{xy}}{\partial x} + \frac{\partial \tau_{yy}}{\partial y}\right) - \left(u_x^n \frac{\partial u_y^n}{\partial x} + u_y^n \frac{\partial u_y^n}{\partial y}\right), \psi_i \rangle, \quad (i = 1, \cdots, N),$$

where， $N$ is the nodes number on each element.

On each node $(x_i, y_i)$ we have：

$$< \frac{\partial u_x}{\partial t}(x_i,y_i), \psi_i(x,y) > = < \frac{1}{\rho}\left(\frac{\partial \tau_{xx}(x_i,y_i)}{\partial x} + \frac{\partial \tau_{xy}(x_i,y_i)}{\partial y}\right), \psi_i(x,y) > - < u_x^n(x_i,y_i)\frac{\partial u_x^n(x_i,y_i)}{\partial x}, \psi_i(x,y) > - < u_y^n(x_i,y_i)\frac{\partial u_x^n(x_i,y_i)}{\partial y}, \psi_i(x,y) > \quad (i = 1, \cdots, N)$$

$$< \frac{\partial u_y}{\partial t}(x_i,y_i), \psi_i(x,y) > = < \frac{1}{\rho}\left(\frac{\partial \tau_{xy}(x_i,y_i)}{\partial x} + \frac{\partial \tau_{yy}(x_i,y_i)}{\partial y}\right), \psi_i(x,y) > - < u_x^n(x_i,y_i)\frac{\partial u_y^n(x_i,y_i)}{\partial x}, \psi_i(x,y) > - < u_y^n(x_i,y_i)\frac{\partial u_y^n(x_i,y_i)}{\partial y}, \psi_i(x,y) > \quad (i = 1, \cdots, N) \tag{2.2}$$

For each element the interpolation $\phi_i(x, y)$ are defined as :

$$u_x^n(x,y) = \sum_{j=1}^{N} u_x(x_j, y_j)\phi_j(x,y) = \sum_{j=1}^{N} u_x(j)\phi_j(x,y)$$

$$u_y^n(x,y) = \sum_{j=1}^{N} u_y(x_j, y_j)\phi_j(x,y) = \sum_{j=1}^{N} u_y(j)\phi_j(x,y)$$

which is the Galerkin method by taking $\psi_i(x, y) = \phi_i(x, y)$. Here $(x_i, y_i)$ is the mesh node of finite element.

If we use explicite form for fixed time from impact the integral form may be written as：

$$\frac{\sum_{j=1}^{N} u_x^{n+1}(j)\iint_e \phi_i(x,y)\phi_j(x_i,y_i)dxdy - \sum_{j=1}^{N} u_x^{n}(j)\iint_e \phi_i(x,y)\phi_j(x_i,y_i)dxdy}{\Delta t} \tag{2.3}$$
$$= \frac{1}{\rho}\left(\frac{\partial \tau_{xx}(x_i,y_i)}{\partial x} + \frac{\partial \tau_{xy}(x_i,y_i)}{\partial y}\right)\iint_e \phi_i(x,y)dxdy - \sum_{j=1}^{N} u_x^n(j)\sum_{k=1}^{N} u_x^n(k)\iint_e \phi_i(x,y)\phi_j(x_i,y_i)\frac{\partial \phi_k(x_i,y_i)}{\partial x}dxdy - \sum_{j=1}^{N} u_y^n(j)\sum_{k=1}^{N} u_x^n(k)\iint_e \phi_i(x,y)\phi_j(x_i,y_i)\frac{\partial \phi_k(x_i,y_i)}{\partial y}dxdy \quad (i = 1, \cdots, N)$$

