Average Pressure Gradient of Swirling Flow Motion of a Viscoelastic Fluid in a Circular Straight Tube with Constant Radius

Fernando Carapau *

Abstract—**Motived by the aim of modelling the behavior of swirling flow motion, we present a 1D hierarchical model for an Rivlin-Ericksen fluid with complexity $n = 2$ flowing in a circular straight tube with constant radius. Integrating the equation of conservation of linear momentum over the tube cross-section, with the velocity field approximated by the Cosserat theory, we obtain a one-dimensional system depending only on time and on a single spatial variable. The velocity field approximation satisfies both the incompressibility condition and the kinematic boundary condition exactly. From this new system, we derive the equation for the wall shear stress and the relationship between average pressure gradient, volume flow rate and swirling scalar function over a finite section of the tube. Also, we obtain the corresponding partial differential equation for the swirling scalar function.**

Keywords: 1D model, average pressure gradient, volume flow rate, swirling flow motion, Rivlin-Ericksen fluids

1 Introduction

In recent years the Cosserat theory approach has been applied in the field of fluid dynamics (see *e.g.* [1], [2], [3], [4], [11]) to reduce the full 3D system of equations of the flow motion into a system of partial differential equations which, apart from the dependence on time, depends only on a single spatial variable. The basis of this theory (see Cosserat [6], Duhen [8]) is to consider an additional structure of deformable vectors (called directors) assigned to each point on a space curve (the Cosserat curve). The relevance of using a director theory related to fluid dynamics is not in regarding the equations as approximations to three-dimensional equations, but rather in their use as independent theories to predict some of the main properties of three-dimensional problems. Several important features of a director theory are: (i) the director theory incorporates all vector components of the equation of linear momentum; (ii) it is a hierarchical theory, making it possible to increase the accuracy of the model; (iii) the system of equations is closed at each order and therefore unnecessary to make assumptions about form of the nonlinear and viscous terms; (iv) invariance under a superposed rigid body motion is satisfied at each order; (v) the wall shear stress enters directly as a dependent variable in the formulation; (vi) the director theory has been shown to be useful for modeling flow in curved tubes, considering many more directors than in the case of a straight tube. We use this theory to predict some of the main properties of a three-dimensional given problem where the fluid velocity field $\boldsymbol{\vartheta}(x_1, x_2, z, t) = \vartheta_i(x_1, x_2, z, t)\boldsymbol{e}_i$ can be approximated by (see Caulk and Naghdi [5]):

$$\boldsymbol{\vartheta} = \boldsymbol{v} + \sum_{N=1}^{k} x_{\alpha_1} \dots x_{\alpha_N} \boldsymbol{W}_{\alpha_1 \dots \alpha_N}, \tag{1}$$

with[1]

$$\boldsymbol{v} = v_i(z,t)\boldsymbol{e}_i, \quad \boldsymbol{W}_{\alpha_1 \dots \alpha_N} = W^i_{\alpha_1 \dots \alpha_N}(z,t)\boldsymbol{e}_i. \tag{2}$$

Here, $\boldsymbol{v}$ represents the velocity along the axis of symmetry z at time t and $x_{\alpha_1} \dots x_{\alpha_N}$ are the polynomial base functions with order k (this number identifies the order of hierarchical theory and is related to the number of directors). Moreover, the vectors $\boldsymbol{W}_{\alpha_1 \dots \alpha_N}$ are the director velocities which are symmetric with respect to their indices and $\boldsymbol{e}_i$ are the associated unit basis vectors. In his work we apply the nine-director theory ($k = 3$ in equation (1)) to study a specific viscoelastic fluid model with swirling motion. Using this theory, we obtain the unsteady relationship between average pressure gradient and volume flow rate over a finite section of a straight circular tube with constant radius and the corresponding equation for the wall shear stress. Also, we obtain the corresponding partial differential equation for the swirling scalar function.

