Proceedings of the World Congress on Engineering 2009 Vol II
WCE 2009, July 1 - 3, 2009, London, U.K.

Particularity Concerning Evaluation of Unguided Rocket Trajectories Deviation under the Disturbance Factors Action

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***Abstract*— To achieve an accurate rocket launch, it is necessary to assess how the perturbations appearing when the rocket leaves the rocket launcher system, influence the rocket flight trajectory and shooting accuracy as well. Therefore, we will analyze the influence of the rocket launcher system oscillations, during the shooting, on the rocket movement. We are taking into account other perturbation factors that can appear during the shooting and the rocket flight in standard atmosphere.**

***Index Terms*— launching device, oscillation, disturbance, mathematical model.**

I. Introduction

We consider a unguided rocket launching device that is viewed as a set of rigid bodies bound together using elastic elements, having three main components [1]: the vehicle chassis (upon which is laid the launching device's basis with the revolving support of the mechanisms), the tilting platform (with the containers for the rockets) and the rockets (including the launching rocket).

In order to highlight the importance of accuracy firing it is necessary to evaluate precisely how the launching oscillations, seen as disturbance factors, influence the rocket flight on the trajectory and implicitly on the firing precision. Consequently, we will analyze the influence of the launching device oscillations during firing on the rocket movement tacking in account the other disturbance factors that act during firing and also during the rocket flight in a standard atmosphere.

Manuscript received the 23 March, 2009. This work was supported in part by the Romanian Space Agency and Military Technical Academy.

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There are used 6 state variables to study the movement of the launching device: the chassis vertical translation „z_S", the chassis pitch motion „γ_y", the chassis rolling motion „γ_x", the tilting platform gyration motion „φ_z", the tilting platform pitching motion „φ_y" and the rocket translation in the container's guiding tube „s". The knowing of these state variables of the launching device is necessary to calculate the initial conditions of the rocket flight on the trajectory [2].

In order to compute the rocket evolution on the trajectory we use a differential equations system that describes the general rocket motion on the trajectory [3], having the following main unknown variables: V - velocity of the rocket center of mass, θ_L, ψ_L - angles which defines the rocket velocity direction in vertical and horizontal planes, $\omega_\xi, \omega_\eta, \omega_\zeta$ - the components of the rocket angular velocity, $\alpha_V, \alpha_H, \gamma$ - the angles which defines the position of the rocket in relation with the system bound up with the rocket velocity, x_p, y_p, z_p - the coordinates for the rocket center of mass.

The mathematical model of the general rocket motion take into account the gas kinetic eccentricities, characteristic for the propulsion system, and the aerodynamic eccentricities. The same, the mathematical model uses all the forces and moments which act on the rocket [4]. The initial conditions for the rocket flight are influenced by the launching device oscillations during the shooting [5]. The differential equation that defines the rocket motion, being so complex, doesn't allow obtaining an analytical solution therefore we need to use a numerical solving method. The numerical methods used must not introduce the considerable errors when we determine the trajectory elements.

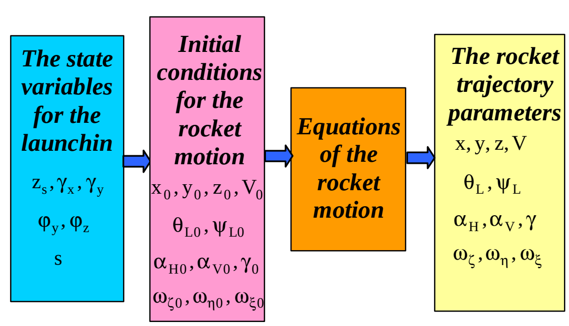

Fig. 1 The block diagram for the equations of the rocket general motion integration

ISBN:978-988-18210-1-0

In order to calculate the rocket trajectory parameters and to evaluate the trajectory deviations we use a numerical application, named ILANPRN [3] and developed by authors. This application is able to calculate automatically the rocket flight initial conditions, using the launching device oscillations during launching (see figure 1). As well we can input the initial conditions to calculate some trajectories.

