# Piecewise Lifting Scheme Based DWT to Model Human Vision Interpolation Phenomenon

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***Abstract*—This paper presents Lifting Scheme as a tool to compute the interpolated versions of an image for modeling Human Vision interpolation phenomenon at optimum computational load. Image can be zoomed in or zoomed out using various interpolation techniques. Here, Lifting Scheme is used as a tool to increase/decrease image size, even though image is not of size $2^n$ x $2^n$ where n is an integer. Zero padding technique is used for images those are not of size $2^n$ x $2^n$ dimensions. Any variable size reduction/magnification of an image can be computed using the proposed lifting scheme. For this, piecewise application of Lifting Scheme based DWT is presented along with the MSE & PSNR of reconstruction for gray and color images.**

***Index Terms*—Discrete Wavelet Transform, Interpolation, Piecewise application of Lifting Scheme**

## I. Introduction

Human vision model to zoom in and zoom out of an image necessitates the computation of interpolated versions of an image. Various interpolation techniques like bi-cubic, bilinear, DWT etc. have been implemented [13],[14],[15]. In our paper, basic Lifting Scheme[16] is modified for variable size image interpolation. For this, piecewise application of Lifting Scheme based DWT is presented and their results on various images are tabulated using MSE and PSNR criteria.

Significant amount of work has already been done on Lifting Scheme. Work presented in [4],[8] is focused on edge detection and image compression using Lifting Scheme. Some of these papers also discuss content based image retrieval and digital imaging [3], [5].

A robust watermarking scheme for digital images based on Lifting Scheme is proposed in [1]. As in [2], authors have proposed an efficient VLSI architecture for the implementation of one-dimensional Lifting Scheme based DWT. Image feature extraction is performed using Haar and Daubechies Lifting Scheme in [3]. Use of a temporal Lifting Scheme to exploit the high temporal coherence in the geometry of successive frames is discussed in [4]. Stereo image coding is proposed in [5] with the help of Lifting Scheme. Generalized Lifting Scheme to the case of multi-wavelets is discussed in [6].A new representation of a surface in a wavelet or lifting scheme basis is discussed in [7]. A method of multi-scale edge detection based on Lifting Scheme and fusion rule is discussed in [8]. The work presented in [9] gives new block based architecture for computing lifting scheme based 2-D DWT coefficients. Work presented in [10] aims at introducing the second generation wavelets and begins with traditional Mallat algorithm. Here, we are presenting the work carried out to obtain variable size of an image to model human vision interpolation phenomenon using piecewise lifting DWT scheme. Section I presents outline of paper and literature survey. Section II presents introduction about Lifting Scheme. Modified piecewise application of Lifting Scheme based DWT is elaborated in section III. Results on various images with MSE and PSNR values in tabulated form are presented in section IV. Conclusion is presented in section V.

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## II. Introduction of Lifting Scheme

The lifting scheme is a well known method for constructing bi-orthogonal wavelets. The main difference with the classical construction is that it does not rely on the Fourier transform.

This way, lifting scheme can be used to construct second generation wavelets, wavelets which are not necessarily translates and dilates of one function. The lifting scheme can be used in situations where no Fourier transform is available. Thus the Fourier transform can thus no longer be used as the construction tool. The lifting scheme provides an alternative. It starts with a trivial wavelet, the “Lazy wavelet"; a function which essentially doesn't do anything, but which has the formal properties of a wavelet. The lifting scheme then gradually builds a new wavelet, with improved properties, by adding in new basis functions. This is the reason for the name “lifting scheme".

The lifting scheme is a technique for both, designing wavelets and performing the discrete wavelet transform. The forward lifting scheme using Haar basis function wavelet transform divides the available data set being processed into an even half and an odd half. Actually, it is worthwhile to merge these steps and design the wavelet filters while performing the wavelet transform. This is then called the second generation wavelet transform. The discrete wavelet transform applies several filters separately to the same signal. In contrast to that, for the lifting scheme, the signal is divided like a zipper. Then a series of convolution-accumulate operations to the divided signals are applied. In this work, mainly we are presenting the results of the second generation wavelets along with some modification on it.

The lifting scheme is an efficient implementation of a wavelet transform algorithm. It was primarily developed as a method to improve wavelet transform, and then it was extended to a generic method to create so-called second-generation wavelets (i.e. wavelets which do not necessarily use the same function prototype at different levels). Second-generation wavelets are much more flexible and powerful than the first generation wavelets. The lifting scheme is an implementation of the filtering operations at each level.