2 System Description

Let x_i $(i = 1, 2, 3)$ be the rectangular cartesian coordinates and for convenience set $x_3 = z$. We consider a

*This work has been partially supported by the research center CIMA/UE, FCT Portuguese funding program. The author Fernando Carapau is a Auxiliar Professor at the Departamento de Matemática and Centro de Investigação em Matemática e Aplicações (CIMA) da Universidade de Évora, Rua Romão Ramalho, N°59, 7001-651, Évora-Portugal (e-mail: flc@uevora.pt)

[1] Latin indices subscript take the values $1, 2, 3$; greek indices subscript $1, 2$, and the usual summation convention is employed over a repeated index.

homogeneous fluid moving within a circular straight and impermeable tube, the domain Ω (see Fig.1) contained in $\mathbb{R}^3$. Also, let us consider the surface scalar function $\phi(z,t)$, that is related with the cross-section of the tube by the following relationship

$$\phi^2(z,t) = x_1^2 + x_2^2. \tag{3}$$

The boundary $\partial\Omega$ is composed by, the proximal cross-

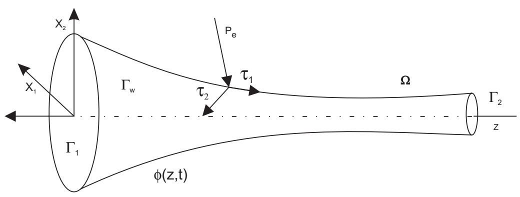

Figure 1: General fluid domain Ω with the tangential components of the surface traction vector τ_1, τ_2 and p_e, where $\phi(z,t)$ denote the radius of the domain surface along the axis of symmetry z at time t.

section Γ_1, the distal cross-section Γ_2 and the lateral wall of the tube, denoted by Γ_w.

The three-dimensional equations governing the motion of an incompressible Rivlin-Ericksen fluid with complexity $n = 2$, without body forces, defined in a straight circular tube Ω with lateral wall Γ_w, is given by (in $\Omega \times (0,T)$)

$$\begin{cases} \rho\Big(\dfrac{\partial \boldsymbol{\vartheta}}{\partial t} + \boldsymbol{\vartheta}\cdot\nabla\boldsymbol{\vartheta}\Big) = \nabla\cdot\boldsymbol{T}, \\ \nabla\cdot\boldsymbol{\vartheta} = 0, \\ \boldsymbol{T} = -p\boldsymbol{I} + \mu\boldsymbol{A}_1 + \alpha_1\boldsymbol{A}_2 + \alpha_2\boldsymbol{A}_1^2, \;\; \boldsymbol{T}_w = \boldsymbol{T}\cdot\boldsymbol{\varpi}, \end{cases} \tag{4}$$

with the initial condition

$$\boldsymbol{\vartheta}(x,0) = \boldsymbol{\vartheta}_0(x) \;\text{ in }\; \Omega, \tag{5}$$

and the homogeneous Dirichlet boundary condition

$$\boldsymbol{\vartheta}(x,t) = 0 \;\text{ on }\; \Gamma_w \times (0,T), \tag{6}$$

where p is the pressure, $-p\boldsymbol{I}$ is the spherical part of the stress due to the constraint of incompressibility and ρ is the constant fluid density. Equation $(4)_1$ represents the balance of linear momentum and $(4)_2$ is the incompressibility condition. In equation $(4)_3$, $\boldsymbol{T}$ is the constitutive equation, $\boldsymbol{T}_w$ denotes the stress vector on the surface whose outward unit normal is $\boldsymbol{\varpi}$. Also, μ is the constant fluid viscosity, α_1 and α_2 are material coefficients usually called the normal stress moduli and the kinematic first two Rivlin-Ericksen tensors $\boldsymbol{A}_1$ and $\boldsymbol{A}_2$ are given by (see Rivlin and Ericksen [10])

$$\boldsymbol{A}_1 = \nabla\boldsymbol{\vartheta} + \big(\nabla\boldsymbol{\vartheta}\big)^T \tag{7}$$

and

$$\boldsymbol{A}_2 = \frac{\partial}{\partial t}\big(\boldsymbol{A}_1\big) + \boldsymbol{\vartheta}\cdot\nabla\boldsymbol{A}_1 + \boldsymbol{A}_1\nabla\boldsymbol{\vartheta} + \big(\nabla\boldsymbol{\vartheta}\big)^T\boldsymbol{A}_1. \tag{8}$$

The classical constitutive equation related with Newtonian fluids is recovered with $\alpha_1 = \alpha_2 = 0$ at condition $(4)_3$.

