Reliability Function Estimator with Exponential Failure Model for Engineering Data

Zuhair A. Al-Hemyari

Abstract **This paper provides estimation of reliability of a component subjected to life testing and the procedure includes essentially polling of two samples of failure data, where the component follows exponential failure model. We assumed that some prior information is available in the form of an initial guess value (θ_o) about the value of the parameter (θ) of exponential distribution from the past, and proposed a two stage shrinkage pooling estimator (TSPE) of reliability function of exponential distribution using complete failure data. The expressions for the bias, mean squared error, expected sample size and relative efficiency are derived. Conclusions regarding the constants involved in the proposed estimator are presented. Simulation, comparisons, and numerical results are reported. The proposed estimator fairs better than the classical two stage pooling estimator.**

Index Terms **exponential failure model, reliability function, relative efficiency, two stage shrunken.**

I. INTRODUCTION

The goal of system modeling is to provide quantitative forecasts of various system performance measures such as downtime, availability, number of failures, capacity, and cost. Evaluation of these measures is important to make optimal decisions when designing a system to either minimize overall cost or maximize a system performance measure within the allowable budget and other performance-based constraints.

In general, the shape or type of failure distribution depends upon the component's failure mechanisms. Similarly, the shape or type of repair distribution depends upon several factors associated with component repairs. Several methods are used to determine the distribution that best fits a given failure or repair pattern. Or, if failures or repairs are known to follow a particular distribution, the specific parameters that define this pattern can be determined by using the known failure and repair times.

As noted earlier, determining the failure and repair distributions of your system and its components is a significant part of evaluating the reliability of your design. If the failure rate is constant, which is generally true for electronic components during the main portion of their useful life, the reliability of the component follows an exponential distribution with p.d.f.

$$f(x \mid \theta) = \theta(\exp(-x\theta)) \;\; ; \;\; x > 0, \;\; \theta > 0. \qquad (1)$$

In the context of reliability evaluation and life testing, a number of engineering data have been examined (see [5] and [14]) and it was shown that the exponential distribution give quite a good fit for most cases. Furthermore, the exponential distribution is a very commonly used distribution in reliability engineering and has a wide range of applications in analyzing the reliability and availability of electronic systems, various queuing networks, and Markov chains; whereas the reliability of a given system (or component) for a given time has been defined as the probability that the system (or component) has a length of life greater than t, i.e.,

$$\begin{aligned} R(t) &= P(x > t) \\ &= \exp(-t\theta) \quad , t > 0, \theta > 0 . \end{aligned} \qquad (2)$$

The classical estimator $\hat{\theta}$ of θ and hence of $R(t)$ can easily be obtained without any complicated mathematical aid. The problem of estimation of θ and $R(t)$ was considered by several authors (see [5] and [14]).

2. INCORPORATING A GUESS VALUE θ_o, AND TSPE

In many practical circumstances, physical experts have some prior information regarding the value of θ due to past experiences, and apply it latently to inference of the actual model. However, in certain situations the prior information is available only in the form of an initial guess value (natural origin) θ_o of θ. For example, a bulb producer may know that the average life time of his product may be close to 1000 hours. Here we may take $\theta_o = 1000.$ In such a situation it is natural to start with an estimator $\hat{\theta}$ (e.g. MLE) of θ and modify it by moving it closer to θ_o, so that the resulting estimator, though perhaps biased, has smaller mean squared error than that of $\hat{\theta}$ in some interval around θ_o. This method of constructing an estimator of θ that incorporates the prior information θ_o leads to what is known as a shrunken estimator.