Lifting scheme consists of three steps:

1. SPLIT 2. PREDICT 3.UPDATE

**1.SPLIT:** In this step, the data is divided into ODD and EVEN elements.

**2.PREDICT:** The PREDICT step uses a function that approximates the data set. The differences between the approximation and the actual data replace the odd elements of the data set. The even elements are left unchanged and become the input for the next step in the transform. The PREDICT step, where the odd value is "predicted" from the even value is described by the equation

$$odd_{j+1,i} = odd_{j,i} - P( even_{j,i} ) \quad (1)$$

**3. UPDATE:** The UPDATE step replaces the even elements with an average. These results in a smoother input for the next step of the wavelet transform. The odd elements also represent an approximation of the original data set, which allows filters to be constructed. The UPDATE phase follows the PREDICT phase. The original values of the odd elements have been overwritten by the difference between the odd element and its even "predictor". So in calculating an average the UPDATE phase must operate on the differences that are stored in the odd elements:

$$even_{j+1,i} = even_{j,i} + U( odd_{j+1,i} ) \quad (2)$$

A simple lifting scheme forward transform is shown in fig. 1.

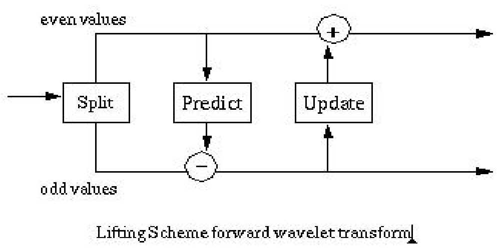


Fig.1 Lifting Scheme Forward Wavelet Transform

**Lifting Scheme Based Wavelet Processing Using Haar Transform**:

In the lifting scheme based on the Haar transform, the prediction step predicts that the odd element will be equal to the even element. The difference between the predicted value (the even element) and the actual value of the odd element replaces the odd element. For the forward transform iteration *j* and element *i*, the new odd element $_{j+1,i}$ would be

$$odd_{j+1,i} = odd_{j,i} - even_{j,i} \quad (3)$$

In the lifting scheme version of the Haar transform the UPDATE step replaces an even element with the average of the even/odd pair (e.g., the even element $s_i$ and its odd successor, $s_{i+1}$):

$$even_{j+1,i} = \frac{even_{j,i} + odd_{j,i}}{2} \quad (4)$$

The original value of the $odd_{j,i}$ element has been replaced by the difference between this element and its even predecessor. Simple algebra lets us recover the original value:

$$odd_{j,i} = even_{j,i} + odd_{j+1,i}$$

Substituting for $odd_{j,i}$ in eq.(4) , we get

$$even_{j+1,i} = \frac{even_{j,i} + even_{j,i} + odd_{j+1,i}}{2}$$

$$even_{j+1,i} = even_{j,i} + \frac{odd_{j+1,i}}{2} \quad (5)$$

The averages (even elements) become the input for the next recursive step of the forward transform. This is shown in fig.2, below.

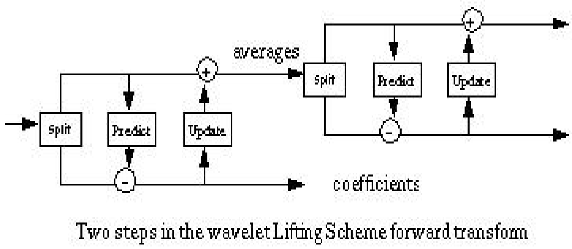


Fig.2 Two steps in Wavelet Lifting Scheme forward transform

If there are $2^n$ x $2^n$ data elements in an image, the first step of the forward transform will produce $2^{n-1}$ averages and $2^{n-1}$ differences (between the prediction and the actual odd element value). These differences are sometimes referred to as wavelet coefficients. Fig. 3 shows a 4-steps forward wavelet transform on a 16-element data set.

