Liquidity Adjusted Intraday Value at Risk

Jun Qi * Wing Lon Ng †

Abstract—**The traditional Value at Risk (VaR) is a very popular tool measuring market risk, but it does not incorporate liquidity risk. This paper proposes an extended VaR model to integrate liquidity risk for intraday trading strategies using high frequency order book data. We estimate the one step ahead liquidity adjusted intraday VaR called(LAIVaR) for both bid and ask positions, considering several threshold trading sizes. We also quantify the liquidity risk premium by comparing our result with the standard VaR approach.**

Keywords: Liquidity adjusted intraday VaR, liquidity risk premium, asymmetric market behaviour

1 Introduction

Risk management has gained much attention over the past two decades. Liquidity risk has lead the cause of many serious market crises. The infamous disaster from the Long Term Capital Management (LTCM), Russian financial crisis in 1998 and unprecedented crisis in the US mortgage in 2007 evidence the dangers of ignoring the effects of liquidity. In September 2007, Northern Rock faced crash due to the absence of liquidity. Big lessons teach us the liquidity plays a very important role in financial markets, especially when it comes to trading. Therefore, a good risk measurement have to take liquidity risk in to account. However, the definition of liquidity is ambiguous and has many versions. "*A liquid market is a market in which a bid-ask price is always quoted, its spread is small enough and small trades can be immediately executed with minimal effect on price* (Black (1971))". Kyle (1985) gives an more formal concept of liquidity that includes the following 3 dimensions: (*a*) The difference of transaction prices deviate from mid-market prices (tightness), (*b*) the amount can be trade with a given market price (depth) and (*c*) the speed of the price recovers to the pre-trade price (resiliency). A concept that is even more difficult to predict and measure is liquidity risk. In a real "frictionless" market, investors are hardly to get the mid-price that is used in many risk applications and a more rigorous risk management is needed. Bangia, Diebold, Schuermann, and Stroughair (1999) argue that the liquidity risk is an important component in order to capture the overall risk. Lawrence and Robinson (1997) assert that failure to consider liquidity may lead to an underestimation of the VaR by 30%.

Although more and more market practitioners have recognized that liquidity risk is a very serious concern for firms, plenty studies have separately analyzed the VaR and liquidity. Only a few studies incorporate liquidity into VaR, not to speak of VaR at intraday level (see, for example, Beltratti and Morana (1999), Dionne, Duchesne, and Pacurar (2006) or Colletaz, Hurlin, and Tokpavi (2007)). The literature reports only a few former studies where researchers have incorporated liquidity risk with conventional VaR by using optimal execution strategy. In general, there are two different methods: the first one is the stochastic horizon methods. Lawrence and Robinson (1997) determine the holding period of VaR according to the size of position and the characteristics of liquidity market. The second method is modelling market price changes induced by the selling off within a fixed time horizon. For example, Glosten, Jagannathan, and Runkle (1997) use this method to derive the optimal strategy of liquidation that will maximize the value over a pre-specified period. Therefore, they consider the impact of the size of the position and the period of execution on the value under liquidation of the position. Bertsimas and Lo (1998) use the similar method to derive the dynamic optimal strategy with the aim of minimizing the expected cost.

Bangia, Diebold, Schuermann, and Stroughair (1999) develop a liquidity adjusted VaR (LAVaR) model (named as the BDSS model after the name of the authors) which is a fundamental framework for integrating liquidity risk into the standard VaR. The BDSS model mainly focuses on exogenous liquidity risk which take the bid-ask spread into account. The LAVaR simply equals the sum of conventional VaR (computed by mid-price) and the liquidity risk adjusted part (computed by ask-bid spread). However, the BDSS has several drawbacks: Firstly, the model is based on the normal distribution which differs from reality. Secondly, the method ignores the endogenous liquidity risk which is also important. Thirdly, the assumption of perfect correlation between liquidity risk and VaR would lead to an overestimation of LAVaR. Erwan (2001) extends the BDSS model by using the weighted average spread which is incorporating the

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endogenous risk effect to instead of the ask-bid spread. He also points out that the endogenous liquidity risk is taking one half part of the total market risk and must not be neglected.

Hisata and Yamai (2000) propose a framework for the quantification of LAVaR model that incorporates the market impact induced by the trader's own liquidation. They derive the optimal execution strategy according to level of market liquidity and the scale of the investor's position. They choose the holding period as an endogenous variable and provide discrete time model and continuous time model for LAVaR measurement.