II. Initial conditions for the rocket flight

Initial conditions for the rocket movement on the trajectory are represented by the rocket center of masses coordinates in the time of launcher quitting, x_{p0}, y_{p0}, z_{p0}, the rocket velocity, V_0, and the initial angles θ_{L0}, ψ_{L0}, α_{V0}, α_{H0}, γ_0, and the initial angular velocity $\omega_{\xi 0}, \omega_{\eta 0}, \omega_{\zeta 0}$:

$$\begin{cases} V_0 = \sqrt{V_{X_0}^2 + V_{Y_0}^2 + V_{Z_0}^2} \\ \theta_{L0} \cong (\delta_y)_0 \\ \psi_{L0} \cong (\delta_z)_0 \\ \omega_{\xi 0} = -\dot{\alpha}_{H0} - \dot{\psi}_{L0}\cos(\theta_{L0} + \alpha_{V0}) \\ \omega_{\eta 0} = \dot{\alpha}_{V0}\cos\alpha_{H0} + \dot{\theta}_{L0}\cos\alpha_{H0} + \\ \qquad + \dot{\psi}_{L0}\sin(\theta_{L0} + \alpha_{V0})\sin\alpha_{H0} \\ \omega_{\zeta 0} = \dot{\gamma}_0 + \dot{\alpha}_{V0}\sin\alpha_{H0} + \dot{\theta}_{L0}\sin\alpha_{H0} - \\ \qquad - \dot{\psi}_{L0}\sin(\theta_{L0} + \alpha_{V0})\cos\alpha_{H0} \\ \alpha_{V0} \cong \arcsin\left[\dfrac{V_v}{V_0}\sin\alpha_0\right] \\ \alpha_{H0} \cong arctg\left[\dfrac{V_L}{V_R}\right] \\ \gamma_0 \cong (\delta_x)_0 \\ x_{p0} = X_0 \\ y_{p0} = Z_0 \\ z_{p0} = -Y_0 \end{cases} \tag{1}$$

where the values X_0, Y_0, Z_0, $V_{X_0}, V_{Y_0}, V_{Z_0}$, $(\delta_x)_0, (\delta_y)_0$, $(\delta_z)_0$, $\omega_{\xi 0}$, $\omega_{\eta 0}$, $\omega_{\zeta 0}$ are determined on the basis of the rocket parameters during the movement on the launching device. The schemas for calculus of the angles α_{V0} and α_{H0} are presented in the figures 2 and 3.

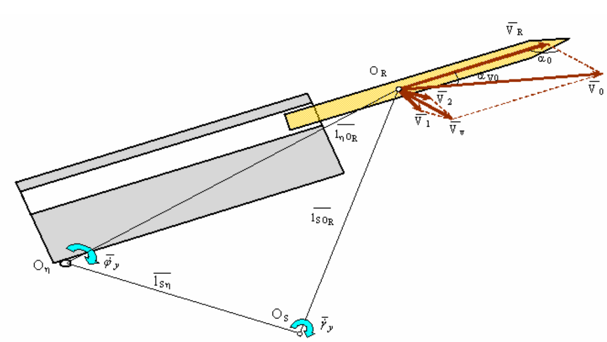

Fig. 2 Calculus schema of the angle α_{V0}

For the calculus relations of the angles α_{V0} and α_{H0} are used the following notations:

$\overline{V}_v$ - launching device vibration velocity

$$V_v = \sqrt{V_1^2 + V_2^2 + 2V_1 V_2 \cos\frac{l_{SO_R}^2 + l_{\eta O_R}^2 - l_{S\eta}^2}{2 l_{SO_R} l_{\eta O_R}}}, \tag{2}$$

where $V_1 = l_{\eta O_R}\dot{\varphi}_{y0}$, $V_2 = l_{SO_R}\dot{\gamma}_{y0}$;

$\overline{V}_L$ - lateral velocity, given by the relation $V_L = l_{\eta O_R}\dot{\varphi}_{z0}$.

