Numerical Simulation of an Induction-Conduction Model Arising in Steel Hardening

J. M. Díaz Moreno*, C. García Vázquez*, M. T. González Montesinos† and F. Ortegón Gallego*

Abstract—**We study a mathematical model for the description of the heating-cooling industrial process of a steel workpiece. The complete thermomechanical model is governed by a nonlinear coupled partial differential system of equations involving the electric potential, the magnetic vector potential, the temperature, the stress tensor and the displacement field together with a system of ordinary differential equations for the steel phase fractions. In this situation, the electric conductivity on the workpiece depends strongly on the temperature $b = b(x, \theta)$, θ being the temperature. Usually one has $b(x, s) \to 0$ when $s \to \infty$ uniformly on the workpiece, leading to degenerate parabolic/elliptic equations. Our analysis considers the case $0 < C_1/(1 + |s|) \le b(x, s) \le C_2$, C_1 and C_2 being constant values. Also we have performed some 2D numerical simulations of the heating-cooling process in a simplified version of the model.**

Keywords: Steel hardening, phase fractions, nonlinear parabolic-elliptic equations, Sobolev spaces, finite elements method.

1 Introduction

The mathematical analysis and numerical simulations of hypoeutectoid steel hardening including phase transitions has been extensively studied during the last years ([1,3-5]). Consider the following system of PDE/ODE:

$$\nabla \cdot (b(\theta)\nabla\varphi) = 0 \text{ in } \Omega_T = \Omega \times (0, T), \quad (1)$$

$$\nabla \cdot A = 0 \text{ in } D_T = D \times (0, T), \quad (2)$$

$$b_0(\theta)A_t + \nabla \times (\frac{1}{\mu}\nabla \times A) + b_0(\theta)\nabla\varphi = 0 \text{ in } D_T, \quad (3)$$

$$A(0) = A_0 \text{ in } \Omega, \quad (4)$$

$$-\nabla \cdot \sigma = f \text{ in } \Omega^s_T = \Omega^s \times (0, T), \quad (5)$$

$$\sigma = K\left(\varepsilon(u) - \theta q(z)I - \int_0^t \gamma(\theta, z, z_t)S \,\mathrm{d}\tau\right), \quad (6)$$

$$z_t = F(\theta, z, \sigma) \text{ in } \Omega^s_T, \quad (7)$$

$$z(0) = z_0 \text{ in } \Omega^s, \quad (8)$$

$$\begin{aligned}\alpha(\theta, z, \sigma)\theta_t - \nabla \cdot (k(\theta)\nabla\theta) + 3\kappa q(z)\theta\nabla \cdot u_t \\ = b_0(\theta)|A_t + \nabla\varphi|^2 \\ +(\rho L + \operatorname{tr}\sigma\theta\bar{q} + 9\kappa\theta^2 q(z)\bar{q})z_t \\ +\gamma(\theta, z, z_t)|S|^2 \text{ in } \Omega^s_T,\end{aligned} \quad (9)$$