The thermodynamics and stability of the fluids related with the constitutive equation $(4)_3$ have been studied in detail by Dunn and Fosdick [12], who showed that if the fluid is to be compatible with thermodynamics in the sense that all motions of the fluid meet the Clausius-Duhem inequality and the assumption that the specific Helmholtz free energy of the fluid is a minimum in equilibrium, then

$$\mu \geqslant 0, \quad \alpha_1 \geqslant 0, \quad \alpha_1 + \alpha_2 = 0. \tag{9}$$

Later, Fosdick and Rajagopal [13], based on the experimental observation, showed that for many non-Newtonian fluids of current rheological interest the reported values for α_1 and α_2 do not satisfy the restriction $(9)_{2,3}$, relaxed that assumption. Also, they showed that for arbitrary values of $\alpha_1 + \alpha_2$, with $\alpha_1 < 0$, a fluid filling a compact domain and adhering to the boundary of the domain exhibits an anomalous behavior not expected on real fluids. The condition $(9)_3$ simplifies substantially the mathematical model and the corresponding analysis. The fluids characterized by (9) are known as second-grade fluids as opposed to the general second-order fluids. It should also be added that the use of Clausius-Duheim inequality is the subject matter of much controversy (see *e.g.* Coscia and Galdi [7]). In the sequel we consider the system $(3) - (8)$ with $\mu \geqslant 0$, $\alpha_1 < 0$ and $\alpha_1 + \alpha_2$ is a arbitrary value.

Using the director theory approach (1) it follows (see [5]) that the approximation of the velocity field $\boldsymbol{\vartheta} = \vartheta_i(x_1, x_2, z, t)\boldsymbol{e}_i$, with nine directors, is given by

$$\begin{aligned} \boldsymbol{\vartheta} \;=\; & \Big[\, x_1(\xi + \sigma(x_1^2 + x_2^2)) - x_2(\omega + \eta(x_1^2 + x_2^2)) \,\Big]\boldsymbol{e}_1 \\ + & \Big[\, x_1(\omega + \eta(x_1^2 + x_2^2)) + x_2(\xi + \sigma(x_1^2 + x_2^2)) \,\Big]\boldsymbol{e}_2 \\ + & \Big[\, v_3 + \gamma(x_1^2 + x_2^2) \,\Big]\boldsymbol{e}_3, \end{aligned} \tag{10}$$

where $\xi, \omega, \gamma, \sigma, \eta$ are scalar functions of the spatial variable z and time t. The scalar functions in (10) have the following physical meaning: γ is related to transverse shearing motion, ω and η are related to rotational motion (also called swirling motion) about $\boldsymbol{e}_3$, while ξ and σ are related to transverse elongation. Also, from [5], the expression for the stress vector (see $(4)_3$) on the lateral surface Γ_w can be rewritten in terms of the tangential components of the surface traction vector and outward unit normal vector by[2]

$$\begin{aligned} \boldsymbol{T}_w \;=\; & \Big[\frac{1}{\phi(1+\phi_z^2)^{1/2}}\Big(\tau_1 x_1\phi_z - p_e x_1 - \tau_2 x_2(1+\phi_z^2)^{1/2}\Big)\Big]\boldsymbol{e}_1 \\ + & \Big[\frac{1}{\phi(1+\phi_z^2)^{1/2}}\Big(\tau_1 x_2\phi_z - p_e x_2 + \tau_2 x_1(1+\phi_z^2)^{1/2}\Big)\Big]\boldsymbol{e}_2 \\ + & \Big[\frac{1}{(1+\phi_z^2)^{1/2}}\Big(\tau_1 + p_e\phi_z\Big)\Big]\boldsymbol{e}_3, \end{aligned} \tag{11}$$

[2]Here a subscripted variable denotes partial differentiation.

where the tangential componente τ_1 is the wall shear stress.