and

（2.6）

（2.7）

$$\frac{\sum_{j=1}^{N} u_y^{n+1}(j)\iint_e \phi_i(x,y)\phi_j(x_i,y_i)dxdy - \sum_{j=1}^{N} u_y^{n}(j)\iint_e \phi_i(x,y)\phi_j(x_i,y_i)dxdy}{\Delta t}$$
$$= \frac{1}{\rho}\left(\frac{\partial \tau_{xy}(x_i,y_i)}{\partial x} + \frac{\partial \tau_{yy}(x_i,y_i)}{\partial y}\right)\iint_e \phi_i(x,y)dxdy$$
$$- \sum_{j=1}^{N} u_x^n(j)\sum_{k=1}^{N} u_y^n(k)\iint_e \phi_i(x,y)\phi_j(x_i,y_i)\frac{\partial \phi_k(x_i,y_i)}{\partial x}dxdy$$
$$- \sum_{j=1}^{N} u_y^n(j)\sum_{k=1}^{N} u_y^n(k)\iint_e \phi_i(x,y)\phi_j(x_i,y_i)\frac{\partial \phi_k(x_i,y_i)}{\partial y}dxdy$$
$$(i=1,\cdots,N)$$

Finally we worked out the matrix form of the coupled Non-Newtonian flow equation：

$$\frac{\{A\}_{N\times N}\cdot\{u_x^{n+1}\}_N - \{A\}_{N\times N}\cdot\{u_x^{n}\}_N}{\Delta t} = \frac{1}{\rho}\left\{\frac{\partial \tau_{xx}}{\partial x} + \frac{\partial \tau_{xy}}{\partial y}\right\}_N \odot \{D\}_N$$
$$- \sum_{j=1}^{N} u_x^n(j)\{E1j\}_{N\times N}\cdot\{u_x^n\}_N - \sum_{j=1}^{N} u_y^n(j)\{E2j\}_{N\times N}\cdot\{u_x^n\}_N$$

and

$$\frac{\{A\}_{N\times N}\cdot\{u_y^{n+1}\}_N - \{A\}_{N\times N}\cdot\{u_y^{n}\}_N}{\Delta t} = \frac{1}{\rho}\left\{\frac{\partial \tau_{xy}}{\partial x} + \frac{\partial \tau_{yy}}{\partial y}\right\}_N \odot \{D\}_N$$
$$- \sum_{j=1}^{N} u_x^n(j)\{E1j\}_{N\times N}\cdot\{u_y^n\}_N - \sum_{j=1}^{N} u_y^n(j)\{E2j\}_{N\times N}\cdot\{u_y^n\}_N$$

It however, can be reversed into the related semi-discrete matrix form for higher order approximation (both in time and space)：

$$\frac{\partial}{\partial t}\{u_x^n\} = \frac{1}{\rho}\{A\}_{N\times N}^{-1}\cdot\left(\left\{\frac{\partial \tau_{xx}}{\partial x} + \frac{\partial \tau_{xy}}{\partial y}\right\}_N \odot \{D\}_N\right)$$
$$- \sum_{j=1}^{N} u_x^n(j)\{A\}_{N\times N}^{-1}\cdot\{E1j\}_{N\times N}\cdot\{u_x^n\}_N$$
$$- \sum_{j=1}^{N} u_y^n(j)\{A\}_{N\times N}^{-1}\cdot\{E2j\}_{N\times N}\cdot\{u_x^n\}_N$$

and

$$\frac{\partial}{\partial t}\{u_y^n\} = \frac{1}{\rho}\{A\}_{N\times N}^{-1}\cdot\left(\left\{\frac{\partial \tau_{xy}}{\partial x} + \frac{\partial \tau_{yy}}{\partial y}\right\}_N \odot \{D\}_N\right)$$
$$- \sum_{j=1}^{N} u_x^n(j)\{A\}_{N\times N}^{-1}\cdot\{E1j\}_{N\times N}\cdot\{u_y^n\}_N$$
$$- \sum_{j=1}^{N} u_y^n(j)\{A\}_{N\times N}^{-1}\cdot\{E2j\}_{N\times N}\cdot\{u_y^n\}_N$$

The 2nd order convergence is guaranteed with the 9 point bi-quadratic elements in the Adini finite element space domain; while keep the model geometry hence, the stiff matrix in a positive definite condition. Therefore it is a fixed time ($0< t < t1 << \infty$) stable numeric scheme (transient LBB). Further more the 2nd and 3rd order Runge-Kutta method have been tested with geometric dimensional controlled time steps to yield the accurate stable solution (the 4th and 3rd order solution are identical) in a sense of evolutionary stability.