$$\theta(0) = \theta_0 \text{ in } \Omega^s. \quad (10)$$

where $\Omega, D \subset \mathbb{R}^N$, $N = 2$ or 3, are bounded, connected and Lipschitz-continuous open sets such that $\bar{\Omega} \subset D$, $\Omega = \Omega^c \cup \Omega^s \cup S$ is the set of conductors, Ω^c the inductor (usually made of copper), Ω^s the steel workpiece, Ω^c and Ω^s being open sets, and $S = \bar{\Omega}^c \cap \bar{\Omega}^s$ is the surface contact between Ω^c and Ω^s, $\Omega^c \cap \Omega^s = \varnothing$ (see Figure 2 below); T stands for the final time of observation; φ the electrical potential; A the magnetic vector potential; σ the stress tensor; $\varepsilon(u) = \frac{1}{2}(\nabla u + \nabla u^T)$ the strain rate tensor; f a given external force; u the displacement field; θ the temperature; $z = (z_1, z_2)$, z_1 and z_2 are the phase fractions ([1,2,6]) of austenite and martensite, respectively, whereas $q(z) = q_1 z_1 + q_2 z_2 + q_0(1 - z_1 - z_2)$, and q_i represents the expansion coefficient of austenite ($i = 1$), martensite ($i = 2$) and of the original mixture of phase fractions ($i = 0$); $F = (F_1, F_2)$ gives the phase fractions model; $k(\theta)$ is the thermal conductivity; $b(\theta)$ the electrical conductivity (by $b(\theta)$ we mean the function $(x, t) \mapsto b(x, \theta(x, t))$, and also for $k(\theta)$, etc.); $b_0(x, s) = b(x, s)$ if $x \in \bar{\Omega}$, $b_0(x, s) = 0$ elsewhere; $\mu = \mu(x)$ is the magnetic permeability; ρ the density; $\kappa = \frac{1}{3}(3\lambda + 2\bar{\mu})$ the bulk modulus of elasticity, λ and $\bar{\mu}$ being the Lamé coefficients; $L = (L_1, L_2)$ is the latent heat; $\bar{q} = \nabla_z q$; $S = \sigma - \frac{1}{3}\operatorname{tr}\sigma I$, that is, the trace free part of the stress tensor σ; the term $\int_0^t \gamma(\theta, z, z_t)S \,\mathrm{d}\tau$ models the transformation induced plasticity. Finally, α is of the form

$$\alpha(\theta, \sigma, z) = \rho c_\epsilon - \operatorname{tr}\sigma q(z) - 9\kappa q(z)^2\theta,$$

where the constant c_ϵ is the specific heat capacity at constant strain. System (1-10) is supplied with suitable boundary conditions.

The thermomechanical model (1)-(10) describes the heating process of a steel workpiece by induction and conduc-

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Figure 1: Car steering rack.

tion. Once the desired high temperature is reached at certain critical parts along the workpiece, the supplied electric current is switched off and the workpiece is then quenched in order to cool it down rapidly. The goal is to produce martensite (hard and brittle steel phase transition) in these critical parts, keeping the rest ductile. Usually, these parts correspond to particular structural components whose surface is going to be highly stressed during its mechanical lifetime. This is the case of the car steering rack (see Figure 1).

The mathematical analysis of a system like (1)-(10) has been done in [6]. In that work it is assumed that the electric conductivity $b(s)$ is bounded above and below far from zero; it also assumes the Coulomb gauge condition for the magnetic vector potential, namely, $\nabla \cdot A = 0$. In our analysis, we have dropped out this condition since this makes appear an undesired pressure gradient in equation (3); this leads us to include a penalty term in this equation of the form $-\delta\nabla(\nabla \cdot A)$, $\delta > 0$ being a small parameter.

2 A simplified model

The heating-cooling process produces a small amount of thermoelastic deformation on the workpiece inherent to the austenite-martensite transformation. In our analysis we consider a simplified model by neglecting mechanical effects.

We split the time interval $[0, T]$ into two intervals: $[0, T] = [0, T_{\mathrm{h}}) \cup [T_h, T_c]$, $T_{\mathrm{h}}, T_{\mathrm{c}} > 0$. The first interval $[0, T_{\mathrm{h}})$ corresponds to the heating process. All along this time interval, a high frequency alternating current is supplied through the conductor which in its turn induces a magnetic field. The combined effect of both conduction and induction gives rise to a production term in the energy balance equation (9), namely $b(\theta)|A_t + \nabla\varphi|^2$. This is Joule's heating. At the instant $t = T_{\mathrm{h}}$, the current is switched off and during the time interval $[T_h, T_c]$ the workpiece is cooled down by means of aqua-quenching.