Further, Agnelidis and Benos (2006) investigate intraday LAVaR in Athens Stock Exchange and extend the model from Madhavan, Richardson, and Roomans (1997) by incorporating trading volume and take both endogenous and exogenous liquidity risk into account. Their result also shows that the liquidity risk must not be neglected. Moreover, the LAVaR exhibits a U-shaped pattern throughout the day. In contrast, Giot and Gramming (2006) introduce a GARCH model to derive LAVaR in an automated auction market. Their empirical model is based on the BDSS model and model the liquidity risk by calculating the weighted average bid price from the real order book data. Their result shows that liquidity in VaR accounts significantly and the liquidity risk exhibits an L-shape pattern throughout the day.

The motivation for our paper is as follows: Firstly, as we claimed in the beginning that liquidity risk is a very important fragment in whole risk system. However the conventional VaR models have not take the liquidity risk in to account. The conventional VaR models heavily rely on the implied assumption that an asset can be traded at a certain price at any quantity within a fixed period of time. This assumption is not realistic under real market conditions, especially in intraday trading, as execution is not always guaranteed, i.e. the conventional VaR models not capture the liquidity risk that traders are exposed to. This paper therefore attempts to measure additional risk due to liquidity in the VaR using intraday data and extends the existing literature in the following way. We consider the endogenous liquidity risk, taking into account the volume effect to model the liquidity adjusted intraday VaR (LAIVaR), which refers to the liquidity fluctuation driven by the size of investors' position.

Secondly, there is an asymmetry in up and down movement in the equity market.Down movement are typically more abrupt than up movement. This is relevant because like hedge funds maybe long assets and need a LAIVaR for both long and short positions. In particular, we are interested in differentiating between both bid and ask sides since different market sides have to face different price movements as well. We estimate the one step ahead LAIVaR of both market sides providing to quantify their real risk position.

The outline of the paper is as follows. Section 2 describes the methodology and Section 3 presents the data and the empirical results. Section 4 concludes.

2 Methodology

Different positions face different risks. We estimate the liquidity adjusted intraday VaR (LAIVaR) model for the bid side, which is for the investor who wants to buy, as well as for the ask side, which is for the investor who wants to sell. Let $v_{i,t}$ denote the corresponding volumes of orders queuing in the book at time t at positions $i = 1, ..., n$. Similar to Giot (2005), we first define for both bid (B) and ask (A) sides the volume-weighted average prices (VWAP) $B_t(v)$ and $A_t(v)$ to trade a volume v in the next short time interval based on the individual bid and ask prices $B_{i,t}(v)$ and $A_{i,t}(v)$, i.e.

$$B_t(v) = \frac{\sum_j B_{i,t} v_{i,t}^{BID}}{\sum_j v_{i,t}^{BID}}$$
$$A_t(v) = \frac{\sum_j A_{i,t} v_{i,t}^{ASK}}{\sum_j v_{i,t}^{ASK}}$$

where v is the pre-specified threshold volume to be traded at time t when executing at least the first j queuing orders on the bid or ask side, such that $v \leq \sum_{min(n)} v_{i,t}$.

This variable is an ex-ante measure of liquidity which indicates an immediate execution trading cost. With a given volume v (inside the depth), we can compute the price impact by using the information of the full limited order book data. In order to capture the liquidity risk we adopt the model from Giot (2005) and define two log ratio return processes as

$$r_t^{BID}(v) = \ln \frac{B_t(v)}{B_{t-1}(v)}$$
$$r_t^{ASK}(v) = \ln \frac{A_t(v)}{A_{t-1}(v)}$$

representing the VWAP returns.

It is reported in former studies that financial intraday data have a consistent diurnal pattern of trading activities over the course of a trading day, due to certain institutional characteristics of organized financial markets, such as opening and closing hours or lunch time. Since it is necessary to take the daily *deterministic* seasonality into account (Andersen and Bollerslev (1999)), smoothing techniques are required to get deseasonalized observations. To remove the seasonality property of high frequency data, Giot and Gramming (2006) assumed a deterministic seasonality in the intraday volatility, and defined the deseasonalized return as

$$D_t^{BID} = \frac{r_t^{BID}}{\sqrt{\phi_t^{BID}}}$$
$$D_t^{ASK} = \frac{r_t^{ASK}}{\sqrt{\phi_t^{ASK}}}$$

where r_t denotes the raw log VWAP-returns and ϕ_t the deterministic seasonality pattern of intraday volatility. We first chose 30 minutes interval raw return as nodes for the whole trading day and then use cubic splines to smooth the average squared sample returns in order to get the intraday seasonal volatility component ϕ_t (see also Giot (2000) and Giot (2005)).