Averaged quantities such as flow rate and average pressure are needed to study 1D models. Consider $S(z,t)$ as a generic axial section of the tube at time t defined by the spatial variable z and bounded by the circle defined in (3) and let $A(z,t)$ be the area of this section $S(z,t)$. Then, the volume flow rate Q is defined by

$$Q(z,t) = \int_{S(z,t)} \vartheta_3(x_1, x_2, z, t)da, \qquad (12)$$

and the average pressure $\bar{p}$, by

$$\bar{p}(z,t) = \frac{1}{A(z,t)} \int_{S(z,t)} p(x_1, x_2, z, t)da. \qquad (13)$$

Now, with the boundary condition (6) and the velocity field (10) on the surface (3), we obtain

$$\xi + \phi^2\sigma = 0, \;\; \omega + \phi^2\psi = 0, \;\; v_3 + \phi^2\gamma = 0. \qquad (14)$$

The incompressibility condition $(4)_2$ applied to the velocity field (10), can be written as

$$(v_3)_z + 2\xi + \big(x_1^2 + x_2^2\big)\big(\gamma_z + 4\sigma\big) = 0. \qquad (15)$$

For equation (15) to hold at every point in the fluid, the velocity coefficients must satisfy the conditions

$$(v_3)_z + 2\xi = 0, \;\; \gamma_z + 4\sigma = 0. \qquad (16)$$

Taking into account $(14)_{1,3}$ these separate conditions (16) reduce to

$$(v_3)_z + 2\xi = 0, \;\; \big(\phi^2 v_3\big)_z = 0. \qquad (17)$$

Now, let us consider a flow in a rigid tube, i.e.

$$\phi = \phi(z). \qquad (18)$$

Conditions (12), (10), $(14)_3$ and $(17)_2$ imply that the volume flow rate Q is just a function of time t, given by

$$Q(t) = \frac{\pi}{2}\,\phi^2(z) v_3(z,t). \qquad (19)$$

Instead of satisfying the momentum equation $(4)_1$ pointwise in the fluid, we impose the following integral conditions

$$\int_{S(z,t)} \Big[\nabla\cdot\boldsymbol{T} - \rho\Big(\frac{\partial\boldsymbol{\vartheta}}{\partial t} + \boldsymbol{\vartheta}\cdot\nabla\boldsymbol{\vartheta}\Big)\Big]da = 0, \qquad (20)$$

$$\int_{S(z,t)} \Big[\nabla\cdot\boldsymbol{T} - \rho\Big(\frac{\partial\boldsymbol{\vartheta}}{\partial t} + \boldsymbol{\vartheta}\cdot\nabla\boldsymbol{\vartheta}\Big)\Big]x_{\alpha_1}\ldots x_{\alpha_N}da = 0, \qquad (21)$$

where $N = 1, 2, 3$.

Using the divergence theorem and integration by parts, equations $(20) - (21)$ for nine directors, can be reduced to the four vector equations:

$$\frac{\partial\boldsymbol{n}}{\partial z} + \boldsymbol{f} = \boldsymbol{a}, \qquad (22)$$

$$\frac{\partial\boldsymbol{m}^{\alpha_1\ldots\alpha_N}}{\partial z} + \boldsymbol{l}^{\alpha_1\ldots\alpha_N} = \boldsymbol{k}^{\alpha_1\ldots\alpha_N} + \boldsymbol{b}^{\alpha_1\ldots\alpha_N}, \qquad (23)$$

where $\boldsymbol{n}$, $\boldsymbol{k}^{\alpha_1\ldots\alpha_N}$, $\boldsymbol{m}^{\alpha_1\ldots\alpha_N}$ are resultant forces defined by

$$\boldsymbol{n} = \int_S \boldsymbol{T}_3 da, \;\; \boldsymbol{k}^{\alpha} = \int_S \boldsymbol{T}_{\alpha} da, \qquad (24)$$

$$\boldsymbol{k}^{\alpha\beta} = \int_S \Big(\boldsymbol{T}_{\alpha}x_{\beta} + \boldsymbol{T}_{\beta}x_{\alpha}\Big)da, \qquad (25)$$

$$\boldsymbol{k}^{\alpha\beta\gamma} = \int_S \Big(\boldsymbol{T}_{\alpha}x_{\beta}x_{\gamma} + \boldsymbol{T}_{\beta}x_{\alpha}x_{\gamma} + \boldsymbol{T}_{\gamma}x_{\alpha}x_{\beta}\Big)da, \qquad (26)$$

$$\boldsymbol{m}^{\alpha_1\ldots\alpha_N} = \int_S \boldsymbol{T}_3 x_{\alpha_1}\ldots x_{\alpha_N}da. \qquad (27)$$