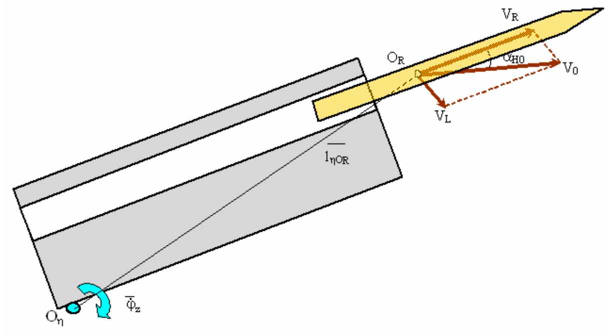

Fig. 3 Calculus schema of the angle α_{H0}

The general rocket movement equations are solved using a calculus module integrated in the numerical application ILANPRN.

III. The „base trajectory”

In order to study the rocket trajectory deviations, we take into account the launching device oscillations during rocket launching. It is necessary to define and to calculate a reference trajectory. This trajectory is named „base trajectory” and represents a theoretical trajectory, obtained by simulation of a rocket launching from a central position of the container and without any disturbances.

So, to obtain the initial conditions for the base trajectory we consider that the system chassis – tilting platform don’t have oscillation motion, being in a static equilibrium. In this case, we will have the following initial conditions: $x_{p0} = 3.373$ m, $y_{p0} = 1.989$ m, $z_{p0} = $ -0.111 m, $V_0 = 56.78$ m/s, $\theta_{L0} = 0°$, $\psi_{L0} = 0°$, $\alpha_{V0} = 0°$, $\alpha_{H0} = 0°$, $\gamma_0 = 173.2°$, $\omega_{\xi 0} = 0$ rad/s, $\omega_{\eta 0} = 0$ rad/s, $\omega_{\zeta 0} = 59.46$ rad/s, and $\theta_0 = 20°$ - imposed parameter.

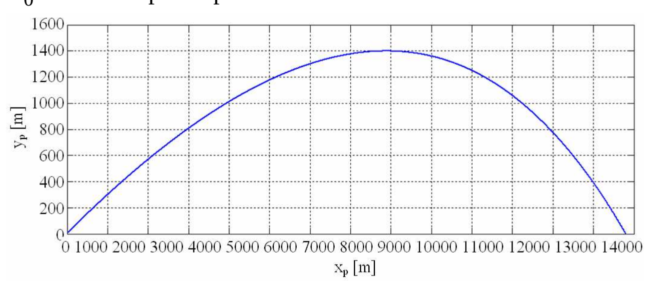

Fig. 4 The base trajectory

Starting from these initial conditions and using the ILANPRN application, were calculated the elements of the base trajectory (figure 4). In table 1 are presented the values for the main parameters of the point of impact considering the base trajectory and comparing with the values given in the shooting table for the same angle of shooting.

Table 1 Trajectory final parameters

	Range	Impact velocity	Angle at impact	Time of flight	Maximum height
	[m]	[m/s]	[degree]	[s]	[m]
Base trajectory	13800	314.43	28.41	34.61	1403
Trajectory shooting table	13800	312	28	34	1360

From table 1 we can see that the values for obtained parameters by numerical simulation are much closed with the values from the shooting table, fact that validates the applied numerical methods.

Considering the fact that the deviations between the calculated trajectory values and those from the shooting table are small, we can have the assurance that the trajectory's elements calculated starting from the initial conditions with small disturbances, will be the same with the reality ones.

IV. The influence of rocket velocity variance on the trajectory

For an increment with 6 m/s we obtain a battle variation of 224 m (see table 2), which is the range of variation for the initial velocity, without taking into account the velocity dispersion caused by the traction force deviations from a rocket to another: $V_0 \in [54.0,\ 60.0]$ m/s.

The initial conditions to integrate the rocket general motion equations are: x_{p0}=3.37m, y_{p0}=1.98m, z_{p0}=-0.11m, θ_{L0}=0°, ψ_{L0}=0°, α_{V0}=0°, α_{H0}=0°, γ_0=173.2°, $\omega_{\xi 0}$=0 rad/s, $\omega_{\eta 0}$=0 rad/s, $\omega_{\zeta 0}$=59.46 rad/s, and for the initial velocity is considered values from the interval presented before.