Heating model

The current passing through the set of conductors $\Omega = \Omega^{\mathrm{c}} \cup \Omega^{\mathrm{s}}$ is modeled with the aid of an auxiliary smooth surface $\Gamma \subset \Omega^{\mathrm{c}}$ cutting the inductor Ω^{c} into two parts, each one of them having a surface contact over the boundary of the workpiece Ω^{s} (see Figure 2).

$$\nabla \cdot (b(\theta)\nabla\varphi) = 0 \text{ in } \Omega \times (0, T_{\mathrm{h}}), \tag{11}$$

$$b(\theta)\frac{\partial\varphi}{\partial n} = 0 \text{ on } \partial\Omega \times (0, T_{\mathrm{h}}), \tag{12}$$

$$\left[b(\theta)\frac{\partial\varphi}{\partial n}\right] = \boldsymbol{j} \text{ on } \Gamma \times (0, T_{\mathrm{h}}), \tag{13}$$

$$b_0(\theta)A_t + \nabla \times (\frac{1}{\mu}\nabla \times A) - \delta\nabla(\nabla \cdot A)$$
$$+ b_0(\theta)\nabla\varphi = 0 \text{ in } D \times (0, T_{\mathrm{h}}), \tag{14}$$

$$A = 0 \text{ on } \partial D \times (0, T_{\mathrm{h}}), \tag{15}$$

$$A(0) = A_0 \text{ in } \Omega, \tag{16}$$

$$z_t = F(\theta, z) \text{ in } \Omega^{\mathrm{s}} \times (0, T_{\mathrm{h}}), \tag{17}$$

$$z(0) = z_0 \text{ in } \Omega^{\mathrm{s}}, \tag{18}$$

$$\rho c_\epsilon \theta_t - \nabla \cdot (k(\theta)\nabla\theta) = b_0(\theta)|A_t + \nabla\varphi|^2$$
$$+ \rho L z_t \text{ in } \Omega \times (0, T_{\mathrm{h}}), \tag{19}$$

$$\frac{\partial\theta}{\partial n} = 0 \text{ on } \partial\Omega \times (0, T_{\mathrm{h}}), \tag{20}$$

$$\theta(0) = \theta_0 \text{ in } \Omega. \tag{21}$$

In (13) $[\cdot]$ stands for the jump across the inner surface Γ. The function $\boldsymbol{j}$ represents the external source current density. The domain D containing the set of conductors is taken big enough so that the magnetic vector potential A vanishes on its boundary ∂D.

Cooling model

Once the heating process ends, aqua-quenching begins. This situation is modeled via the Robin boundary condition given in (25).

We put $z_{T_{\mathrm{h}}} = z(T_{\mathrm{h}})$, that is, $z_{T_{\mathrm{h}}}$ is the phase fraction distribution at the final heating instant T_{h} obtained from (17). In the same way, we define $\theta_{T_{\mathrm{h}}} = \theta(T_{\mathrm{h}})$. Obviously, these functions will be taken as the initial phase fraction distribution and temperature, respectively, in the cooling model.

$$z_t = F(\theta, z) \text{ in } \Omega^{\mathrm{s}} \times (T_{\mathrm{h}}, T_{\mathrm{c}}), \quad (22)$$
$$z(T_{\mathrm{h}}) = z_{T_{\mathrm{h}}} \text{ in } \Omega^{\mathrm{s}}, \quad (23)$$

$$\rho c_\epsilon \theta_t - \nabla \cdot (k(\theta)\nabla\theta) = \rho L z_t \text{ in } \Omega \times (T_{\mathrm{h}}, T_{\mathrm{c}}), (24)$$
$$-k(\theta)\frac{\partial\theta}{\partial n} = \beta(\theta - \theta_{\mathrm{e}}) \text{ on } \partial\Omega \times (T_{\mathrm{h}}, T_{\mathrm{c}}), \quad (25)$$
$$\theta(T_{\mathrm{h}}) = \theta_{T_{\mathrm{h}}} \text{ in } \Omega. \quad (26)$$

In (25), the constant value θ_{e} stands for the temperature of the spray water quenching the workpiece during the cooling time interval $[T_{\mathrm{h}}, T_{\mathrm{c}}]$. Also, the function β is a heat transfer coefficient and is given by

$$\beta(x,t) = \begin{cases} 0 & \text{on } \partial\Omega \cap \partial\Omega^{\mathrm{c}}, \\ \beta_0(t) & \text{on } \partial\Omega \cap \partial\Omega^{\mathrm{s}}. \end{cases}$$

where $\beta_0(t) > 0$ (usually taken to be constant).