A standard problem in life testing deals with estimation of the parameter θ and $R(t)$ on the basis of less time and minimum cost of experimentation. The cost of experimentation can be achieved by using any prior information available about θ and devising a two stage shrunken pooling estimator (TSPE) in which it is possible to

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obtain an estimator from a small first stage sample and an additional second stage sample is required only if this estimator is not reliable. A TSPE of θ is defined as follows: let $X_{1i},\ i=1,2,\ \ldots\ ,n_1$ be a random sample of size $n_1 < n,$ where the variables are distributed with p.d.f. (1) and $\hat{\theta}_1$ be a good estimator of θ based on n_1 observations. Construct a region R in the space of $\theta,$ based on the prior value θ_O and an appropriate criterion. If $\hat{\theta}_1 \in R,$ use the estimator $k(\hat{\theta}_1-\theta_o)+\theta_o,$ for θ where $0 \le k \le 1.$ But if $\hat{\theta}_1 \notin R$, obtain $X_{2i},\ i=1,2,\ \ldots\ ,n_2,\ \ n_2=n-n_1$,compute $\hat{\theta}_2$, and then pool $\hat{\theta}_1$ and $\hat{\theta}_2$ to find $\hat{\theta}=(n_1\hat{\theta}_1+n_2\hat{\theta}_2)/n.$ The TSPE of θ is thus given by

$$\tilde{\theta}=\left\{\ [k(\hat{\theta}_1-\theta_o)+\theta_o]I_R+\hat{\theta}\ \ I_{\bar{R}}\ \ \right\}, \qquad (3)$$

where I_R and $I_{\bar{R}}$ are respectively the indicator functions of the acceptance region R and the rejection region $\overline{R}$. It may be noted here that TSPE of the parameter θ for complete, type I and type II censored data has also been considered by several authors (see [1]-[4], [6]-[9], [12] and [13]). In this paper we consider the problem of estimation of the reliability function $R(t)$ in the exponential distribution when the information regarding θ is available in the form of initial guess value θ_O. The general case of TSPE of $R(t)$ has been proposed and studied. The expressions for the bias, mean squared error, expected sample size and relative efficiency are obtained. Some numerical results and conclusions drawn therefrom are presented.

3. ESTIMATOR ASSUMPTIONS AND DERIVATION

As mentioned, the exponential distribution serves as a very useful model in analyzing the life testing and reliability data. Among the different type of data, interestingly, the complete and censored data (type I and type II) have received a considerable attention particularly in the reliability analysis. In this section first we define the general proposed estimator based on complete failure data, then we describe a choice of the region R , and finally we obtain the necessary expressions of the proposed estimator.

3.1 FAILURE DATA

Suppose $n_j,\ \ i=1,2$ items are subjected to life test and the test is terminated after all items have failed. Let $X_{1j}, X_{2j},\ \ldots\ ,X_{nj}$ be the random failure times of size n_j and suppose the failure times are exponentially distributed with p.d.f. (1). The MLE $\hat{\theta}_j$ of θ based on the above items is given by

$$\hat{\theta}_j = 1/\overline{X}_j \quad j=1,\ \ 2, \qquad (4)$$

where $2r_j\ \theta\ /\hat{\theta}_j$ distributed as chi square random variable with $2r_j$ degrees of freedom (see [10]). The proposed TSPE of $R(t)$ in this case is defined by:

$$\tilde{R}(t)=\begin{Bmatrix}\left[\exp(-t\left(k(\hat{\theta}_1)-\theta_o\right)+\theta_o)\right]I_R + \\ \left[\exp(-t\left((n_1(\hat{\theta}_1)+n_2(\hat{\theta}_2))/n\right)I_{\bar{R}}\right]\end{Bmatrix} \qquad (5)$$

