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Simulation, Analyze and Program Support for Pneumatic Cylinder System

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Abstract—*The paper describes simulation of pneumatic cylinder systems with long pneumatic pipes. Problem is considered from parameter distribution and time delay perspective. It is applied the control for special group of distributed parameter systems, with distributed control, where control depends of one space and one time coordinate, and finite spectrum assignment method is implemented for time delay system. The stability on finite space interval is analyzed and efficient program support is developed in symbolic Maple program language.*

***Index Terms*— distributed control systems, distributed control, program support**

I. Introduction

A main scope of this paper, is a simulation, an analyze and program support of pneumatic cylinder systems. For complete description of pneumatic system cylinder system it is very important to analyze the characteristics of the pipes connected to a cylinder, which behavior can't be found without taking into account an influence from transient in long pneumatic lines. Methods of analyse of control systems and simulation methods, which are used for observing dynamic behavior of linear dynamic systems with time delay, and distributed parameter systems, based on linear algebra, operation calculus, functional analyse, integral differential equations and linear matrix non-equations are applied. Signal transient in long pneumatic lines is analysed from time delay and parameter distribution view of point. The pressure or flow changing phenomena in pneumatic control systems is very complex, and has a significant effect on the stability, response and construction issues of the system and its components. Up to now, the published papers have not been shown complete analyze of this phenomena and as well have not presented the adequate control system.

It is obvious that phenomena of transient of the pressure and the flow in pneumatic control systems, especially with long pneumatic lines have character of time delay and parameter distribution, and that further analyze should be implemented. This paper describes the simulation of pneumatic cylinder system, observing the problem from time delay and parameter distribution perspective. Mathematical models of these systems are described by partial different equations, but apart from distributed phenomena we can't neglect system time delay.

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II. Mathematical Model

The Figure 1 shows a schematic diagram of pneumatic cylinder system. The system consists of cylinder, inlet and outlet pipes and two speed control valves at the charge and discharge sides. Detailed procedure of creating this mathematical model is described in [1].

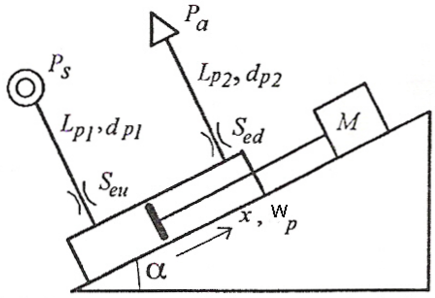

Figure 1: Schematic diagram of pneumatic cylinder system

A. Cylinder model

For describing behavior of pneumatic cylinder, the basic equations that are used are: state equation of gases, energy equation and motion equation.

$$\frac{dP}{dt} = \frac{1}{V} \cdot \left(\frac{P \cdot V}{\theta} \frac{d\theta}{dt} + R \cdot \theta \cdot G - P \cdot \frac{dV_d}{dt} \right) \tag{1}$$

where P is pressure (kPa), V- is volume (m^3), θ- temperature (K), R- universal gas constant (J/kgK), and V_d- is dead volume (m^3).

The temperature change of the air in each cylinder chamber, from the first law of thermodynamics, can be written as:

$$\frac{d\theta_d}{dt} = \frac{1}{C_v \cdot m_d} \cdot \left\{ S_{hd} \cdot h_d \left(\theta_a - \theta_d \right) + R \cdot \dot{m}_d \cdot \theta_d - P_d \cdot \frac{dV_d}{dt} \right\} \tag{2}$$

$$\frac{d\theta_u}{dt} = \frac{1}{C_v \cdot m_u} \cdot \left\{ S_{hu} \cdot h_u \left(\theta_a - \theta_u \right) + C_p \cdot \dot{m}_u \cdot T_1 - C_v \cdot \dot{m}_u \cdot \theta_u - P_u \cdot \frac{dV_u}{dt} \right\} \tag{3}$$

where C_v- is specific heat at constant volume (J/kgK), m- mass of the air (kg), S_h –heat transfer area (m^2), $\dot{m}$ - mass flow rate (kg/s), and subscript d denotes downstream side, and subscript u denotes upstream side.

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Taking into account that thermal conductivity and the heat capacity of the cylinder are sufficiently large compared with them of the air, the wall temperature is considered to be constant.