The quantities $\boldsymbol{a}$ and $\boldsymbol{b}^{\alpha_1\ldots\alpha_N}$ are inertia terms defined by

$$\boldsymbol{a} = \int_S \rho\Big(\frac{\partial\boldsymbol{\vartheta}}{\partial t} + \boldsymbol{\vartheta}\cdot\nabla\boldsymbol{\vartheta}\Big)da, \qquad (28)$$

$$\boldsymbol{b}^{\alpha_1\ldots\alpha_N} = \int_S \rho\Big(\frac{\partial\boldsymbol{\vartheta}}{\partial t} + \boldsymbol{\vartheta}\cdot\nabla\boldsymbol{\vartheta}\Big)x_{\alpha_1}\ldots x_{\alpha_N}da, \qquad (29)$$

and $\boldsymbol{f}$, $\boldsymbol{l}^{\alpha_1\ldots\alpha_N}$, which arise due to surface traction on the lateral boundary, are defined by

$$\boldsymbol{f} = \int_{\partial S} \sqrt{1+\phi_z^2}\,\boldsymbol{T}_w ds, \qquad (30)$$

$$\boldsymbol{l}^{\alpha_1\ldots\alpha_N} = \int_{\partial S} \sqrt{1+\phi_z^2}\,\boldsymbol{T}_w\, x_{\alpha_1}\ldots x_{\alpha_N}ds. \qquad (31)$$

3 1D Results

Let us consider a flow in a rigid tube with constant radius, i.e. $\phi = cts$. Now, taking into account the velocity (10), the stress vector (11), the volume flow rate (19), the average pressure (13), the incompressiblity condition $(4)_2$, the boundary condition (6) and the results quantities $(24) - (31)$ on equations $(22) - (23)$, we obtain

$$\begin{aligned} \bar{p}_z &= -\Big(\frac{8\mu}{\pi\phi^4}\Big)Q(t) - \Big(\frac{4\rho\phi^2 + 24\alpha_1}{3\pi\phi^4}\Big)Q_t(t) \\ &+ \omega\omega_z\Big(2\alpha_1 + \frac{2}{3}\alpha_2 + \frac{1}{20}\phi^2\rho\Big) - \omega\omega_{zzz}\frac{\phi^2}{30}\Big(\alpha_2 + 2\alpha_1\Big) \\ &+ \omega_z\omega_{zz}\frac{\phi^2}{20}\Big(\alpha_2 + 4\alpha_1\Big) \end{aligned} \qquad (32)$$

and the wall shear stress

$$\begin{aligned} \tau_1 &= \frac{4\mu}{\pi\phi^3}Q(t) + \frac{\rho}{6\pi\phi}\Big(1 + 24\frac{\alpha_1}{\rho\phi^2}\Big)Q_t(t) + \frac{\omega\omega_z\phi^3\rho}{40} \\ &+ (2\alpha_1 + \alpha_2)\Big[\frac{\omega\omega_{zzz}\phi^3}{60} - \frac{\omega\omega_z\phi}{6} + \frac{\omega_z\omega_{zz}\phi^3}{30}\Big] \end{aligned} \qquad (33)$$

where the scalar function $\omega(z,t)$ satisfies the following partial differential equation

$$\begin{aligned} 0 &= 16\omega + \omega_z Q(t)\Big(\frac{16\alpha_1}{\pi\mu\phi^2} + \frac{6\rho}{5\pi\mu}\Big) - \omega_{zz}\phi^2 \qquad (34) \\ &- \omega_{zzz}Q(t)\frac{6\alpha_1}{5\pi\mu} + \omega_t\Big(\frac{16\alpha_1}{\mu} + \frac{\phi^2\rho}{\mu}\Big) - \omega_{tzz}\frac{\alpha_1\phi^2}{\mu}. \end{aligned}$$

Considering the material coefficients $\alpha_1 = \alpha_2 = 0$ on equation (32) − (34), we recover the swirling Navier-Stokes solution obtained and validated by Caulk and Naghdi [5].