3 An existence result

We consider the system (11)-(21) describing the heating process by conduction-induction. Besides the assumptions on data already mentioned along the Introduction, we will considered the following hypotheses.

(H.1) $\partial\Omega^{\mathrm{c}}$ is piecewise C^1.

(H.2) On the electric conductivity.

$$b(x,s) = \begin{cases} b^{\mathrm{c}}(s) & \text{if } x \in \Omega^{\mathrm{c}},\ s \in \mathbb{R}, \\ b^{\mathrm{s}}(s) & \text{if } x \in \Omega^{\mathrm{s}},\ s \in \mathbb{R}. \end{cases}$$

where $b^{\mathrm{c}}, b^{\mathrm{s}} \in C(\mathbb{R})$ and there exist positive constant values b_1, b_2, C_1 and C_2 such that

$$0 < b_1 \le b^{\mathrm{c}}(s) \le b_2, \quad \text{for all } s \in \mathbb{R},$$
$$0 < \frac{C_1}{1+|s|} \le b^{\mathrm{s}}(s) \le C_2, \quad \text{for all } s \in \mathbb{R}.$$

(H.3) $j \in L^\infty(0,T; H^{-1/2}(\Gamma))$ and

$$\langle j(t), 1\rangle_\Gamma = 0, \text{ almost everywhere } t \in (0, T_h).$$

Here, $\langle\cdot,\cdot\rangle_\Gamma$ stands for the duality pair between $H^{-1/2}(\Gamma)$ and $H^{1/2}(\Gamma)$.

(H.4) $A_0 \in H^1(\Omega)$.

(H.5) $F \in L^\infty(\mathbb{R}^2)^N \cap C(\mathbb{R}^2)$ and there exists a constant L_F such that

$$|F(s_1,s_2) - F(s_1,s_3)| \le L_F|s_2 - s_3|,$$
$$\text{for all } s_1, s_2, s_3 \in \mathbb{R}.$$

(H.6) $z_0 = (z_{01}, z_{02}) \in L^\infty(\Omega^{\mathrm{s}})^2$ and

$$0 \le z_{01}, z_{02}, z_{01} + z_{02} \le 1.$$

(H.7) $k \in C(\mathbb{R})$ and there exist two positive constant values k_1 and k_2 such that

$$0 < k_1 \le k(s) \le k_2, \quad \text{for all } s \in \mathbb{R}.$$

(H.8) $\mu \in L^\infty(D)$ and there exists a constant value μ_* such that $0 < \mu_* \le \mu$ in D.

(H.9) $\theta_0 \in L^1(\Omega)$, $\theta_0 \ge 0$.

(H.10) ρ and c_ϵ are strictly positive constant values. $L \in L^\infty(L^\infty(\Omega^{\mathrm{s}})^2)$ and $L = 0$ if $x \in D \setminus \bar{\Omega}$.

Remark We could assume more general hypotheses, for instance ρ, c_ϵ and L may depend on the temperature θ. Anyway the main difficulty is still found in the nonuniformly elliptic character of the function b^{s} leading to a very complex analysis from a mathematical standpoint.

Remark In the situation described here, we are just considering the evolution of two phase fractions which correspond to austenite and martensite. Of course, we may consider a more general setting which includes other phase fractions like bainite, pearlite and ferrite or a mixing of them all (see [4]).

Variational formulation

As it has been pointed out, the main difficulty in the mathematical analysis of the system (11)-(21) is found in the nonuniformly elliptic character of the diffusion coefficient b_0. In particular, the gradient of φ may not lie in the Sobolev space $H^1(\Omega)$ with respect to spatial variable. The same comment may be said about the time derivative of A appearing in the equation (14).