3.2 CHOICE FOR REGION R

Estimator (5) is completely obtained by specifying the region R. A choice for the region R is considered here. In this choice we follow Katti's (see [10]) approach. That is, let R be the region which minimizes $MSE(\tilde{\theta}\mid\theta_o, R\ )$ (see (3)). This gives

$$\begin{aligned}R=\{\ &\hat{\theta}_1:(k^2-\frac{n_1^{\ 2}}{n^2})(\hat{\theta}_1-\theta_0)^2-\frac{2n_1n_2}{n^2}x \\ &xB(\hat{\theta}_1\mid\theta_0)(\hat{\theta}_1-\theta_0)-\frac{n_1^{\ 2}}{n^2}MSE(\hat{\theta}_1\mid\theta_0)\le 0\ \} \qquad (6)\\ =[\ &Max.(0,\theta_0(1-v)),\theta_0(1+v)\ \ ],\end{aligned}$$

where

$$v=(\sqrt{p}\ \ +n_1n_2)/(n^2k-n_1^{\ 2})(n_2-1)\sqrt{n_2-2}\ \ ,$$

$$p=(n_1n_2)^2+n_2^{\ 2}(n^2k^2-n_1^{\ 2})(n^2+n-2),\ \ k>n_1/n_2,$$

and $n_2 > 2.$

3.3 EXPRESSIONS

Let $R\ =[a_j,\ \ b_j]$ The expressions for the bias, bias ratio, expected sample size, and relative efficiency of $\tilde{R}(t)$ are obtained as follows:

$$\begin{aligned}&B(\tilde{R}(t)\mid R)=E(\tilde{R}(t)-R(t))/e^{-t\theta}\\ &=2e^{(-t\theta((1-k)\lambda-1)}(n_1t\theta\ \ k)^{r_1/2}\,\overline{\beta}_{n_1}(2\sqrt{n_1t\theta\ \ k})\\ &+(n_1^{\ 2}t\theta/(n_1+n_2))^{r_1/2}((4e^{t\theta}/\Gamma n_1\Gamma n_2)x\\ &x(n_2^{\ 2}t\theta/(n_1+n_2))^{r_2/2}x \qquad (7)\\ &x\beta_{n_2}(2\sqrt{(n_2^{\ 2}t\theta/(n_1+n_2))}\\ &+\beta_{n_1}(2\sqrt{n_1^{\ 2}t\theta/(n_1+n_2))}\ \ )\\ &-\overline{\beta}_{n_1}(2\sqrt{n_1^{\ 2}t\theta/(n_1+n_2)\ }\ ))-1,\end{aligned}$$

$$
\begin{aligned}
&MSE(\tilde{R}(t) \mid R) = e^{-2t\theta}\{\ [\ (2e^{-t\theta((1-k)\lambda-1)}x \\
&x(1/\Gamma(n_1)))(2n_1 t\theta\ k)^{r/2}\,\bar{\beta}_{n_1}(2\sqrt{2n_1 t\theta\ k}) \\
&-4e^{(-t\theta(1-k)\lambda-1))}(n_1 t\theta\ k)^{n_1/2}\,\bar{\beta}_{n_1}(2\sqrt{n_1 t\theta k}) \\
&+4e^{t\theta}(n_1{}^2 t\theta/(n_1+n_2))^{n_1/2}\,x \\
&x(n_2{}^2 t\theta/(n_1+n_2))^{n_2/2}(1/\Gamma(n_1)\Gamma(n_2))x \\
&x\big[2^{(n_1+n_2)/2}e^{t\theta}\beta_{n_2}(2\sqrt{(2n_2{}^2 t\theta/(n_1+n_2))}) \qquad (8) \\
&x[\ \beta_{n_1}(2\sqrt{(2n_1{}^2 t\theta/(n_1+n_2))}\ \ ) \\
&-\bar{\beta}_{n_1}(2\sqrt{(2n_1{}^2 t\theta/(n_1+n_2))})\ \ ] \\
&-2\beta_{n_2}(2\sqrt{n_2{}^2 t\theta/(n_1+n_2)}\ \ )x \\
&x[\ \beta_{n_1}(2\sqrt{(n_1{}^2 t\theta/(n_1+n_2))}) \\
&-\bar{\beta}_{n_1}(2\sqrt{(n_1{}^2 t\theta/(n_1+n_2))}\ \ ]+1\ \},
\end{aligned}
$$

