Proceedings of the World Congress on Engineering 2009 Vol I
WCE 2009, July 1 - 3, 2009, London, U.K.

Spectral Filtering Highly Chirped Pulses in All Normal Dispersion Fiber Lasers

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Abstract—**We present a theoretical description of the generation of ultra-short, high-energy pulses in an all-normal dispersion laser cavity with spectral filtering. A reduced variational model based upon the Haus master mode-locking equations with quintic saturation is shown to characterize the experimentally observed dynamics. Critical in driving the intra-cavity dynamics is the nontrivial phase profiles generated and their periodic modification from the spectral filter. The theory gives a simple geometrical description of the intra-cavity dynamics and possible operation modes of the laser cavity. Further, it provides a simple and efficient method for optimizing the laser cavity performance.**

Keywords: mode-locked lasers, fiber lasers, laser theory

1 Introduction

Great effort and progress has been made experimentally to achieve mode-locked fiber lasers that produce high-energy, ultra-short pulses. Interest in high-power lasers has led to considering cavities with net normal group-velocity dispersion. Recently, a new class of high powered femtosecond fiber lasers has been demonstrated in which pulse-shaping is based on spectral filtering highly-chirped pulses in an all-normal dispersion (ANDi) laser cavity [1]. The ANDi laser exhibits a variety of pulse shapes and evolutions, which distinguish it from the typical soliton (anomalous) mode-locked lasers. The importance of the dissipative processes and the lack of a dispersion map in the ANDi laser make it an attractive physical realization of an averaged model capable of accurately describing the pulse solutions of the laser. Indeed, the ANDi laser [2] shows spectral profiles which are solutions to the cubic-quintic Ginzburg-Landau equation (CQGLE) [3]. However, these are static solutions and not capable of describing the observed intra-cavity pulse evolutions. Here we provide an analytical description of the dynamics of the ANDi laser using a variational method of the averaged evolution equations which highlights the action of spectral filtering. The reduction demonstrates the underlying stable node structure of the mode-locked solution, and highlights how phase profiles, which have been experimentally observed, are critical in the mode-locking dynamics. Further, the variational model provides an excellent theoretical framework for optimizing cavity performance [4].

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2 Governing equation and reduced model

The CQGLE

$$i\frac{\partial u}{\partial z}+\frac{d}{2}\frac{\partial^2 u}{\partial t^2}+(1-i\beta)|u|^2u+i\mu|u|^4u +i\delta u-ig\left(1+\tau\frac{\partial^2}{\partial t^2}\right)u=0, \qquad (1)$$

is a general evolution equation used to model mode-locked lasers [3]. Here z is the propagation distance, t is the retarded time, u is the complex envelope of the electric field, and all parameters except d (<0) are taken to be positive. Pulse profiles in the ANDi laser show spectral profiles which are solutions to Eq. (1) of the form [3]

$$u(z,t)=\frac{\eta}{\sqrt{B+\cosh(\omega t)}}e^{-i[A\ln(B+\cosh(\omega t))+\varphi z]}. \qquad (2)$$

These pulse solutions allow for distinct spectral structures depending on the offset phase parameter B. These exact stationary solutions to Eq. (1) accurately model a wide variety of operating modes of the ANDi laser [2]. However, these are static solutions and not capable of describing the observed intra-cavity pulse evolutions.

The variational method can be used to describe the complete evolution problem with ordinary differential equations that govern the evolution of a finite set of pulse parameters. To fully capture the varying phase profiles which have been observed in the ANDi laser cavity [2], we assume a solution-based mode-locking ansatz of the form (2) where all parameters η, ω, A, B and φ depend on z. The specific form of the phase profile is essential to capture the different spectral profiles observed in the ANDi laser [2]. Using this ansatz leads to a (5×5) reduced system of nonlinear ordinary differential equations that can be solved (numerically) to characterize the dynamics of the laser [4]. For certain parameter regimes, attracting fixed points exist which represent stable mode-locking.