The variational formulation corresponding to the system (11)-(21) is as follows:

To find $\varphi, \theta : \Omega \times (0, T_h) \mapsto \mathbb{R}$, $A : D \times (0, T_h) \mapsto \mathbb{R}^N$ and $z : \Omega^{\mathrm{s}} \times (0, T_h) \mapsto \mathbb{R}^2$ measurables, such that

$$\left\{ \begin{array}{l} \varphi \in L^1((0,T_{\mathrm{h}}) \times \Omega),\ b(\theta)^{1/2}\varphi \in L^\infty(L^2(\Omega)), \\ b(\theta)^{1/2}|\nabla\varphi| \in L^\infty(L^2(\Omega)), \text{ and} \\ \displaystyle\int_\Omega b(\theta)\nabla\varphi\nabla v \,\mathrm{d}x + \langle j, v\rangle_\Gamma = 0, \\ \qquad \text{for all } v \in H^1(\Omega) \text{ and} \\ \qquad \text{almost everywhere } t \in (0, T_h); \end{array} \right. \quad (27)$$

$$\left\{ \begin{array}{l} A \in L^\infty(H_0^1(D)^N), \\ b(\theta)^{1/2}A_t \in L^2(L^2(\Omega)^N), \text{ and} \\ \displaystyle\int_\Omega b(\theta)A_t \boldsymbol{v} \,\mathrm{d}x + \int_D \frac{1}{\mu}\nabla \times A \nabla \times \boldsymbol{v} \,\mathrm{d}x \\ \quad \displaystyle +\varepsilon \int_D \nabla \cdot A \nabla \cdot \boldsymbol{v} \,\mathrm{d}x + \int_\Omega b(\theta)\nabla\varphi \boldsymbol{v} \,\mathrm{d}x = 0, \\ \quad \text{for all } \boldsymbol{v} \in H_0^1(D)^N \text{ and} \\ \quad \text{almost everywhere } t \in (0, T_h); \end{array} \right. \quad (28)$$

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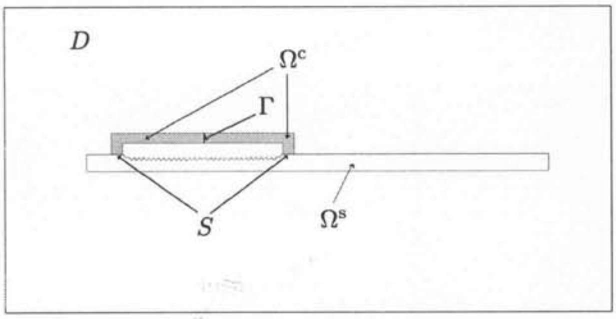

Figure 2: Domains D, $\Omega = \Omega^s \cup \Omega^c \cup S$ and the interface $\Gamma \subset \Omega^c$. The inductor Ω^c is made of copper. The workpiece contains a toothed part to be hardened by means of the heating-cooling process described above. It is made of a hypoeutectoid steel.

$$\left\{\begin{array}{l} \theta \in L^q(W^{1,q}(\Omega)), \text{ for all } q \in [1, \frac{N+2}{N+1}) \text{ and} \\ -\int_0^{T_h}\int_\Omega \rho c_\epsilon \theta \zeta_t \, dx\, dt + \int_0^{T_h}\int_\Omega k(\theta)\nabla\theta\nabla\zeta \, dx\, dt \\ = \int_0^{T_h}\int_\Omega (b_0(\theta)|A_t + \nabla\varphi|^2 + \rho L z_t)\zeta \, dx\, dt \\ -\int_\Omega \rho c_\epsilon \theta_0(x)\zeta(0,x)\, dx, \\ \quad \text{for all } \zeta \in \mathcal{D}([0,T_h]\times\bar{\Omega}) \text{ such that} \\ \quad \zeta(T,\cdot) = 0 \text{ in } \Omega. \end{array}\right. \tag{29}$$

$$\left\{\begin{array}{l} z \in W^{1,\infty}(L^\infty(\Omega^s)^2), \text{ and} \\ z_t = F(\theta, z) \text{ in } [0,T_h]\times\Omega^s, \\ z(0) = z_0 \text{ in } \Omega^s. \end{array}\right. \tag{30}$$

The following existence result for the variational formulation (27)-(30) holds (see [5]).