where $\beta_{p-1}(2\sqrt{ab})$ (see[14]) is the Bessel function of order

$p,\ p = r_j + 1,\quad j = 1,2,\quad \beta_p(.) = \beta_{-p}(.),\quad p = 1,2,...,.,$

given by

$$
\begin{aligned}
&\int_0^\infty X^{-p}\quad e^{-(aX+b/X)}dX = 2(a/b)^{(p-1)/2}x \\
&x\beta_{p-1}(2\sqrt{ab}) = 2(a/b)^{(p-1)/2}\ \sum_{p=0}^{\infty}\ (-1)^p x \qquad (9) \\
&xt^{n+2p}/2^{n+1}\Gamma(p+1)\Gamma(n+p+1),
\end{aligned}
$$

and $\bar{\beta}_{p-1}(2\sqrt{ab})$, is an incomplete Bessel's function, where the integration (summation) is over the interval

$R^* = [a_j{}^*, b_j{}^*],\ \ a_j{}^* < b_j{}^*,\quad a_j{}^* = 2n_j\quad \theta/a_j,$

$b^*{}_j = 2n_j\quad \theta/b_j.$

The efficiency of $\tilde{R}_2(t)$ relative to $\hat{R}(t)$ is given by:

$$
RE(\tilde{R}(t) \mid \hat{R}(t)) = MSE(\hat{R}(t))/MSE(\tilde{R}(t) \mid R), \quad (10)
$$

where

$$
\begin{aligned}
&MSE(\hat{R}(t)) = e^{-2t\theta}\{\ [\ (4e^{t\theta}(n_1{}^2 t\theta/(n_1+n_2))^{n_1/2}x \\
&x(n_2{}^2 t\theta/(n_1+n_2))^{n_2/2}(1/\Gamma(n_1)\Gamma(n_2))x \\
&x\big[2^{n_1/2}2^{n_2/2}e^{t\theta}\beta_{n_2}(2\sqrt{(2n_2{}^2 t\theta/(n_1+n_2))}\ \ )x \\
&x\beta_{n_1}(2\sqrt{(2n_1{}^2 t\theta/(n_1+n_2))}\ \ ) \qquad (11) \\
&-2\beta_{n_1}(2\sqrt{(n_1{}^2 t\theta/(n_1+n_2))})x \\
&\beta_{n_2}(2\sqrt{n_2{}^2 t\theta/(n_1+n_2)}\ \ )\ ]+1\ \}.
\end{aligned}
$$

The expected sample size required to obtain $\tilde{R}(t)$ is given by

$$
E(n \mid \tilde{R}(t), R) = n - n_2(G(2n_1, b) - G(2n_1, a)), \quad (12)
$$

where $G(2n_1,.)$ is the cumulative distribution function of a chi-square random variable with $2n_1$ degrees of freedom.

4. SIMULATION AND NUMERICAL RESULTS

For the testimator $\tilde{R}(t)$, the relative efficiency, bias ratio, expected sample size and percentage of the overall sample size saved $100(n_2/n)x\Pr(\hat{\theta}_1 \in R)$ were computed for $\tilde{R}(t)$ by taking $n_1 = 4(2)12,\ \ n_2 = 2(2)12,\ \ t\theta = 0.3(0.3)3,$ and $0.1 \le \lambda \le 10(\lambda = (\theta_0/\theta))$. The Katti's type region R (as in (6)) is defined for $k > n_1/n_2$. Relative efficiency of $\tilde{R}(t)$ is computed for $k = (n_1/n_2,)\ \ +10^{-i}$, $i = 1(1)10,$ and it has been observed that the relative efficiency is high for $i = 7,$ therefore, when using the region R the value $k = (n_1/n_2)\ \ +10^{-7}$ is recommended. Some of these results are presented in tables 1 to 4. The following observations are drawn on the basis of these computations.