Having computed the deseasonalized VWAP return process, we apply a GARCH(1,1) model

$$h_t = \alpha_0 + \sum_{i=1}^{q} \alpha_i \varepsilon_{t-i}^2 + \sum_{i=1}^{p} \beta_i h_{t-i} \tag{1}$$

for both market sides with h_t as the conditional variance for the (deseasonalized) VWAP-returns and ε_t as normally distributed innovations. The LAIVaR at time t for the two return process given confidence level α can be modelled as

$$\text{LAIVaR}_t = \mu_t + Z_\alpha \sigma_t \tag{2}$$

with σ_t as the volatility component. Based on the estimated conditional variance, the standard deviation of the raw return at time t is $\sigma_t = \sqrt{h_t \phi_t}$. From (2), we can estimate the LAIVaR for both bid and ask sides which can be displayed as LAIVaR_t^{BID} and LAIVaR_t^{ASK} respectively.

In the "frictionless" market, the frictionless VaR is computed by the mid-price. In order to quantify the liquidity risk premium, we also need to compute the intraday VaR ($IVaR^{MID}$) based on the mid-price and compare it with the LAIVaR. We define the log ratio return of mid price $r_{mid,t}$ as

$$r_t^{MID} = \ln \frac{P_t^{MID}}{P_{t-1}^{MID}} \tag{3}$$

where P_t^{MID} is the mid-price at time t and model the mid-price return process using a GARCH(1,1) volatility process. Similarly, the IVaR of mid-price returns at time $t-1$ is given by:

$$IVaR_t^{MID} = \mu_t^{MID} + Z_\alpha \sigma_t^{MID} \tag{4}$$

To compare the difference of the liquidity risk, we translate our results back to price IVaR which means the worst $\alpha\%$ predict price of asset if one execute his product at time t. Most studies in the literature ignore upside risk and onle focus on the downside risk, however in our paper the upside risk is a measure for traders who intend to long asset. The higher upside risk means the higher cost. We define the liquidity risk premium λ_t as the difference between mid-price IVaR and LAIVaR

$$\lambda_t = \begin{cases} \frac{1}{T}\sum_{t=1}^{T}(PVaR_{m(t)} - LaIVaR_{(t)}) \\ \frac{1}{T}\sum_{t=1}^{T}(LaIVaR_{(t)} - PVaR_{m(t)}) \end{cases} \tag{5}$$

Finally, we are also interested in the relative liquidity risk cost and the difference of the LAIVaR between the bid and ask side. To capture the LAIVaR of VWAP-prices for different levels on both bid and ask side of the order book *jointly*, we apply the dynamic conditional correlation (DCC) multivariate GARCH model proposed by Engle (2002). Consider the the bivariate filtrated normally distributed return process

$$r_t \mid \Psi_{t-1} \sim N(0, H_t) \tag{6}$$

with the covariance matrix

$$H_t = D_t R_t Dt \tag{7}$$

where R_t represents the correlation matrix of the returns on both market sides. Further, Engle (2002) assumes that

$$\begin{aligned} D_t &= diag(\sqrt{h_t}) && (8)\\ Q &= (1-a-b)\overline{Q} + a\varepsilon_{t-1}\varepsilon_{t-1}' + bQ_{t-1} && (9)\\ R_t &= (diag(Q_t))^{-\frac{1}{2}} Q_t (diag(Q_t))^{-\frac{1}{2}} && (10) \end{aligned}$$

where

$$\overline{Q} = T^{-1}\sum_{t=1}^{T} \varepsilon_t \varepsilon_t' \quad . \tag{11}$$