For the trajectory deflection is obtained a variation of 38.4 m, and for the time of flying a variation of 0.46 s. Therefore, the initial velocity variance, according to the launching device oscillations, goes to a semnificative modification for the battle and the maximum trajectory deflection, the other parameters having a small variation.

Table 2 Rocket initial velocity influence on the impact point

V_0	X	Y	t_c	V_c	$(\theta_L)_c$
[m/s]	[m]	[m]	[s]	[m/s]	[degree]
54	13695	1385.4	34.39	314.41	-48.23
55	13734	1392.2	34.47	314.42	-48.29
56	13771	1398.8	34.55	314.43	-48.36
56.7	***13800***	***1403.9***	***34.61***	***314.44***	***-48.41***
57	13808	1405.3	34.63	314.44	-48.42
58	13845	1411.8	34.70	314.45	-48.49
59	13881	1418.2	34.78	314.46	-48.55
60	13919	1424.8	34.86	314.48	-48.61

Taking into account the initial velocity variation, was determined the variation in time for the rocket center of mass δx_p (figure 5), δy_p (figure 6), and for the rocket velocity variation, δV , in relation to the determined parameters for the base trajectory.

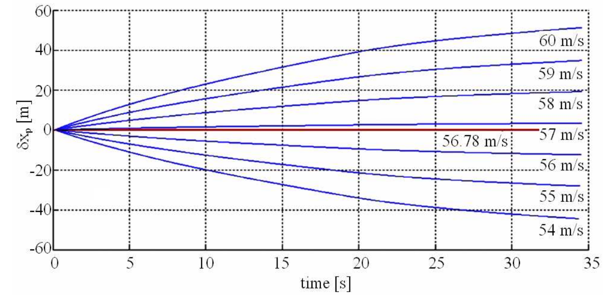

Fig. 5 The influence of velocity variation on x_p coordinate

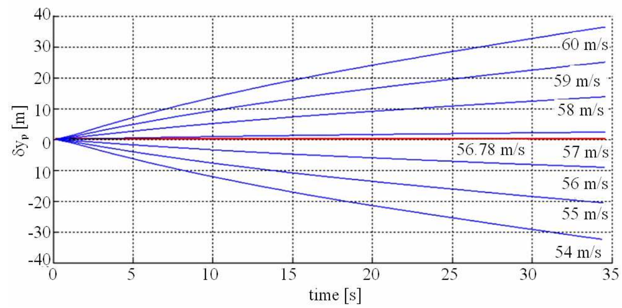

Fig. 6 The influence of velocity variation on y_p coordinate

We are observed that in the same time with the rise of the initial, δx_p and δy_p become bigger. As well, δV have a descending evolution to value 0, having a slope major on the final trajectory portion, which is the same with the situation when the rocket velocity becomes too big because of the gravity acceleration.

V. The influence of vertical angle variation on the trajectory elements

To study the influence of the θ_{L0} angle variation (angle which defines the initial orientation of the rocket in vertical plane) on the trajectory elements, it is considered a variation interval of [-0.3 0] degree.

The initial conditions to integrate the rocket general motion equations are: x_{p0}=3.37m, y_{p0}=1.98m, z_{p0}=-0.11m, V_0=56,78m/s, ψ_{L0}=0°, α_{V0}=0°, α_{H0}=0°, γ_0=173.2°, $\omega_{\xi 0}$=0 rad/s, $\omega_{\eta 0}$=0 rad/s, $\omega_{\zeta 0}$=59.46 rad/s, and for θ_{L0} is considered values from the interval presented before.