1. The testimator $\tilde{R}(t)$ is biased. The bias ratio is approximately zero (to the third decimal point) of $\tilde{R}(t)$ for $0.1 \le \lambda \le 1,\ n_1,\ n_2$, and $t\theta$, and increasing very slowly with increases of λ. In Table 1 we have presented some sample values of the bias ratio.
2. The testimator $\tilde{R}(t)$ for $0.1 \le \lambda \le 10,$ has smaller mean squared error than the pooled estimator $\hat{R}(t)$ (see Table 2).
3. Relative efficiency of $\tilde{R}(t)$ is a concave function of λ, i.e., the proposed testimators have maximum efficiency in the neighborhood of $\lambda \cong 1.4$ .Some of these computations are given in Table 2.

4. Efficiency of $\tilde{R}(t)$ relatively to the classical estimator $\hat{R}(t)$ is a decreasing function of $t\theta, n_2$, and n_1. i.e., $n_1 \cong 4,\ \ n_2 \cong 4,\ \ t\theta = 0.3,$ and gives higher relative efficiency than for other values of $t\theta,\ n_2$, and n_1 (Table2).

5. $E(n \mid \tilde{R}(t))$ is generally smaller than n when $\lambda \cong 1.4$ and increases very slowly with decreases or increases of λ (also see Table 3).

6. The percentage of the overall sample saved of $\tilde{R}(t)$ is maximum when θ_O is close to θ, the percentage is about (5%-15%) for small values of n_1. i.e., $n_1 \cong 4, n_2 \cong 2(2)12,\ \ 0.5 \le \lambda \le 1.7,$ and gives smaller percentage than for other values of λ, n_2, and n_1 (see Table 4).

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Table 1: Showing $B(\tilde{R}(t) \mid R)$ when $t\theta = 0.3$ and different values of $n_1,\ n_2$ and λ.

n_1	λ / n_2	$0.1 \le \lambda \le 1.1$	1.4	2	8
4	4	0.000	-0.0004	-0.0012	-0.0034
	6	0.000	0.000	0.000	-0.002
	10	0.000	0.000	0.000	0.000
	12	0.000	0.000	0.000	0.000
6	4	0.000	0.000	0.000	-0.001
	6	0.000	0.000	0.000	0.000
	10	0.000	0.000	0.000	0.000
	12	0.000	0.000	0.000	0.000
8	4	0.000	0.000	-0.001	-0.004
	6	0.000	0.000	0.000	0.000
	10	0.000	0.000	0.000	0.000
	12	0.000	0.000	0.000	0.000
10	4	0.000	0.000	-0.002	-0.003
	6	0 .000	0.000	0.000	0.000
	10	0.000	0.000	0.000	0.000
	12	0.000	0.000	0.000	0.000
12	4	0.000	0.000	-0.001	-0.002
	6	0.000	0.000	0.000	0.000
	10	0.000	0.000	0.000	0.000
	12	0.000	0.000	0.000	0.000

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Table 2: Showing $RE(\widetilde{R}(t) \mid \hat{R}(t))$ when $t\theta = 0.3$ and different values of n_1, n_2 and λ.