In equation of motion, the friction force is considered as sum of the Coulomb and viscous friction, and force of viscous friction is considered as linear function of piston velocity, and other parameters have constant effect to friction force of cylinder. Then, equation of motion may be presented in following form:

$$M \cdot \frac{dw_p}{dt} = P_u \cdot S_u - P_d \cdot S_d + P_a \cdot (S_d - S_u) - M \cdot g \cdot \sin\alpha - c \cdot w_p - F_q \quad (4)$$

where S- cylinder piston area (m^2), w_p- piston velocity (m/s), M- load mass (kg), c-cylinder viscous friction (Ns/m), P_a- atmospheric pressure (kPa), F_q- Coulomb friction (N), g- acceleration of gravity (m/s^2).

B. Pipe model

By using the finite difference method, it can be possible to calculate the airflow through the pneumatic pipe. The pipe is divided into n partitions.

Applying the continuity equation, and using relation for mass of the air $m = \rho \cdot A \cdot \partial z$ and mass flow $\dot{m} = \rho \cdot A \cdot w$, it can be obtained:

$$\frac{\partial m_i}{\partial t} = \dot{m}_{i-1} - \dot{m}_i . \quad (5)$$

Starting from the gas equation, and assuming that the volume of each part is constant, deriving the state equation it follows:

$$\frac{dP_i}{dt} = \frac{R \cdot \theta_i}{V}\left(\dot{m}_{i-1} - \dot{m}_i\right) + \frac{R \cdot m_i}{V}\frac{d\theta_i}{dt} . \quad (6)$$

The motion equation of the air is derived from Newton's second law of motion and is described as:

$$\frac{\partial w}{\partial t} = \frac{P_i - P_{i+q}}{\rho_i \cdot \delta z} - \frac{\lambda}{2d} \cdot w_i \cdot |w_i| - |w_i| \cdot \frac{\partial w_i}{\partial z} \quad (7)$$

where λ is pipe viscous friction coefficient and is calculated as a function of the Reynolds number:

$$\lambda = \frac{64}{\mathrm{Re}}, \qquad \mathrm{Re} < 2.5x10^3 \quad (8)$$

$$\lambda = 0.3164 \cdot \mathrm{Re}^{-0.25}, \quad \mathrm{Re} < 2.5x10^3 \quad (9)$$

The respective energy can be written as:

$$\Delta E_{st} = E_{1i} - E_{2i} + L_{1i} - L_{2i} + Q_i \quad (10)$$

where E_{1i} is input energy, E_{2i} is output energy, L_{1i} is cylinder stroke in downstream side, and L_{2i} is cylinder stroke in upstream side of pipe model, and the total energy is calculated as sum of kinematic and potential energy.

Deriving the total energy ΔE_{st} , it is obtained the energy change ΔE_{st}:

$$\Delta E_{st} = \frac{d}{dt}\left\{ C_v \cdot m_i \cdot \theta_i + \frac{1}{2} \cdot m_i \cdot \left(\frac{w_{i-1} + w_i}{2}\right)^2 \right\} \quad (11)$$

In equation (10), the inflow and outflow energy as well as the work made by the inflow and outflow air can be presented with following:

$$w_{i-1} \geq 0 \quad E_1 = C_v \cdot \dot{m}_{i-1}\, \theta_{i-1} + \frac{1}{2} \cdot \dot{m}_{i-1} \cdot w_{i-1}^2, \qquad L_1 = R \cdot \theta_{i-1} \cdot \dot{m}_{i-1}$$

$$w_{i-1} < 0 \quad E_1 = C_v \cdot \dot{m}_{i-1}\, \theta_i + \frac{1}{2} \cdot \dot{m}_{i-1} \cdot w_{i-1}^2, \qquad L_1 = R \cdot \theta_i \cdot \dot{m}_{i-1}$$

$$w_i \geq 0 \quad E_2 = C_v \cdot \dot{m}_i\, \theta_{i-} + \frac{1}{2} \cdot \dot{m}_i \cdot w_i^2, \qquad L_1 = R \cdot \theta_i \cdot \dot{m}_i$$

$$w_i < 0 \quad E_2 = C_v \cdot \dot{m}_i\, \theta_{i+1} + \frac{1}{2} \cdot \dot{m}_i \cdot w_i^2, \qquad L_1 = R \cdot \theta_{i+1} \cdot \dot{m}_i \quad (12)$$

From the following equation the heat energy Q can be calculated:

$$Q = h_i \cdot S_h \cdot \left(\theta_a - \theta_i\right) \quad (13)$$

where h is heat transfer coefficient which can be easily calculated from the Nusselt number Nu, and thermal conductivity k:

$$h_i = \frac{2Nu_i \cdot k_i}{d_p} \quad (14)$$

where d_p is pipe diameter.