### 2.2 The boundary-layer analysis of the stress on the contact interface

For contact thin-layer near boundary an anisotropic visco-elastic P-T/T equation is studied to analyse (an exponential impact term has been added to the UCM equation) the following semi-discrete equations, the Galerkin-Runge-Kutta (2nd order G-RK or higher) scheme

$$\{\dot{\tau}_{xx}^{n+1}\} = \frac{1}{2}\left(\{Fp_1\}^n + \{Fp_1\}_{pre}^{n+1}\right)$$
$$\{\dot{\tau}_{yy}^{n+1}\} = \frac{1}{2}\left(\{Fp_2\}^n + \{Fp_2\}_{pre}^{n+1}\right)$$
$$\{\dot{\tau}_{xy}^{n+1}\} = \frac{1}{2}\left(\{Fp_3\}^n + \{Fp_3\}_{pre}^{n+1}\right)$$

where

$$Fp_1 = \frac{2\eta}{\lambda}\cdot\{A\}_{N\times N}^{-1}\cdot\{B1\}_{N\times N}\cdot\{u_x\}_N + 2(1-\zeta)\cdot\sum_{j=1}^{N}\tau_{xx}(j)\{A\}_{N\times N}^{-1}\cdot\{E1j\}_{N\times N}\cdot\{u_x\}_N$$
$$-\frac{1}{\lambda}\cdot\{\tau_{xx}\}_N + (2-\zeta)\cdot\sum_{j=1}^{N}\tau_{xy}(j)\{A\}_{N\times N}^{-1}\cdot\{E2j\}_{N\times N}\cdot\{u_x\}_N$$
$$-\zeta\cdot\sum_{j=1}^{N}\tau_{xy}(j)\{A\}_{N\times N}^{-1}\cdot\{E1j\}_{N\times N}\cdot\{u_y\}_N - \frac{\varepsilon}{\eta_0}\cdot\sum_{j=1}^{N}[\tau_{xx}(j)+\tau_{yy}(j)]\{A\}_{N\times N}^{-1}\cdot\{G\}_{N\times N}\cdot\{\tau_{xx}\}_N$$
$$-\sum_{j=1}^{N}u_x(j)\{A\}_{N\times N}^{-1}\cdot\{E1j\}_{N\times N}\cdot\{\tau_{xx}\}_N - \sum_{j=1}^{N}u_y(j)\{A\}_{N\times N}^{-1}\cdot\{E2j\}_{N\times N}\cdot\{\tau_{xx}\}_N$$

$$Fp_2 = \frac{2\eta}{\lambda}\cdot\{A\}_{N\times N}^{-1}\cdot\{B2\}_{N\times N}\cdot\{u_y\}_N + 2(1-\zeta)\cdot\sum_{j=1}^{N}\tau_{yy}(j)\{A\}_{N\times N}^{-1}\cdot\{E2j\}_{N\times N}\cdot\{u_y\}_N$$
$$-\frac{1}{\lambda}\cdot\{\tau_{yy}\}_N + (2-\zeta)\cdot\sum_{j=1}^{N}\tau_{xy}(j)\{A\}_{N\times N}^{-1}\cdot\{E1j\}_{N\times N}\cdot\{u_y\}_N$$
$$-\zeta\cdot\sum_{j=1}^{N}\tau_{xy}(j)\{A\}_{N\times N}^{-1}\cdot\{E2j\}_{N\times N}\cdot\{u_x\}_N - \frac{\varepsilon}{\eta_0}\cdot\sum_{j=1}^{N}[\tau_{xx}(j)+\tau_{yy}(j)]\{A\}_{N\times N}^{-1}\cdot\{G\}_{N\times N}\cdot\{\tau_{yy}\}_N$$
$$-\sum_{j=1}^{N}u_x(j)\{A\}_{N\times N}^{-1}\cdot\{E1j\}_{N\times N}\cdot\{\tau_{yy}\}_N - \sum_{j=1}^{N}u_y(j)\{A\}_{N\times N}^{-1}\cdot\{E2j\}_{N\times N}\cdot\{\tau_{yy}\}_N$$