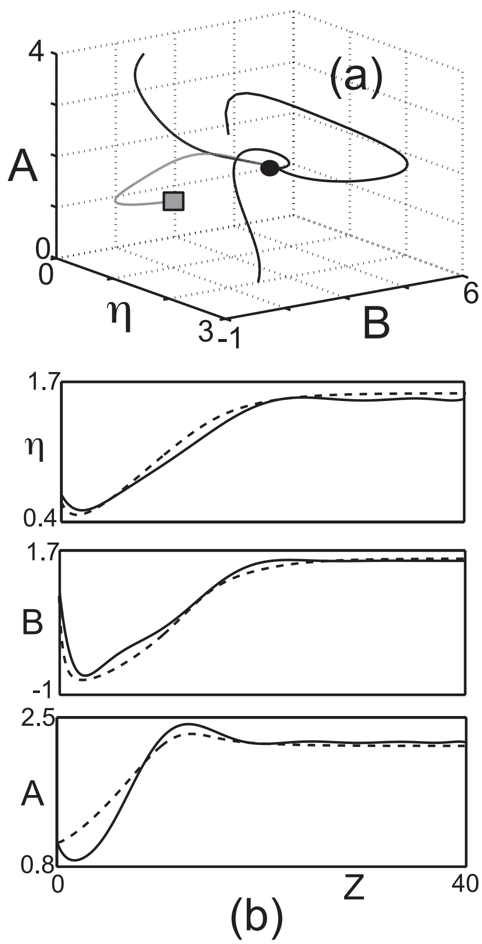

Figure 1: Reduced CQGLE model with the ansatz Eq. (2) showing a stable node $(\eta_0, B_0, A_0) = (1.6, 1.6, 2.2)$ as the attracting state for the laser. The physical parameters are $d = -0.4, \tau = 0.2, \delta = 1$, $g_0 = 1.5$, $e_0 = 1$, $\beta = 0.7$, and $\mu = 0.2$. (b) Pulse parameters from full numerical simulation of Eq. (1) (solid) compared with those obtained from the reduced model (dashed). The initial condition for both the full model and reduced model (gray square) is $u(0,t) = 0.5(1 + \cosh(t/2))^{-1/2}\exp\left[i/2 \log(1 + \cosh(t/2))\right]$.

For example, Fig. 1(a) shows the characteristic behavior associated with the reduced equations. For convenience of visualization, here we let $\omega = \eta$ in Eq. (2). The reduced system can be found explicitly in Ref. [4]. A stable node $(\eta_0, B_0, A_0) = (1.6, 1.6, 2.2)$ is the attracting state for the laser. Fig. 1(b) shows the agreement between the pulse parameters from full numerical simulation of Eq. (1) (solid) with those obtained from the reduced model (dashed). The remarkable agreement shows that this ansatz is valid in describing a mode-locked fiber laser with a cubic-quintic saturable absorber. The location of the fixed point as well as its stability depend on the parameters in the equations. This reduced model illuminates the dynamics in the laser cavity and can further allow us to consider the mechanism of the spectral filter in the laser cavity.

3 Action of spectral filtering

The reduced model gives us steady state solutions which are characterized by the phase profiles observed in the ANDi laser. Although the fixed points of the reduced model have the correct temporal and spectral profiles seen in the ANDi laser, it fails to capture the round trip cavity dynamics [1]. To capture the intra-cavity pulse dynamics we must consider the operation of the spectral filter.

The spectral filter can be assumed to be a Gaussian function with full width half maximum typically less than the gain bandwidth. The strength of the spectral filter can be characterized by the ratio filter to pulse bandwidth Γ. To observe the spectral profile evolution per round trip in the laser cavity, the filter action cannot be averaged into Eq. (1) and must be considered as a discrete forcing on the governing equations. Here we consider its effects in the context of the reduced model where a stable node acts as the attracting state of the laser. When the spectral filter is introduced in the laser cavity, it modifies this attracting state in some way. Here we assume that the round-trip cavity length is long enough so that after filter application, the pulse returns to its attracting state before encountering the spectral filter again. Thus the filter acts only on the fixed point $(\eta_0, \omega_0, B_0, A_0)$ of the dynamical system and will modify the parameters of the fixed point so that $(\eta_0, \omega_0, B_0, A_0) \rightarrow (\eta_f, \omega_f, B_f, A_f)$, where the f subscript denotes the post-filtered solution.

Figure 2 shows the effect of a gaussian spectral filter on the pulse ansatz solution Eq. (2) with $(\eta_0, \omega_0, B_0, A_0) = (1, 1, -0.5, 3.3)$ for various ratios Γ. The insets show the Gaussian filter along with the spectral profile of the solution ansatz Eq. (2) corresponding to the particular fixed point. Also shown are the resultant spectral and temporal profiles of the pulse solution after the filter is applied (solid), along with a fitted pulse solution Eq. (2) with (η_f, B_f, A_f) (dashed). The accurate spectral and temporal fit between the post-filtered pulse and the pulse solution Eq. (2) with modified parameters clearly illustrates that the application of the spectral filter on the fixed point solution effectively changes the pulse solution parameters.

4 Dynamics and cavity optimization

Combining the reduced model, which is based on averaged evolution equations, with the essential discrete element in the laser, the spectral filter, we obtain a geometrical interpretation of the intra-cavity dynamics of the ANDi laser. It is by examining the evolution along these flow lines that we can understand the dynamics of