THEOREM 1 *There exists a solution to the system (11)-(21) in the sense of the variational formulation (27)-(30).*

Moreover, there exists $r > 1$ such that $\varphi \in L^r(W^{1,r}(\Omega))$, $A_t \in L^r((0,T_h)\times\Omega)^N$ and $1/b(\theta) \in L^r((0,T_h)\times\Omega)$.

4 Numerical simulation

We have carried out some numerical simulations for the approximation of the solution to the problem (27)-(30). We want to describe the hardening treatment of a car steering rack during the heating-cooling process. The goal is to produce martensite along the tooth line together with a thin layer in its neighborhood inside the steel workpiece.

Figure 2 shows the open sets D, $\Omega = \Omega^s \cup \Omega^c \cup S$ and the inferface Γ which intervene in the setting of the problem. The inductor Ω^c is made of copper. The workpiece contains a toothed part to be hardened by means of the heating-cooling process described above. It is made of a hypoeutectoid steel. The open set $D \setminus \bar{\Omega}$ is air. The magnetic permeability μ in (28) is then given by

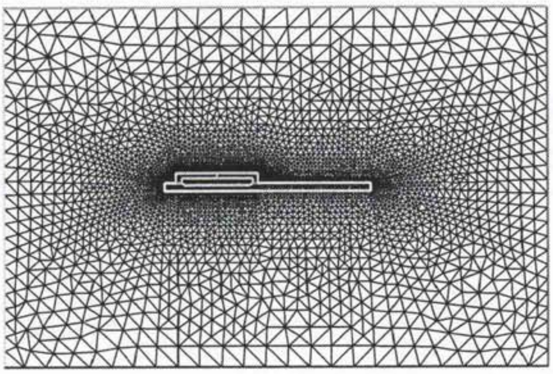

Figure 3: Domain triangulation. The triangulation of D contains 72088 triangles and 36095 vertices.

$$\mu(x) = \begin{cases} \mu_0 & \text{if } x \in D\setminus\bar{\Omega}, \\ 0.99995\mu_0 & \text{if } x \in \Omega^c, \\ 2.24\times 10^3\mu_0 & \text{if } x \in \Omega^s, \end{cases}$$

where $\mu_0 = 4\pi \times 10^{-7}$ (N/A^2) is the magnetic constant (vacuum permeability).

The martensite phase can only derive from the austenite phase. Thus we need to transform first the critical part to be hardened (the tooth line) into austenite. For our hypoeutectoid steel, austenite only exists in a temperature rank close to the interval [1050, 1670] (in °K degrees). During the first stage, the workpiece is heated up by conduction and induction (Joule's heating) which renders the tooth line to the desired temperature. In order to transform the austenite into martensite, we must cool it down at a very high rate. This second stage is accomplished by spraying water over the workpiece (this is called aquaquenching).

In this simulation, the final time of the heating process is $T_h = 5.5$ seconds and the cooling process extends also for 5.5 seconds, that is $T_c = 11$.

We have used the finite elements method for the space approximation and a Crank-Nicolson scheme for the time discretization. Figures 3 and 4 show the triangulation of D in our numerical simulations. We have used $P2$-Lagrange approximation for φ, A and θ and $P1$ for z.

In Figure 5 we can see the evolution of the temperature distribution of the rack along the tooth line. The initial temperature is $\theta_0 = 300$ °K. At $t = 5.5$ the heating process ends and the computed temperature shows that

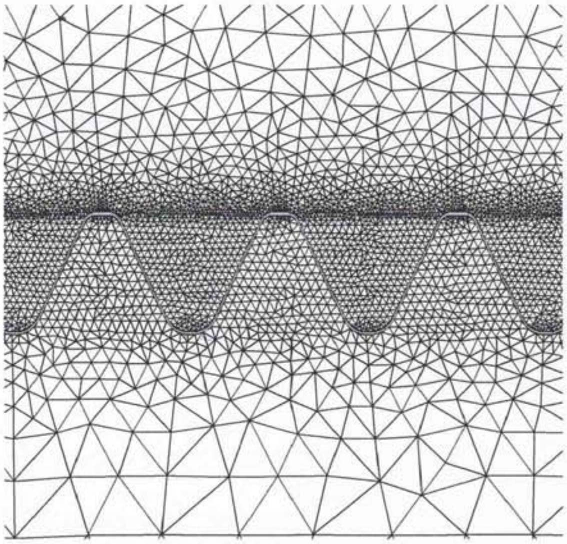

Figure 4: Domain triangulation: zoom around three teeth.

the temperature along the rack tooth line lies in the interval [1050, 1670].