n_1	λ / n_2	0.1	0.4	0.5	0.6	0.7	0.8	0.9	1.0	1.1	1.4	2	8
4	4	1.00	1.05	1.41	1.82	2.08	2.46	2.85	3.47	3.47	5.94	2.12	1.03
	6	1.00	1.05	1.28	1.35	2.18	2.51	2.77	3.15	3.31	5.68	2.09	1.03
	10	1.00	1.07	1.19	1.69	2.08	2.31	2.75	2.33	2.78	5.58	2.08	1.03
	12	1.00	1.16	1.38	1.69	1.91	1.82	2.27	3.21	2.75	5.58	2.08	1.03
6	4	1.00	1.08	1.23	1.33	1.48	1.74	2.07	2.12	2.12	5.58	2.08	1.03
	6	1.00	1.03	1.23	1.44	1.49	1.72	2.17	2.22	2.24	5.20	1.92	1.01
	10	1.00	1.02	1.11	1.34	1.69	1.69	2.32	2.45	2.46	4.96	1.89	1.01
	12	1.00	1.03	1.11	1.27	1.49	1.76	2.02	2.25	2.29	4.96	1.89	1.01
8	4	1.00	1.04	1.07	1.27	1.41	1.42	1.56	1.54	1.54	4.96	1.89	1.01
	6	1.00	1.03	1.12	1.26	1.41	1.46	1.65	1.65	1.69	4.96	1.89	1.01
	10	1.00	1.02	1.12	1.26	1.49	1.51	1.81	1.01	1.71	4.90	1.74	1.00
	12	1.00	1.02	1.12	1.27	1.33	1.53	1.91	1.91	1.99	4.88	1.74	1.00
10	4	1.00	1.01	1.07	1.10	1.18	1.28	1.29	1.30	1.31	4.88	1.74	1.00
	6	1.00	1.00	1.06	1.15	1.25	1.28	1.33	1.37	1.38	4.88	1.74	1.00
	10	1.00	1.00	1.03	1.10	1.23	1.44	1.49	1.50	1.53	3.56	1.60	1.00
	12	1.00	1.00	1.07	1.10	1.23	1.38	1.48	1.51	1.53	3.50	1.60	1.00
12	4	1.00	1.00	1.04	1.06	1.13	1.15	1.16	1.17	1.17	3.50	1.59	1.00
	6	1.00	1.00	1.02	1.09	1.12	1.19	1.21	1.21	1.28	3.50	1.59	1.00
	10	1.00	1.00	1.03	1.07	1.14	1.27	1.31	1.31	1.34	3.50	1.59	1.00
	12	1.00	1.00	1.02	1.11	1.16	1.31	1.35	1.35	1.37	3.37	1.59	1.00

Table 3: Showing $E(n \mid \widetilde{R}(t), R)$ when $t\theta = 0.3$ and different values of n_1, n_2 and λ.

n_1	λ n_2	0.1	0.4	0.5	0.6	0.7	0.8	0.9	1.0	1.1	1.4	2	8
4	4	8.00	7.93	7.85	7.74	7.63	7.54	7.46	7.39	7. 36	7.23	7.44	7.56
	6	10.00	9.91	9.79	9.64	9.49	9.36	7.25	9.17	9.11	9.00	9.20	9.52
	10	14.00	13.86	11.68	13.44	13.20	12.98	12.81	12.69	12.61	12.43	12.61	12.62
	12	16.00	15.85	15.84	15.62	15.34	15.05	14.79	14.44	14.35	14.10	14.39	14.41
6	4	10.00	9.98	9.93	9.86	9.78	9.71	9.46	9.59	9.57	9.32	9.52	9.61
	6	12.00	11.97	11.91	11.81	11.71	11.59	11.51	11.44	11.41	11.22	11.42	11.54
	10	16.00	15.96	15.87	15.71	15.53	15.35	15.21	15.12	15.07	14.93	14.65	14.76
	12	18.00	17.99	17.84	17.66	17.46	17.23	17.08	16.96	16.90	16.71	16.93	17.11
8	4	12.00	11.99	11.96	11.92	11.86	11.79	11.73	11.69	11.67	11.34	11.57	11.63
	6	14.00	13.99	13.97	13.91	13.86	13.79	13.74	13.69	13.67	13.43	13.65	13.77
	10	18.00	17.99	17.94	17.83	17.69	17.55	17.42	17.34	17.17	16.99	17.12	17.25
	12	20.00	19.98	19.92	19.80	19.63	19.46	19.32	19.22	19.17	19.02	19.25	19.36
10	4	14.00	13.99	13.89	13.95	13.89	13.84	13.79	13.76	13.74	13.85	13.91	13.99
	6	16.00	15.99	15.98	15.93	15.86	15.78	15.71	15.67	15.64	15.63	15.75	15.86
	10	20.00	19.99	19.97	19.89	19.78	19.86	19.55	19.47	19.45	19.23	19.45	19.56
	12	22.00	21.99	21.96	21.88	21.74	21.59	21.46	21.38	21.35	21.21	21.46	21.58
12	4	16.00	15.99	15.99	15.96	15.92	15.87	15.83	15.79	15.78	15.52	15.72	15.84
	6	18.00	17.99	17.99	17.95	17.89	17.82	17.76	17.76	17.72	17.61	17.82	17.89
	10	22.00	21.99	21.98	21.93	21.83	21.72	21.62	21.56	21.54	21.35	21.55	21.65
	12	24.00	23.99	23.92	23.92	23.81	23.67	23.56	23.48	23.46	23.24	23.47	23.53