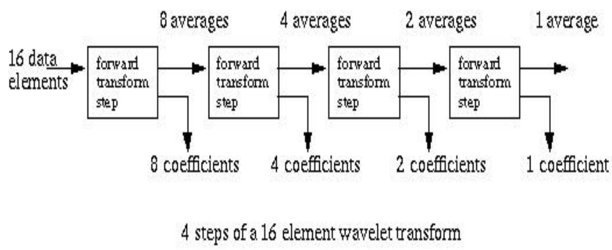


Fig. 3 Four step Forward Lifting Scheme

The split phase that starts each forward transform step moves the odd elements to the second half of the array, leaving the even elements in the lower half. At the end of the transform step, the odd elements are replaced by the differences and the even elements are replaced by the averages. The even elements become the input for the next step, which again starts with the split phase.

One of the elegant features of the lifting scheme is that the inverse transform is a mirror of the forward transform. Inverse Lifting Scheme block schematic is shown in fig.4. In the case of the Haar transform, additions are substituted for subtractions and subtractions for additions. The merge step replaces the split step.

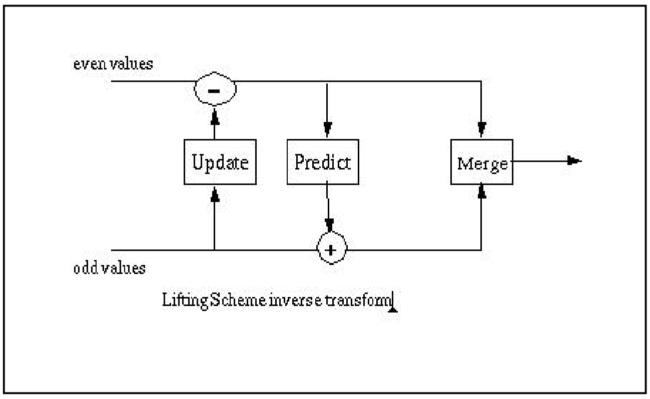


Fig.4 Inverse Lifting Scheme block schematic

## III. Modified Piecewise Application Of Lifting Scheme DWT

In conventional Lifting Scheme based DWT, complete image is divided into two parts that is even and odd image pixels. One even and one odd image pixel leads to PREDICT and UPDATE step as discussed. Here, in modified version of Lifting Scheme based DWT, image is not divided into even and odd sections, but the complete image is windowed. Windowing technique is applied throughout the complete image so as to have equal number of pixels in each window. Number of windows formed depends on the percentage interpolation required to be calculated. For example, if an image of size 256 x 256 is to be interpolated with 10% of reduction of original image size, then overall 26 x 26 pixels are to be reduced from original image. To achieve this from the original image of 256x256, 26x26 rows and columns are to be dropped such that resultant image formation is of size 230x230. To achieve this, the image is divided into n number of windows each having size as 256/26=9.86 rounded off to 10. Then, Lifting scheme is applied on a window of size 10 pixels. Thus, 26 windows are formed each containing 10 pixels for an image size of 256x256 for 10% reduction in image size. To equalize the last window containing 6 samples, complete image is padded by 2 rows of zeros at the top and bottom and 2 columns of zeros at left and right side of the image and then Lifting Scheme is applied on each window of 10 samples. Thus PREDICT and UPDATE step application on each window throughout the complete image yields reduction in size of an image. Thus, 10% reduction in image size is computed. Magnification of image so as to increase image size by 10% can be achieved using inverse Lifting Scheme. For this the difference components obtained at every stage during forward Lifting Scheme procedure are stored and are used here in inverse lifting scheme procedure. Currently available average component and the stored difference components undergo inverse lifting scheme procedure to yield magnification of an image. The only difference remains in the application of PREDICT and UPDATE steps. These steps are interchanged and magnification of an image is obtained. Thus, piecewise application of Lifting Scheme based DWT technique results in reduction and magnification of an image.

Fig. 5 shows piecewise application of Lifting scheme DWT. In this original image of size 30x30 is taken into consideration which is divided into 3 windows each containing 10 samples. To each window individually modified Lifting Scheme is applied so as to achieve required reduction. Similarly, reverse procedure that is Inverse Lifting Scheme is applied to obtain magnification of an image.

For generalized Lifting scheme it was necessary to divide data into two parts i.e. even values and odd values and process it for Lifting Scheme. Here, in modified piecewise lifting scheme procedure, image is divided into number of windows as shown in fig.5.