The residuals are assumed to be

$$\varepsilon_{it} = r_{it}/\sqrt{h_{it}} \tag{12}$$

with $h_{i,t} = \alpha_0 + \alpha_i \varepsilon_{i,t-1}^2 + \beta_i h_{i,t-1}$ where i stand for different asset. Following Engle (2002), the log-likelihood function can be written as

$$\begin{aligned} L(\theta,\varphi) &= \sum_{t=1}^{T} L_t(\theta,\varphi) \\ &= -\tfrac{1}{2}\textstyle\sum_{t=1}^{T}(log|D_t R_t D_t| + r_t' D^{-1} R_t^{-1} D^{-1} r_t) \\ &= -\tfrac{1}{2}\textstyle\sum_{t=1}^{T}(\underbrace{2log|D_t| + r_t' D^{-1} r_t}_{L_v(\theta)} \underbrace{- \varepsilon_t'\varepsilon_t + log|R_t| + \varepsilon_t' R_t \varepsilon_t}_{L_c(\theta,\varphi)}) \end{aligned}$$

allowing a two step estimation approach as it can be decomposed into a volatility part

$$\begin{aligned} L_v(\theta) &= -\frac{1}{2}\sum_{t=1}^{T}(2log|D_t| + r_t' D^{-2} r_t) && (13)\\ &= \frac{1}{2}\sum_{t=1}^{T}\sum_{i=1}^{n}(log(h_{i,t} + \frac{r_{i,t}^2}{h_{i,t}})) && (14) \end{aligned}$$

and a correlation part

$$L_c(\theta,\varphi) = -\frac{1}{2}\sum_{t=1}^{T}(log|R_t| + \varepsilon_t' R_t \varepsilon_t - \varepsilon_t'\varepsilon_t) \quad . \tag{15}$$

Hence, we first estimate the parameters $\theta = (\alpha_0, \alpha_i, \beta)$ in (14) in the univariate GARCH models, and then substitute θ into (15) to estimate the parameter $\varphi = (a, b)$.

Table 1: Data description

Average volume of ...	NR	RBS	HSBC
Best ask	2979	2038	28386
Best bid	2802	2039	18450
Best three ask orders	7504	8762	57074
Best three bid orders	6654	9015	41743
Total ask side	345420	1077160	3939042
Total bid side	346030	1116740	4348526
Threshold Size	2000;20000	10000;100000	50000;200000

3 The Empirical Analysis

The historical order book using empirical data extracted from the SETS (Stock Exchange Trading System) that is operated by the London Stock Exchange. The SETS is a powerful platform providing a electronic market for the trading of the constituents of the FTSE All Share Index, Exchange Traded Funds, Exchange Traded Commodities. Trading in the SETS system is continuous during the opening hours and is based on the so-called continuous double auction mechanism. A computer keeps track of all submitted orders and order changes. The matching of supply and demand is automatically performed, generally based on the usual algorithms following a strict price-time order priority. This study only considers the continuous trading phase, where the order book is open and visible for all registered market participants. It starts after the opening auction at 8 am, where the opening price is determined as the price which maximizes the volume that can be traded, and ends at 4.30 pm with the launch of the daily closing auction. The sample period of our data ranges from 1^{st} March 2007 to 31^{st} March 2007. The data set contains full order book information including all events recorded in the order book (limit orders, market orders, iceberg orders, cancelations, changes, full/partial executions) and their matching outcomes.

We assume that different volume sizes executed have different liquidity risk effect. However for simple case, we present two liquidity executions in this paper which are based on big and small size of volume. Executing big volume orders has bigger liquidity risk than small volume. We measure the investor's risk on both downside and upside risk which depend on investors' trading strategy (short or long position). In this paper, we choose three different liquidity stocks from the SETS limit order book which are Northern Rock (NR), Royal Bank of Scotland (RBS) and HSBC. Table 1 gives a list of several average volumes which reflect the liquidity activity for the three selected stocks and shows that HSBC have the largest trade size in every situations. If we compare the average cumulated volume of total ask and bid, the NR is the smallest. According to these facts we choose two different threshold volume sizes to reflect different liquidity positions for each stock indicated in the last in the table.

We filter every 5 minuets snapshots of the order book to get an equally spaced time series data. Table 2 presents the GARCH model parameter estimates (with the standard errors in brackets) based on the VWAP returns for the three stocks with different threshold volume values. For stock NR and HSBC, all α parameters are as expected smaller than β which means that the updated variance is mainly based on the past variance and less effected by "news".