Figure 7 shows the austenization along the tooth line at time instants $t = 0.8$, 2, 4 and 5.5 seconds, and the austenite transformation attained at the final heating instant $t = 5.5$ along the rack tooth line.

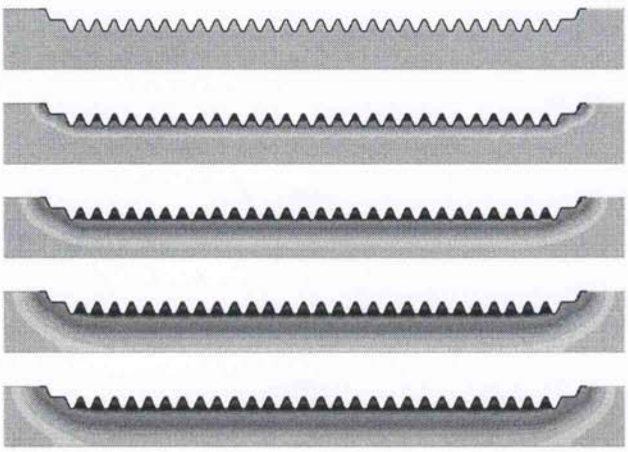

Figure 5: Heating process: Temperature at time instants $t = 0$, 1, 3, 5, and 5.5 seconds.

Figure 8 shows the formation of martensite from austenite along the rack tooth line during the cooling stage at time instants $t = 6.02545$, 7.08227, 8.55636 and 11 seconds, and the martensite transformation attained at the final cooling instant $t = 11$ along the rack tooth line. We have good agreement versus the experimental results obtained in the industrial process.

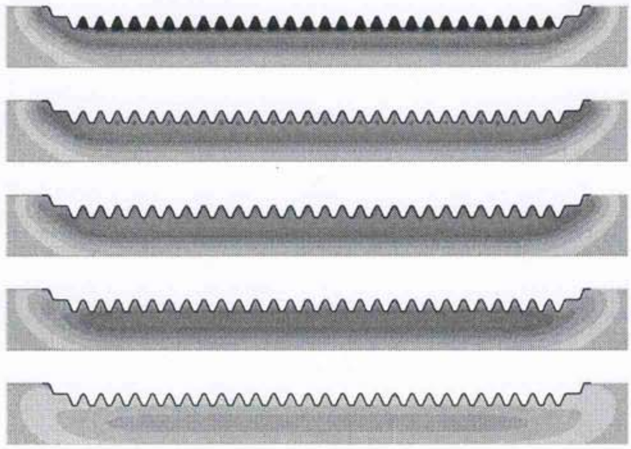

Figure 6: Cooling process: Temperature at time instants $t = 5.57$, 5.97, 6.23, 7.08, and 11 seconds.