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Table 4: Showing $100(n_2/n)x\Pr(\hat{\theta}_1 \in R)$ when $t\theta = 0.3$ and different values of $n_1, n_2,$ and λ.

n_1	λ / n_2	0.1	0.4	0.5	0.6	0.7	0.8	0.9	1.0	1.1	1.4	2	8
4	4	0.01	4.14	6.33	8.01	9.14	9.81	10.11	10.11	9.90	9.72	9.60	6.10
	6	0.01	4.97	7.59	9.61	10.97	11.77	12.13	12.14	11.88	11.77	11.65	7.12
	10	0.01	5.92	9.04	11.44	13.06	14.01	14.44	14.45	14.15	14.00	13.88	8.31
	12	0.01	6.21	9.49	12.01	13.71	14.71	15.16	15.17	14.86	14.70	14.58	9.01
6	4	0.00	0.54	1.21	2.11	3.08	3.95	4.36	5.12	5.43	5.26	5.15	1.22
	6	0.00	0.68	1.51	2.64	3.85	4.64	5.79	6.40	6.79	6.39	6.28	1.55
	10	0.00	0.85	1.89	3.30	4.82	6.17	7.24	8.00	8.49	8.20	8.14	1.93
	12	0.00	0.91	2.02	3.52	5.14	6.58	7.72	8.53	9.05	8.82	8.70	2.32
8	4	0.00	0.10	0.30	0.61	1.05	1.59	2.14	2.63	3.02	2.84	2.73	1.00
	6	0.00	0.13	0.38	0.78	1.35	2.04	2.75	3.38	3.88	3.55	3.44	1.31
	10	0.00	0.17	0.49	1.01	1.75	2.65	3.56	4.38	5.03	4.65	4.55	1.52
	12	0.00	0.18	0.53	1.09	1.89	2.86	3.85	4.73	5.43	5.21	5.16	1.71
10	4	0.00	0.02	0.09	0.22	0.41	0.68	1.02	1.39	1.74	1.43	1.31	0.34
	6	0.00	0.02	0.12	0.28	0.54	0.90	1.34	1.82	2.28	2.00	1.93	0.52
	10	0.00	0.03	0.15	0.38	0.72	1.20	1.79	2.43	3.04	2.78	2.65	0.61
	12	0.00	0.03	0.17	0.41	0.78	1.30	1.95	2.65	3.31	3.08	2.99	0.70
12	4	0.00	0.00	0.03	0.09	0.19	0.33	0.52	0.77	1.03	0.88	0.76	0.16
	6	0.00	0.00	0.03	0.12	0.25	0.44	0.70	1.02	1.38	1.09	0.89	0.22
	10	0.000	0.01	0.05	0.16	0.34	0.60	0.95	1.40	1.88	1.53	1.41	0.30
	12	0.000	0.01	0.05	0.18	0.37	0.66	1.05	1.54	2.06	1.77	1.65	0.31

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