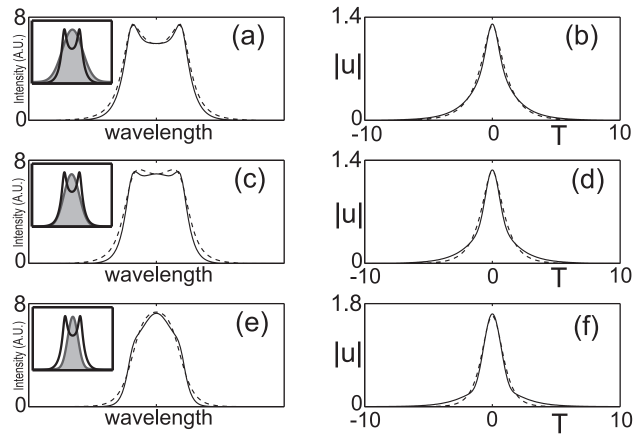

Figure 2: For three different ratios of the filter-to-pulse bandwidth, $\Gamma = 0.87$ (a,b), $\Gamma = 0.62$ (c,d), and $\Gamma = 0.5$ (e,f), the resultant spectral and temporal profiles is displayed. In each row of subfigures, the post-filtered pulse (solid) profile and spectrum is shown along with a fitted solution Eq. (2) with $(\eta_f, \omega_f, B_f, A_f)$ (dashed). The inset shows the Gaussian filter (shaded) relative to the pre-filtered pulse spectrum.

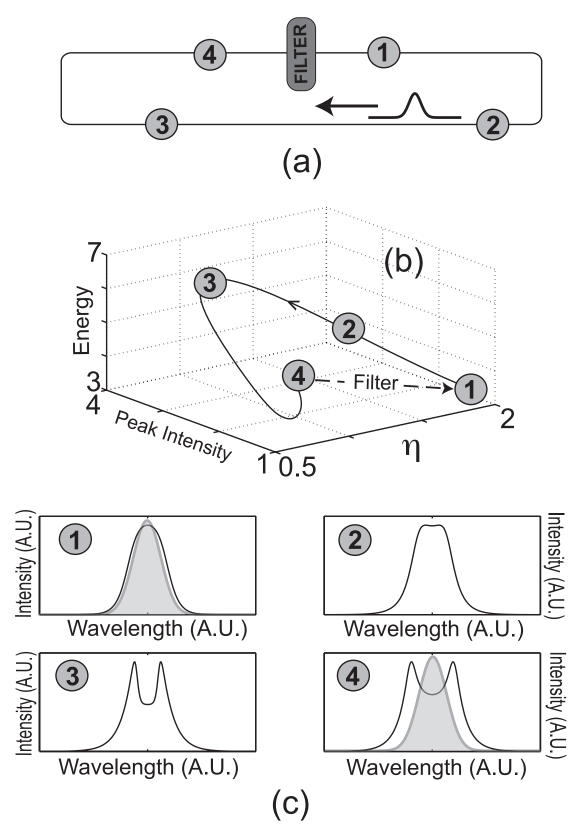

Figure 3: (a) Illustration of the laser cavity with discrete filter and four labeled positions in the cavity. (b) The intra-cavity mode-locked evolution in the experimentally relevant variables along with the action of the spectral filtering (dotted line). Note that high energy, high peak intensity pulses can be obtained if the output coupler is placed at position 3. (c) The output spectral profiles at the labeled intra-cavity positions of (a) and (b). The Gaussian spectral filter (shaded) is shown in the pre- and post-filtered positions.

the ANDi laser. Figure 3 shows an example of the laser dynamics. Figure 3(a) shows the laser configuration with the spectral filter as the primary discrete element. Figure 3(b) illustrates the phase line (whose initial condition is specified by the spectral filter width) in a relevant phase plane. Figure 3(c) shows the spectral profiles at the various positions labeled in the laser set-up and phase plane. The fixed point is denoted by 4, where the pulse directly after filtering is denoted by the 1 position. The periodic application of the filter (once per round trip) actively controls the parameters of the mode-locked pulse, changing the pulse solution parameters in Eq. (2) from the fixed point 4 to position 1. Along the flow line the pulse evolution contains different spectral profiles that have been observed experimentally in the ANDi laser cavity [1].

Using the variational model developed here, the laser performance is analyzed with varying pump power and spectral filter band-width. The reduced model shows that the effect of a narrow filter bandwidth is similar to increasing the pump power, which agrees with experimental results. Further, Fig. 3(b) elucidates a geometrical procedure for obtaining a high energy, high peak intensity pulse for a given configuration. Inserting the output coupler at the fold of the flow line where the energy and peak intensity is maximized results in a desired pulse output. Thus there is a specific distance required for pulse propagation directly after filtering to the output coupling which optimizes laser performance. Indeed, at position 3 both the pulse energy and peak intensity are significantly increased from the corresponding values at position 4.

5 Conclusion

The variational model used here provides a geometrical interpretation that completely describes the intra-cavity dynamics of the ANDi fiber laser. The resulting intra-cavity temporal and spectral profiles are in good agreement with observed numerical and experimental results. Laser performance is analyzed with varying pump power and spectral filter band-width. The reduced model shows that the effect of a narrow gain bandwidth is similar to increasing the pump power. Again, this is in agreement with numerical and experimental results. Finally, the laser can be engineered to take advantage of the intra-cavity pulse dynamics by placing the output coupler at positions where the pulse has the desired temporal and spectral profile.

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ISBN: 978-988-17012-5-1