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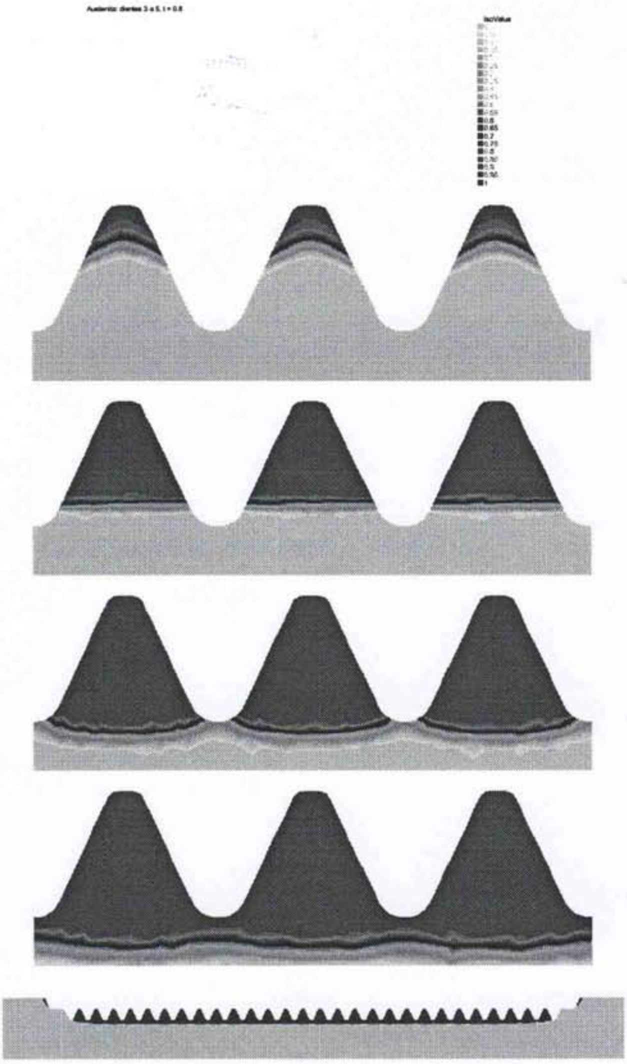

Figure 7: Heating process. Austenite evolution at time instants $t = 0.8$, 2, 4, and 5.5 seconds. The last figure shows the attained austenite at $t = 5.5$ along the rack tooth line.

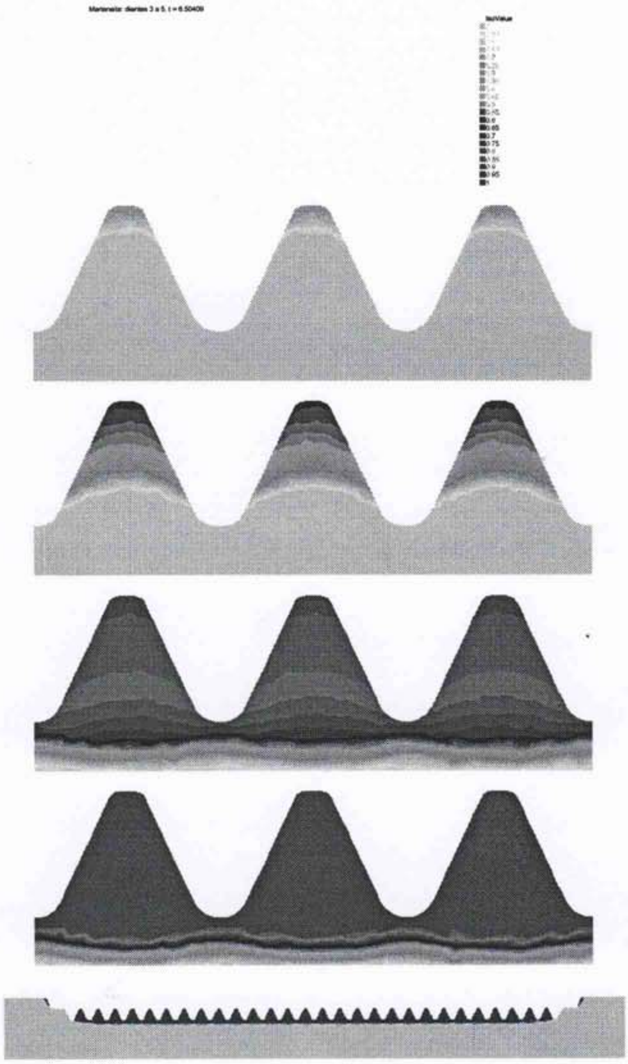

Figure 8: Cooling process. Martensite transformation at time instants $t = 6.50$, 7.08, 8.56, and 11 seconds, and martensite at the final instant of the aquaquenching stage, $t = 11$, along the rack tooth line.