and Fp3 has been defined in the previors paper [9],

note that,

$$\{A\}_{N\times N} = \left\{\iint_e \phi_i(x,y)\phi_j(x,y)dxdy\right\}_{N\times N}$$
$$\{B1\}_{N\times N} = \left\{\iint_e \phi_i(x,y)\frac{\partial \phi_j(x,y)}{\partial x}dxdy\right\}_{N\times N} \quad (2.16)$$

$$\{B2\}_{N\times N}=\left\{\iint_e \phi_i(x,y)\frac{\partial\phi_j(x,y)}{\partial y}dxdy\right\}_{N\times N}$$

$$\{C\}_{N\times N\times N}=\left\{\iint_e \phi_i(x,y)\phi_j(x,y)\phi_k(x,y)dxdy\right\}_{N\times N\times N}$$

$$\{D\}_N=\left\{\iint_e \phi_i(x,y)dxdy\right\}_N$$

$$\{E1\}_{N\times N\times N}=\{E1(i,j,k)\}_{N\times N\times N}=\left\{\iint_e \phi_i(x,y)\phi_j(x,y)\frac{\partial\phi_k(x,y)}{\partial x}dxdy\right\}_{N\times N\times N}$$

$$\{E2\}_{N\times N\times N}=\{E2(i,j,k)\}_{N\times N\times N}=\left\{\iint_e \phi_i(x,y)\phi_j(x,y)\frac{\partial\phi_k(x,y)}{\partial y}dxdy\right\}_{N\times N\times N}$$

with

$$\{E1j_0\}_{N\times N}=\{E1j_0(i,k)\}_{N\times N}=\{E1(i,j,k)|_{j=j_0}\}_{N\times N}$$

$$\{E2j_0\}_{N\times N}=\{E2j_0(i,k)\}_{N\times N}=\{E2(i,j,k)|_{j=j_0}\}_{N\times N}$$

Where the initial value (n=0) is from the test and documented in the FEA boundary condition data base.

Therefore the stress corrected Cauchy flow is

$$\underline{\dot{u}}=\frac{1}{\rho}\nabla\cdot\underline{\underline{\tau}}-\underline{u}\cdot\nabla\underline{u} \quad .$$

With the discrete component form：

$$\dot{u}_x^{n+1}=\frac{1}{\rho}\left(\frac{\partial\tau^{n+1}{}_{xx}}{\partial x}+\frac{\partial\tau^{n+1}{}_{xy}}{\partial y}\right)-\left(u_x^n\frac{\partial u_x^n}{\partial x}+u_y^n\frac{\partial u_x^n}{\partial y}\right),$$

$$\dot{u}_y^{n+1}=\frac{1}{\rho}\left(\frac{\partial\tau^{n+1}{}_{xy}}{\partial x}+\frac{\partial\tau^{n+1}{}_{yy}}{\partial y}\right)-\left(u_x^n\frac{\partial u_y^n}{\partial x}+u_y^n\frac{\partial u_y^n}{\partial y}\right),$$

The Runge Kutta-Galerkin method yields the higher precision solution with at least 2nd order of convergence.

where $\underline{\underline{\tau}}$ is the stress, $\underline{\underline{D}}$ is the strain, $\underline{u}$ is the elastic/plastic deformation velocity, $\eta$ is the viscosity, $\lambda$ is the relaxation constant, $\varepsilon$ is the elongation rate, $\xi$ is the shear rate, $\rho$ is the density. It enabled us to calculate the microscopic overstritching, i.e. elongation - shear stress.

To solve the stability problem for the element over-stretch limite in the complex contacting boundary, we developed a technique with the combination of the cubic element and the shell element. It simulated large deformation successfully.