Table 3 Influence of angle θ_{L0} variation on the falling point

θ_{L0}	X	Y	t_c	V_c	$(\theta_L)_c$
[degree]	[m]	[m]	[s]	[m/s]	[grade]
-0.30	13684	1362	34.13	315.18	-47.87
-0.25	13703	1369	34.21	315.05	-47.96
-0.20	13723	1376	34.29	314.93	-48.05
-0.15	13742	1383	34.37	314.8	-48.14
-0.10	13761	1390	34.45	314.68	-48.23
-0.05	13781	1396	34.53	314.56	-48.32
0	***13800***	***1403***	***34.61***	***314.44***	***-48.41***

In virtue of the data presented in table 3, can be seen that for a variation of θ_{L0} angle with 0.3 degree we obtain a semnificative variation of battle, approximately 116 m. As well, the trajectory deflection is modified with 41.7 m, the remained velocity with 0.72 m/s, the falling angle with 0.538 degree, and the flight time with 0.485 s.

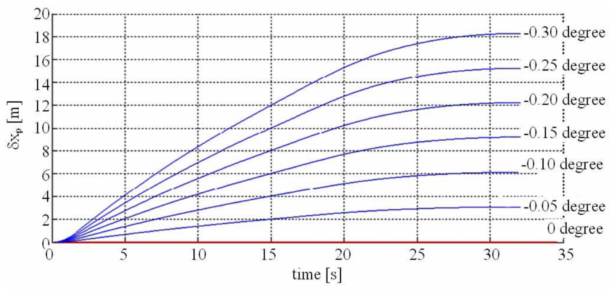

Fig. 7 The influence of initial θ_{L0} angle variation on x_p coordinate

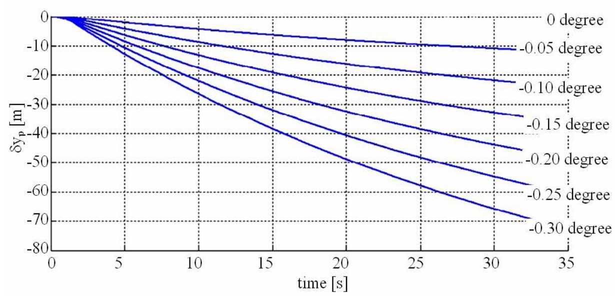

Fig. 8 The influence of initial θ_{L0} angle variation on y_p coordinate

In figures 7 and 8 we have presented the evolutions in time for the rocket center of mass coordinates δx_p and δy_p. We consider that on the trajectory final portion the amplification of variation δx_p is smaller than the one for the variation δy_p.

VI. THE INFLUENCE OF HORIZONTAL ANGLE VARIATION ON THE TRAJECTORY ELEMENTS

Studying the influence on the trajectory elements, for variation of ψ_{L0} angle which defines the rocket orientation in horizontal plane, is made considering a variation interval of [-0.1 0.1] degrees.

The initial conditions to integrate the rocket general motion equations are: x_{p0}=3.37m, y_{p0}=1.98m, z_{p0}=-0.11m, V_0=56,78m/s, θ_{L0} =0°, α_{V0}=0°, α_{H0}=0°, γ_0=173.2°, $\omega_{\xi 0}$=0 rad/s, $\omega_{\eta 0}$=0 rad/s, $\omega_{\zeta 0}$=59.46 rad/s, and for ψ_{L0} is considered values from the interval presented before.

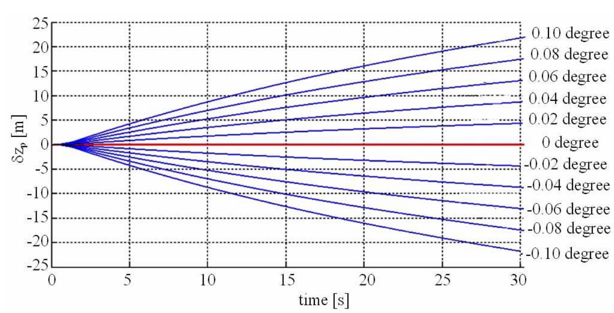

Fig. 9 The influence of initial ψ_{L0} angle variation on the z_p coordinate

In table 4 are presented the trajectory parameters in the falling point under the influence of ψ_{L0} angle**.** If we consider a variation of 0.1 degree for ψ_{L0} angle is obtained a semnificative variation the side deviation level, of approximate 24 m, the rest of parameters being constants. In conclusion the variation of the initial ψ_{L0} has an influence on the side dispersion of the rocket trajectory.