Nusselt number can be calculated from Ditus and Boelter formula for smooth tubes, and for fully developed turbulent flow:

$$Nu_i = 0.023 \cdot \mathrm{Re}_i^{0.8} \cdot \mathrm{Pr}^{0.4} \quad (15)$$

and thermal conductivity k can be calculated as a linear function of temperature:

$$k_i = 7.95 \cdot 10^{-5} \cdot \theta_i + 2.0465 \cdot 10^{-3} \quad (16)$$

III. APPLICATION OF DISTRIBUTED CONTROL

Control of distributed parameter systems, which depends of time and space coordinate is called distributed control. If we choose control U, for pressure difference in two close parts of pneumatic pipe, and for state X, if we choose air velocity through the pneumatic pipe, with assumptions that are shown during derivation of mathematical model of pneumatic pipe, finally it is obtained:

$$\frac{\partial X}{\partial t} + |X| \cdot \frac{\partial X}{\partial z} + a \cdot X \cdot |X| = b \cdot U,\ z \in [0, L] \quad (17)$$

where $a = \frac{\lambda}{2d}$, $b = \frac{1}{\rho \cdot \delta z}$.

Nominal distributed control can be solved by using procedure which is described in [5], and result of that control is nominal state $w_N(t,z)$ of chosen system. In that case it yields:

$$L(X_N(t,z)) = \frac{1}{b} \cdot \frac{\partial X_N}{\partial t} + \frac{1}{b} \cdot |X| \cdot \frac{\partial X}{\partial z} + \frac{1}{b} \cdot a \cdot X \cdot |X| = U(t,z) \quad (18)$$

where L is appropriate operator.

System (18) is exposed to many disturbances, so the real dynamic must be different from nominal. It is applied deviation from nominal system state, and then the nominal system state can be realized as:

$$x(t,z) = X(t,z) - X_N(t,z),\ 0 < z \leq L . \quad (19)$$

Time derivation of deviation from nominal system state, can be presented by following equation:

$$\frac{\partial x(t,z)}{\partial t} = \frac{\partial X(t,z)}{\partial t} - \frac{\partial X_N(t,z)}{\partial t} \quad (20)$$

and from equations (17), it yields:

$$\frac{\partial x(t,z)}{\partial t} = r(t,z) + |X| \cdot \frac{\partial X}{\partial z} + a \cdot X \cdot |X| - b \cdot U \quad (21)$$

where $r = \frac{\partial X_N}{\partial t}$

IV. PRACTICAL STABILITY

Using the concept of extern linearization, which is described in, [5], we can include distributed control in the following form:

$$U(t,z)=\left[(a-k)\cdot X\cdot|X|+k\cdot X_N\cdot|X|+|X|\cdot\frac{\partial X}{\partial z}+r\right]/b, \quad (22)$$
$$0\le z\le L$$

Including the equation (44) in the equation (43), it yields:

$$\frac{\partial x(t,z)}{\partial t}=-k\cdot x(t,z),\ 0\le z\le L \quad (23)$$

Functional V is chosen in the form:

$$V(x)=\frac{1}{2}\cdot\int_0^L[x(t,z)]^2\cdot dz=\frac{1}{2}\cdot\|x(t,z)\|^2 \quad (24)$$

Derivation of functional V is given as:

$$\begin{aligned}\frac{dV(x)}{dt}&=\int_0^L x\cdot\frac{\partial x}{\partial t}\cdot dz\\&=-k\cdot\int_0^L[x(t,z)]^2\cdot dz=-2\cdot k\cdot V(x)\end{aligned} \quad (25)$$

Taking into account that $V(x)$ is positive defined functional, time derivation of functional given by equation (47) will be negative defined function for $k>0$, and in that way all necessary conditions from Ljapunov theorem applied to functional V, are fulfilled.

V. TIME DELAY APPROACH

Pneumatic cylinder systems significantly depend on behavior of pneumatic pipes, and thus it is necessary to further investigate influence of long pipes. It is obvious that phenomena of transient of the pressure and the flow in pneumatic control systems, especially with long pneumatic lines have character of time delay and parameter distribution, and that further analysis should be implemented.

Let consider the case of transient of the pressure signal through connected pneumatic lines, by using modal approximation of each pneumatic line, as in [12]. Taking into account that delays are neglected, it is obtained system transfer matrix:

$$W(s)=\sum_{k=0}^{N}\frac{n_k}{s-p_k} \quad (26)$$

where p are poles and n system nulls, and k is index.