Further more, in the boundary-layer, a two dimensional correction (modify) scheme is developed to overcome the three dimensional non-Newtonian complex boundary problem. Therefore reduce the dimensional computing and saved CPU time.

The asymptotic analysis is followed to discover the numeric feasibility of the non-Newtonian solution by the multiscale perturbation method in the complex space.

The above initial matrix form can be written as the simple nonlinear parabolic conservative type

$$\frac{\partial u}{\partial t}-a\frac{\partial u}{\partial x}=f(u) \qquad ,u(x,0)=u^0(x)$$

or the discontinuous Galerkin form

$$\frac{dU_h}{dt}=F_h(U_h) \qquad ,t>0$$

$$U_h(x,0)=P_0u^0(x)\in V_h,$$

where $P_0\in L^2(0,T;H^r(\Omega))$ is the local projecting operator: $S_n\to V_h$; $F_h(U_h)$ is the approximation of $(-\partial_x f(u))$ in the discontinuous Galerkin discretization. A minmode slope control function maybe introduced to strengthen the nonlinear stability which is based on the dimensional control of the time step and the mean value of jump conditions across the discontinuouse contact surfaces. To solve the above semi-discretized form, we adopt the 2nd and 3rd order Runge-Kutta methods which kept the original precision from the finite element approximation. The convergent solutions of velocity and stress are shown in the following figures (figure 1 (a,b)). The corresponding numerical Runge-Kutta correction process based upon the Galerkin prediction is as follows (k = 2, 3):

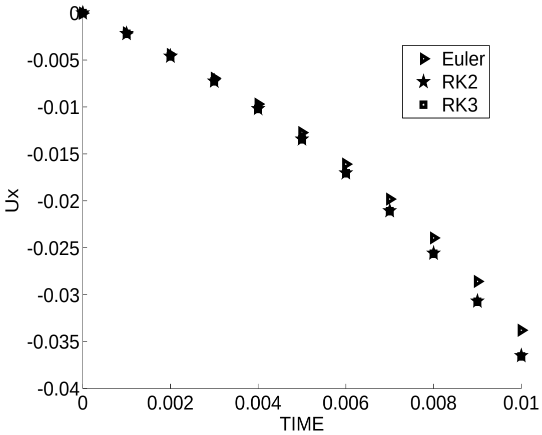
(1) Let $u_h^{(0)}=u_h^n$; （2.1

(2) For i = 1,2,..,k, （2.2

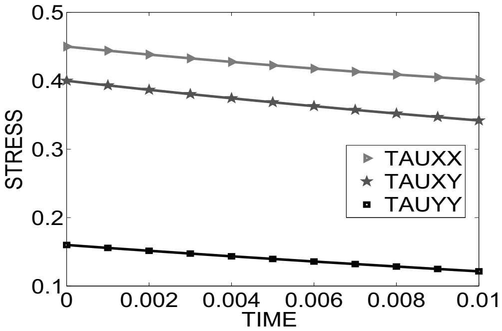
$$u_h^{(i)}=\sum_{l=1}^{i-1}\alpha_{il}u_h^{(l)}+\beta_{il}\Delta tL_h(u_h^{(l)}) \quad ;$$

(3) Let $u_h^{n+1}=u_h^{(k)}$;

This is so called convex combination of Runge-Kutta method, where $\alpha_{il}$, $\beta_{il}$ are the non-negative coefficients.




(a)




(b)

Figure **1** (a) velocity of 3 numerical scheme show the high precision of the RK method; (b) stress components result of stable G-RK scheme.

## III. Conclusion

The semi-discrete scheme for the effective numeric correction yields the 2nd order convergent solutions in this paper. The posteriori estimate will be given in another paper for the numerical analysis and modeling. The dissipative flow and stress in the impact boundary-layer show the impact hardening and shear thinning behaviour correctly in the rheological sense. The velocity of 3 numerical scheme showing the high precision of the RK method which result in the coupled stable stress components of the G-RK scheme. It is integrated towards the currently validated macro-scopic non-Newtonian models based on our 3 diomensional FEA solver with a super-convergence interpolation.

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