Table 4 Influence of ψ_{L0} angle variation on the impact point

ψ_{L0}	$(\theta_L)_c$	X	Z	Y	t_c	V_c
[degree]		[m]			[s]	[m/s]
-0.10	-48.4	13800	-24.1	1403.9	34.61	314.4
-0.08	-48.4	13800	-19.3	1403.9	34.61	314.4
-0.06	-48.4	13800	-14.5	1403.9	34.61	314.4
-0.04	-48.4	13800	-9.7	1403.9	34.61	314.4
-0.02	-48.4	13800	-4.9	1403.9	34.61	314.4
0	***-48.4***	***13800***	***-0.1***	***1403.9***	***34.61***	***314.4***
0.02	-48.4	13800	4.7	1403.9	34.61	314.4
0.04	-48.4	13800	9.5	1403.9	34.61	314.4
0.06	-48.4	13800	14.3	1403.9	34.61	314.4
0.08	-48.4	13800	19.1	1403.9	34.61	314.4
0.10	-48.4	13800	23.9	1403.9	34.61	314.4

In figure 9 is presented the evolution in time for coordinate z_p of the center of mass and we can observe that it is growing to the value of 24 m.

VII. TRAJECTORIES DEVIATION FOR THE SINGLE FIRING

Here were studied the trajectories deviations obtained under the influence of the launching device oscillations for the single firing case, considering the shooting from different positions from the rocket container.

There were simulated 40 trajectories, considering the firing from each existing position. The initial position used were obtained based on the equations described above (1), using the rocket movement elements at the end of time. The initial conditions are described in the table 5.

Table 5 Initial conditions

	Minimum	Maximum	Mean
x_{p0} [m]	3.3282	3.4775	3.403
y_{p0} [m]	1.7086	2.1185	1.9136
z_{p0} [m]	-0.6910	0.69089	0.000101
V_0 [m/s]	54.591	59.957	57.364
θ_{L0} [degree]	-0.2252	-0.09864	-0.1608
ψ_{L0} [degree]	-0.0016	0.019567	0.008947
α_{H0} [degree]	-1.119	1.1598	0.044708
α_{V0} [degree]	-2.1298	2.696	0.45668
γ_0 [degree]	173.29	173.61	173.4
$\omega_{\zeta 0}$ [rad/s]	59.468	59.581	59.508
$\omega_{\eta 0}$ [rad/s]	-0.00629	0.011181	0.0014605
$\omega_{\xi 0}$ [rad/s]	-0.00992	0.009712	-0.00069103

The results of the simulations gave us some information concerning the deviation of the impact points of the rocket.

All the points were found in a rectangle (figure 10) having the horizontal dimension 173 m and the vertical dimension

ISBN:978-988-18210-1-0

235 m. Considering the firing from the same conditions, the only different being the position of the rocket in the container, the deviation of the trajectories obtained is bigger in the range direction than in lateral direction.

The figure 10 shows the distribution of the impact points. We can see how the impact points describe a rectangle in the single firing case, the coordinates of the impact points being influenced only by the position of the rocket in the launching container, and of course by the oscillations of the container in the launching time.