Table 2: Estimated Parameters at 5 minutes frequency

NR	v=2000		v=20000	
	Ask	Bid	Ask	Bid
$a0$	$2.8047e-7$ $(3.6838e-8)$	$2.6218e-7$ $(3.5863e-8)$	$2.6298e-7$ $(2.6819e-8)$	$4.3733e-7$ $(5.0698e-8)$
$a1$	0.2367 (0.0121)	0.2013 (0.0100)	0.2631 (0.0087)	0.1564 (0.0103)
$b1$	0.7202 (0.0193)	0.7570 (0.0169)	0.7148 (0.0128)	0.7630 (0.0189)
RBS	$v = 10000$		$v = 100000$	
	Ask	Bid	Ask	Bid
$a0$	$7.6274e-7$ $(3.6187e-8)$	$1.2803e-6$ $(4.715e-8)$	$1.376e-6$ $(1.9968e-5)$	$1.5055e-6$ $(4.094e-8)$
$a1$	0.2706 (0.0102)	0.4580 (0.0264)	0.4953 (0.0142)	0.7377 (0.0245)
$b1$	0.5710 (0.0168)	0.3041 (0.0204)	0.5046 (0.0106)	0.2318 (0.0132)
HSBC	$v = 50000$		$v = 200000$	
	Ask	Bid	Ask	Bid
$a0$	$1.3154e-7$ $(7.0776e-9)$	$1.5111e-7$ $(7.2394e-9)$	$1.3685e-7$ $(7.5151e-9)$	$1.4553e-7$ $(5.9531e-9)$
$a1$	0.3012 (0.0168)	0.2579 (0.0157)	0.2781 (0.0164)	0.2932 (0.0126)
$b1$	0.6371 (0.0124)	0.6429 (0.0171)	0.6429 (0.0161)	0.6381 (0.0126)

Figure 1 displays both upside and downside the LAIVaR (with α=5%) of prices and compares this with the frictionless IVaR, all based on a 5 minutes sampling frequency. Those graphs demonstrate the comparison of the conventional VaR result and our result. The volume choice can make a big different of the estimation of VaR. For huge size of the volume execution of all three assets, the LAIVaR is always above the conventional VaR for upside risk and lower for downside risk, and the difference is obvious. The LAIVaR also displays asymmetric between upside and downside position. For algorithmic trader who always adjust their position in short time period, it is important to take liquidity risk in to account. The upside and downside LAIVaR allow traders know exactly how large the risk of long and short position. As shown in the Figure 1, the huge volume gain more liquidity risk and higher cost. Hence, the conventional method which use mid-price to measure IVaR is underestimating the risk.

Figure 2 shows the dynamic conditional correlation and the conditional variance for bid and ask position of three assets. For each asset, there are results for two different volumes. The most fluctuant correlation is the sample volume equal to 2000 of Northern Rock, which is from -0.7 to 1.

We examined the effect of our liquidity risk by liquidity risk premium λ. Figure 3 displays the forecasted risk premium λ of different volume for both ask and bid side. Liquidity risk is higher when volume size are bigger for all three assets. For larger volume size there are more big jumps of risk premium which can effect the traders who plan to execute large volumes in short time. The risk premium also shows different with same volume but different trading positions.

Table 3 reports the liquidity risk premium of price LAIVaR for the three stocks. The values in brackets are the mean liquidity risk premium in percentage. The results shows how large the conventional VaR methods will underestimate the risk. We are also interested in the asymmetric effect of liquidity risk in ask and bid side. For example, the liquidity risk premia of different volumes for the NR stock are bigger on bid side. However in the case of RBS, the liquidity risk premium of ask side is larger than bid side when the volume equal to 10000. For HSBC the liquidity risk premium is roughly equal in both sides.

General speaking, by examining the liquidity risk premium, one can emphasize that the importance of the liquidity risk component when measure the VaR model. An investor, especially for the one who have to execute large size volume of asset, must take into account the effect of liquidation in order to trade rationally.

Table 3: Liquidity Risk Premium (λ) for three stocks. VaR

5 minutes		Ask	(%)	Bid	(%)
NR	v=2000	0.3370	(0.0005)	0.8781	(0.0008)
	v=20000	1.6755	(0.0014)	2.9364	(0.0025)
RBS	v=10000	1.9241	(0.0011)	1.3305	(0.0007)
	v=100000	9.4878	(0.0048)	11.9636	(0.0054)
HSBC	v=50000	0.9624	(0.0010)	0.7133	(0.0008)
	v=200000	1.4567	(0.0017)	1.4827	(0.0017)

4 Conclusion

This paper extends the conventional VaR measurement methodology by taking the liquidity risk and trade position in to account. We use the information of limited order book data to study the asymmetric risk effect for bid and ask side. Our paper improve the BDSS model by incorporating the endogenous liquidity risk effect to instead of the ask-bid spread. Compared with Giot and Gramming (2006), we use different real return process which can reflect the real market information to measure LAIVaR, and we also consider both upside and downside VaR and liquidty risk premium.