Now, let us consider just the steady case. Using the following dimensionaless variables[3]

$$\hat{z} = \frac{z}{\phi},\ \hat{Q} = \frac{2\rho}{\pi\phi\mu}Q,\ \hat{w} = fw,\ \hat{\alpha}_1 = \frac{\alpha_1}{\phi^2\rho},\ \hat{\alpha}_2 = \frac{\alpha_2}{\phi^2\rho} \qquad (35)$$

on equation (32) − (34), where f is the Coriolis frequency, we obtain

$$\begin{aligned} \hat{\hat{p}}_{\hat{z}} &= -4\hat{Q} + \mathcal{R}_0\Big[\hat{w}\hat{w}_{\hat{z}}(2\hat{\alpha}_1 + \frac{2}{3}\hat{\alpha}_2 + \frac{1}{20}) \\ &- \hat{w}\hat{w}_{\hat{z}\hat{z}\hat{z}}(\frac{\hat{\alpha}_2}{30} + \frac{\hat{\alpha}_1}{15}) + \hat{w}_{\hat{z}}\hat{w}_{\hat{z}\hat{z}}(\frac{\hat{\alpha}_2}{20} + \frac{\hat{\alpha}_1}{5})\Big] \qquad (36) \end{aligned}$$

and the wall shear stress

$$\begin{aligned} \hat{\tau}_1 &= 2\hat{Q} + \mathcal{R}_0\Big[\frac{\hat{w}\hat{w}_{\hat{z}}}{40} + (2\hat{\alpha}_1 + \hat{\alpha}_2)\Big(\frac{\hat{\omega}\hat{\omega}_{\hat{z}\hat{z}\hat{z}}}{60} \\ &- \frac{\hat{\omega}\hat{\omega}_{\hat{z}}}{6} + \frac{\hat{\omega}_{\hat{z}}\hat{\omega}_{\hat{z}\hat{z}}}{30}\Big)\Big] \qquad (37) \end{aligned}$$

where

$$\mathcal{R}_0 = \frac{\rho^2\phi^4}{f^2\mu^2}$$

is the Rossby number: a small Rossby number signifies a system which is strongly affected by Coriolis forces, and a large Rossby number signifies a system in which inertial and centrifugal forces dominate. Also, the scalar function $\hat{\omega}(\hat{z})$ satisfies the following ODE

$$0 = 16\hat{\omega} + \hat{\omega}_{\hat{z}}\hat{Q}\Big(8\hat{\alpha}_1 + \frac{3}{5}\Big) - \hat{\omega}_{\hat{z}\hat{z}} - \hat{\omega}_{\hat{z}\hat{z}\hat{z}}\hat{Q}\frac{3\hat{\alpha}_1}{5}. \qquad (38)$$

Now, integranting condition (36) over a finite section of the tube $[\hat{z}_1, \hat{z}]$, with $\hat{z}_1$ fixed, we obtain the nondimensional average pressure gradient

$$\begin{aligned} \hat{p}\hat{\hat{p}}(\hat{z}) &= \hat{\hat{p}}(\hat{z}_1) - \hat{\hat{p}}(\hat{z}) \\ &= 4\hat{Q}(\hat{z} - \hat{z}_1) + \mathcal{R}_0\Big[(\frac{\hat{\alpha}_2}{30} + \frac{\hat{\alpha}_1}{15})\int_{\hat{z}_1}^{\hat{z}} \hat{w}\hat{w}_{\hat{z}\hat{z}\hat{z}}d\hat{z} \\ &- (2\hat{\alpha}_1 + \frac{2}{3}\hat{\alpha}_2 + \frac{1}{20})\int_{\hat{z}_1}^{\hat{z}} \hat{w}\hat{w}_{\hat{z}}d\hat{z} \\ &- (\frac{\hat{\alpha}_2}{20} + \frac{\hat{\alpha}_1}{5})\int_{\hat{z}_1}^{\hat{z}} \hat{w}_{\hat{z}}\hat{w}_{\hat{z}\hat{z}}d\hat{z}\Big]. \qquad (39) \end{aligned}$$

[3] In cases where the steady flow rate is specified, the nondimensional flow rate $\hat{Q}$ is identical to the classical Reynolds number used for flow in tubes, see [11].