When we consider the system with time delay caused by transitient of pneumatic signal through the long pipes, then the transfer matrix by using finite spectrum assignment method, are presented as:

$$W(s)=\sum_{k=0}^{N}\frac{1}{s-p_k}\cdot\beta_k \quad (27)$$

$$\beta_k=n_k\cdot\exp(-\tau_k\cdot p_k) \quad (28)$$

where τ is time delay.

It is possible for given positive defined matrix P and Q, determine matrix P_c, which consists set of pre-dominant poles, that can be assigned by using appropriate control law, as well as matrix F, as it is descibed in [3]:

$$F=P-P_C \quad (29)$$

and transfer matrix with time delay can be factorized, by using appropriate transformation:

$$W(s)=R(s)\cdot P^{-1}(s) \quad (30)$$

The Figure 2 describes connected pneumatic lines:

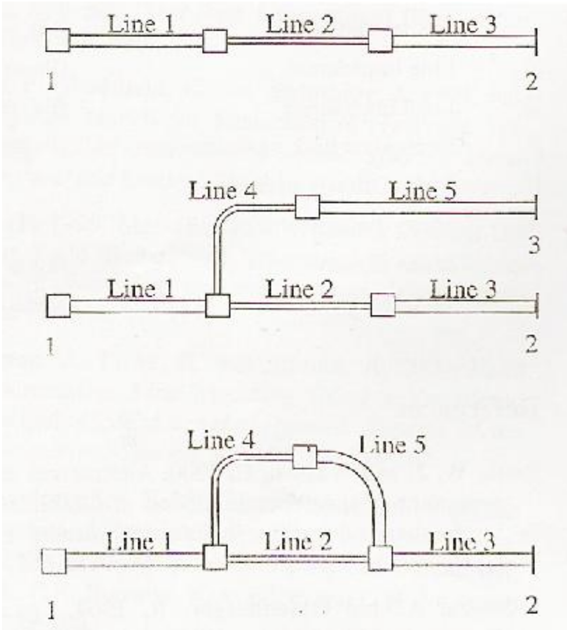

Figure 2: Connected pneumatic lines

Linear time delay systems have infinite number of poles and nules of cvazi-characteristic equation, and it has been shown that it is impossible by using state feedback, assign them to appropriate places in left plane s.

By using finite spectrum assignment method in frequent domain, which is given in [3], poles from the system described by equation (26), can be transfered to determined places.

VI. PROGRAM SUPPORT

Here is presented the program support for finite spectrum assignment method in frequent domain, applied to pneumatic transition signal in long pipes.

Program support is developed in symbolic program language Maple.

```
fcsa = prog(G, Pc, Q)
local u, j, k, rd, cd;
#Function fsam returns transfer matrix and
Desired poles given by matrix Pc and  Q
#
wuth(lunalg):
uu : = array(udentuty, 1..2, 1..2);
l  := scalarmul(uu, -1);
cd :=coldum(W);
rd :=rowdum(W);
for u from 1 to cd d0
        c[u,j] :=solve(denom(W[u,j]);
uf u=j then
        p[u,j] :=p[u,j]*1/numer(W[u,j]);
else
        p[u,j] :=0
fu:
r[u,j] := lcoeff(numer(W([u,j]);
l*c[u,j];
        od;
od;
W0 :=multuply(r,p);
for u from 1 to cd do
        for j from 1 to rd do
        If u=j then
         gu[u,j] :=scalarmul(G,1);
        else
         gu[u,j] :=0;
        fu;
        od;
od;
ee :=multiply(l, Pc);
f :=add(p,ee);
pj := scalarmul(uu,s);
mo :=multiply(u,pj);
p0 :=add(p,m0);
q0 :=add(Q,m0);
for u from 1 to cd do
        for j from 1 to rd do
        uf u=j then
         r0[u,j] :=scalarmul(r, -1);
        else
         r0[u,j] :=0;
        fu;
        od;
od;
k0 :=add(f, r0);
gf:=multiply(q0,f);
kp:=scalarmul(k0, p0);
ru :=unverse(r);
h0 :=  multiply(ru0,hr0);
h :=add(pj, h0);
gy := scalarmul(g,p);
y :=unverse(Pc);
Wyv :=scalarmul(gy, y);
```

VII. CONCLUSION

Further investigation would lead into scientific integration approach of model design, mathematical-software interpretation, developing of control algorithm in the function of the model, choosing construction solutions depending on required performances, developing of integral control and production, optimal control algorithm developing, and exchanging the information and knowledge with the other experts through the Internet.

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