Our method provide an new practical empirical technique which can help the algorithmic traders to quantify their risk depending on their market position. We establish the liquidity risk premium to quantify the liquidity risk between different volume sizes which provide a specified structure of liquidity risk.

Our results show that the liquidity risk is a crucial factor in estimating VaR. Negligence of liquidity cost lead to under estimate risk as the conventional VaR model. We further contribute by studying and contrasting the patterns of LAIVaR and liquidity risk premium between bid side and ask side of an order drive stock market. We provide significant and specific information for investors who want to long or short. In consequence the modeling of the LAIVaR allows investors to adjust positions with a benchmark for the decision making.

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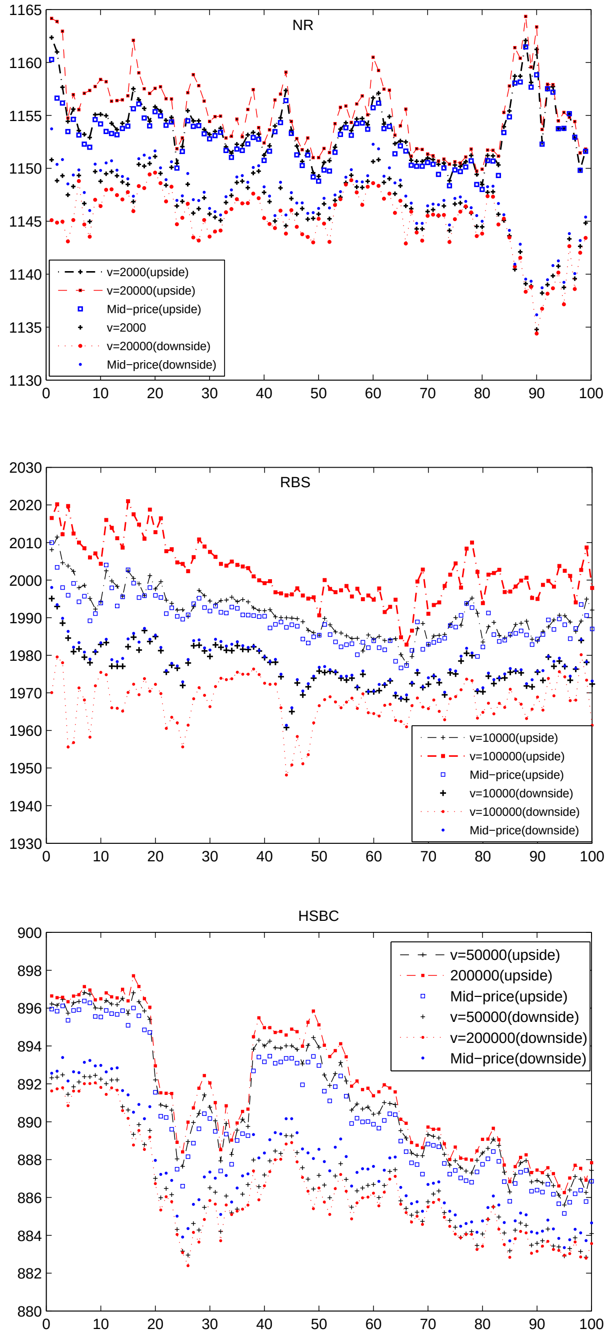

Figure 1: Price IVaR with 5 minutes frequency of three companies (α=5%)