Finally, we observe that the behavior of the average pressure gradient, wall shear stress and swirling effects given by equations (39), (37) and (38) can be numerically illustrated for different values of $\mathcal{R}_0$, $\hat{Q}$, $\hat{\alpha}_1$ and $\hat{\alpha}_2$.

4 Conclusions and Future Work

A nine director theory has been used to derive a 1D model in a circular straight rigid and impermeable tube with constant radius to predict some of the main properties of the 3D Rivlin-Ericksen fluid model with swirling motion. Unsteady relationship between average pressure gradient (wall shear stress, respectively), volume flow rate and the swirling function over a finite section of the tube has been obtained. Also, we obtain a PDE relate with the scalar swirling function. One of the possible extensions of this work is the numerically simulation of the steady/unsteady equations (32) − (34) and also, the application of this hierarchical theory for specific tube geometries and/or other fluid models.

References

[1] Carapau, F., Sequeira, A., "Unsteady flow of a generalized Oldroyd-B fluid using a director theory approach", *WSEAS Transactions on Fluid Mechanics*, V1, pp. 167-174, 2/06

[2] Carapau, F., Sequeira, A., "Axisymmetric motion of a second order viscous fluid in a circular straight tube under pressure gradients varying exponentially with time", *WIT Transaction on Enginneering Science*, V52, pp. 409-419, 1/06.

[3] Carapau, F., Sequeira, A., Janela, J., "1D simulations of second-grade fluids with shear-dependent viscosity", *WSEAS Transactions on Mathematics*, V6, pp. 151-158, 1/07.

[4] Carapau, F., Sequeira, A., "1D Models for blood flow in small vessels using the Cosserat theory", *WSEAS Transactions on Mathematics*, V5, pp. 54-62, 01/06.

[5] Caulk, D.A., Naghdi, P.M., "Axisymmetric motion of a viscous fluid inside a slender surface of revolution", *Journal of Applied Mechanics*, V54, pp. 190-196, 01/87.

[6] Cosserat, E., Cosserat, F., "Sur la théorie des corps minces", *Compt. Rend.*, V146, pp. 169-172, 1/1908.

[7] Coscia, V., Galdi, G.P., "Existence, uniqueness and stability of regular steady motions of a second-grade fluid", *Int. J. Non-Linear Mechanics*, V29, N4, pp.493-506, 01/94.

[8] Duhem, P., "Le potentiel thermodynamique et la pression hydrostatique", *Ann. École Norm*, V10, pp. 187-230, 01/1893.

[9] Galdi, G.P., Sequeira, A., "Further existence results for classical solutions of the equations of a second-grade fluid", *Arch. Rational Mech. Anal.*, V128, pp. 297-312, 01/94.

[10] Rivlin, R.S., Ericksen, J.L., "Stress-deformation relations for isotropic materials", *J. Rational Mech. Anal.*, V4, pp. 323-425, 01/55.

[11] Robertson, A.M., Sequeira, A., "A director theory approach for modeling blood flow in the arterial system: An alternative to classical 1D models", *Mathematical Models & Methods in Applied Sciences*, V15, N6, pp. 871-906, 01/05.

[12] Dunn, J.E., Fosdick, R.L., "Thermodynamics, stability and boundedness of fluids of complexity 2 and fluids of second grade", *Arch. Rational Mech. Anal.*, V56, pp.191-252, 01/74.

[13] Fosdick, R.L., Rajagopal,K.R., "Anomalous features in the model of second order fluids", *Arch. Rational Mech. Anal.*, V70, pp.145-152, 01/79.