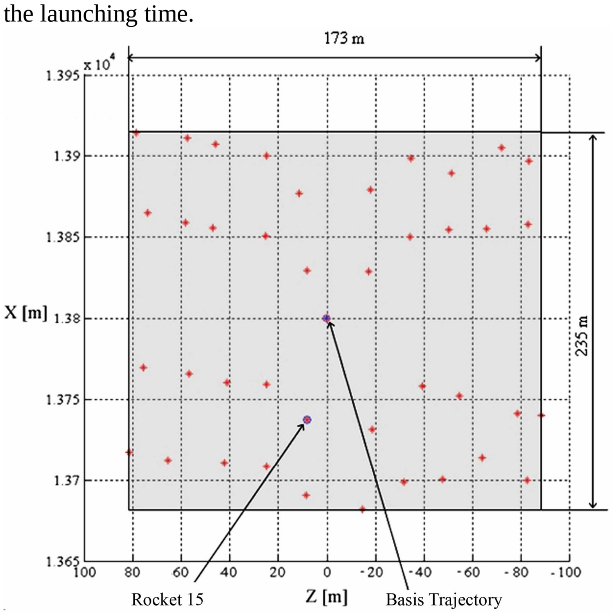

Fig. 10 Trajectory deviations

All the impact points are represented in relation with the basis trajectory impact point. This trajectory isn't a real trajectory and it isn't corresponding to a real rocket firing. The real trajectory which is the most appropriated to the basis trajectory is the trajectory obtained from the 15 positions from the launching container.

We can see that the oscillations of the launching device influence the position of the rocket 15 impact points with 62 m in range direction and with 8.29 m in lateral direction. The impact point of the basis trajectory is placed in the central area of the deviation surface.

VIII. Conclusion

In conclusion, to make a good appreciation of the launching device performances is needed to take in account the oscillations of it by the time of shooting.

In this context it is required to know the variation of the rocket motion parameters by the time of releasing the guide way, this variation being determined by the systematic and random perturbations which acts on the launching device and rocket by the time of launching. Also, it is necessary that the launching device parameters are known (the stiffness for different subassembly and their parts).

So, to design a launching device which can provide the execution of combat missions in optimal conditions it is necessary, to be designed in some way which take into account that the amount of rocket motion parameters deviation must be minimal when it leaves the launching device.

References

[1] P. ŞOMOIAG, F. MORARU, D. SAFTA, C. MOLDOVEANU, *A Mathematical Model for the Motion of a Rocket-Launching Device System on a Heavy Vehicle*, WSEAS TRANSACTIONS on APPLIED and THEORETICAL MECHANICS, April 2007, Issue 4, Volume 2, pp. 95-101, ISSN 1991-8747;

[2] P. ŞOMOIAG, D. SAFTA, F. MORARU, C. MOLDOVEANU, *A Scheduling Algorithm for the Motion of a Rocket-Launching Device System on a Heavy Vehicle*, WSEAS TRANSACTIONS on APPLIED and THEORETICAL MECHANICS, April 2007, Issue 4, Volume 2, pp. 102-107, ISSN 1991-8747;

[3] P. ŞOMOIAG, D. SAFTA, C. MOLDOVEANU, *Particularty of the Rocket Movement upon the Launcher under the Disturbance Factors Action which Appear During the Firing*, World Congress on Engineering 2008, The 2008 International Conference of Mechanical Engineering, 2-4 iulie 2008, Londra, United Kingdom, Lecture Notes in Engineering and Computer Science, WCE2008 Proceedings, Volumul II, 1396-1401 pp., ISBN 978-988-17012-3-7;

[4] P. SOMOIAG, C. MOLDOVEANU, *Application numerique pour la determination des oscillations du lanceur pendant le tirage*, The 31th Internationally attended scientific conference of the Military Technical Academy – MODERN TECHNOLOGIES IN THE 21st CENTURY , Bucharest, 3–4 November, 2005;

[5] P. SOMOIAG, *Cercetari privind determinarea oscilatiilor la lansare si influenta acestora asupra zborului rachetei*, PhD thesis, Military Technical Academy, Bucharest, 2007;

[6] R VOINEA., *Mecanica si vibratii mecanice*, Educational and Pedagogical Publishing House, 1999;

[7] V. KRYLOV, *Computation of Ground Vibrations Generated by Accelerating and Braking Road Vehicles*, Journal of Vibration and Control, SAGE Publications, Vol. 2, No. 3, 299-321, 1996;

[8] O. ZHUA, I. MITSUAKI., *Chaotic vibration of a nonlinear full-vehicle model*, International Journal of Solids and Structures Volume 43, Issues 3-4, February 2006, pages 747-759.