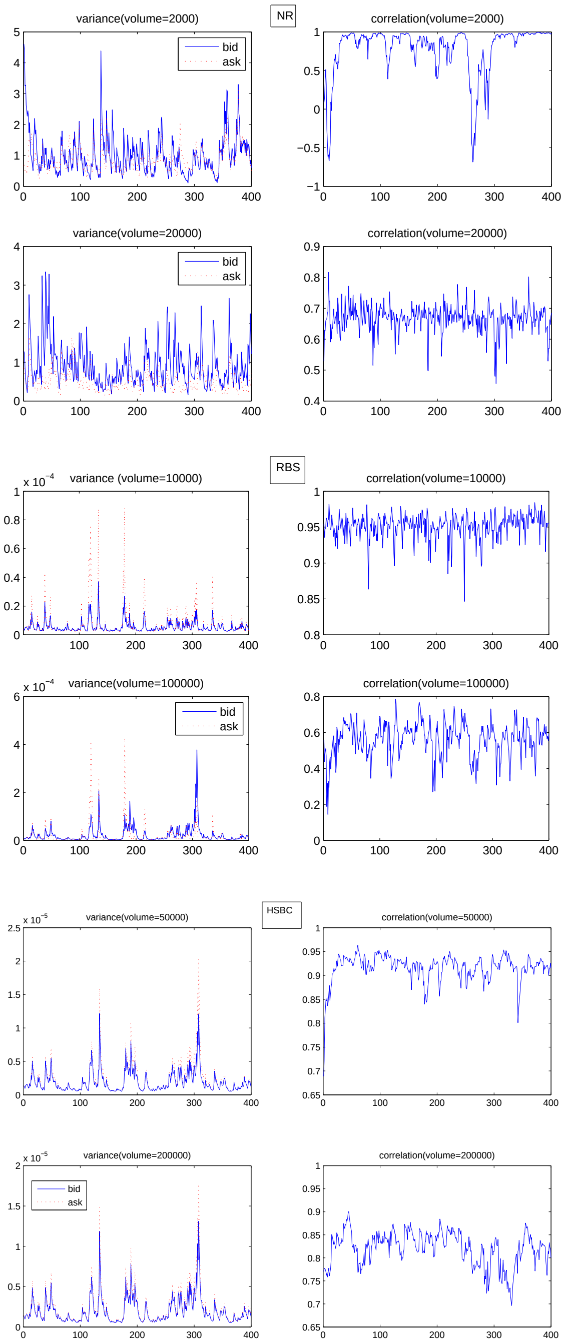

Figure 2: Variance and correlation three companies with different volume size for 5 minutes frequency

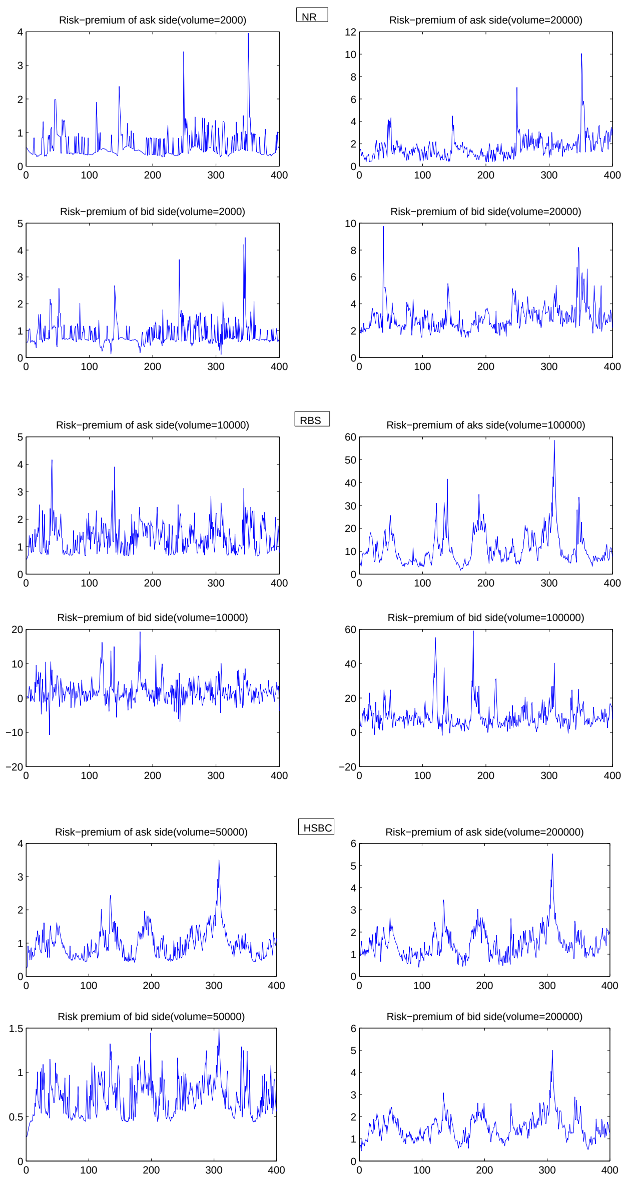

Figure 3: Risk-premium with 5 minutes frequency of three companies with different volume size

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