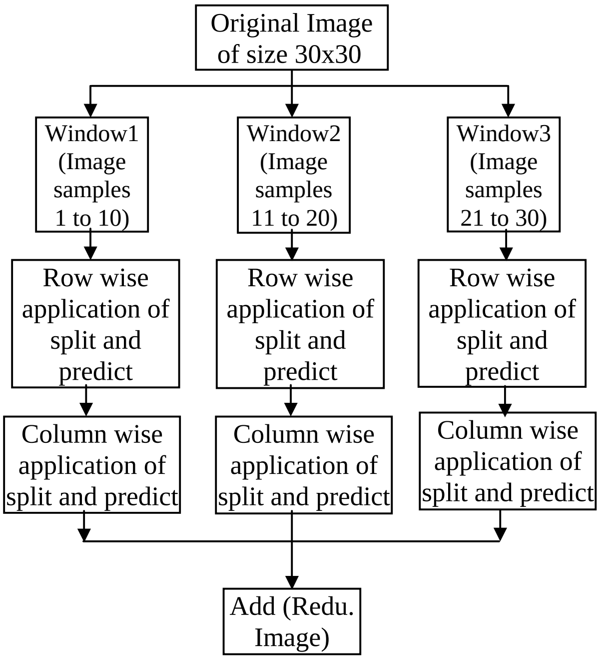


Fig. 5 Piecewise application of Lifting scheme DWT

If original image is of size 30x30 pixels, then it is divided into 3 windows for 10% reduction in size. To each window lifting scheme procedure is applied. In average step, 2 consecutive samples in each window are added to result into total 9 samples as shown in fig.6.

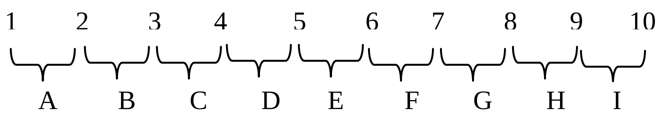


Fig. 6 Update step in modified Lifting Scheme DWT on a window of 10 samples

Samples 1 and 2 are added to yield a new sample A, samples 2 and 3 are added to yield a new sample B, samples 3 and 4 are added to yield a new sample C and so on till 10th sample.

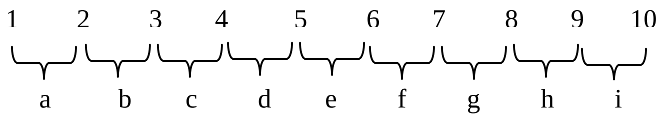


Fig. 7 Predict step in modified Lifting Scheme on a window of 10 samples.

Similarly, UPDATE or difference step procedure is implemented to result in new samples as a, b, c, d, e, f, g, h, i as in fig. 7. Thus first window underwent Lifting Scheme procedure and overall from 10 samples earlier present in first window, now the newly created first window has 9 sample (pixel) points. Similar procedure is applied to second window and third window. Thus, at the end, from total 30x30 available samples now 3 windows each containing 9x9 samples are obtained so as to reduce the size to 27x27. Thus, image resizing using piecewise Lifting Scheme is achieved.

## IV RESULTS

MSE and PSNR are used as evaluation parameters to measure performance of this scheme. For various images, their Haar based Lifting Scheme DWT is computed. Image reduction in size is achieved till it reaches two pixels- one as average pixel and the other as difference pixel as shown in fig. 8 below. Various intermediate components to reduce cameraman image to two pixels is shown in fig.8. The experimentation is done on several gray and color images but a few results are tabulated using the parameters MSE and PSNR as shown in table1 for color images. Image size not equal to $2^n$ x$2^n$ is also experimented and results are presented in table1. Original image is reconstructed using inverse lifting scheme block schematic and reconstructed image of cameraman is obtained as in fig. 9. Piecewise application of Lifting Scheme based DWT for 10% and 20% reduction in original image is experimented and results are tabulated in table 2 and 3 respectively. The original, interpolated and reconstructed images are presented in fig. 10, 11 & 12. Fig. 10 shows application of Lifting Scheme on colored image onion. Fig 11 shows original cell image, row wise and column wise application of piecewise Lifting scheme DWT to achieve 10% reduction in size. Fig 12 presents 20% reduction in size of circles. Original circles, 20% interpolated circles and reconstructed circles are shown. Similar other variable size reduction can be achieved using the same technique of piecewise application of Lifting scheme. The magnification of the images is possible using the piecewise lifting scheme IDWT applied on zeroeth level DWT of the image as in [12].

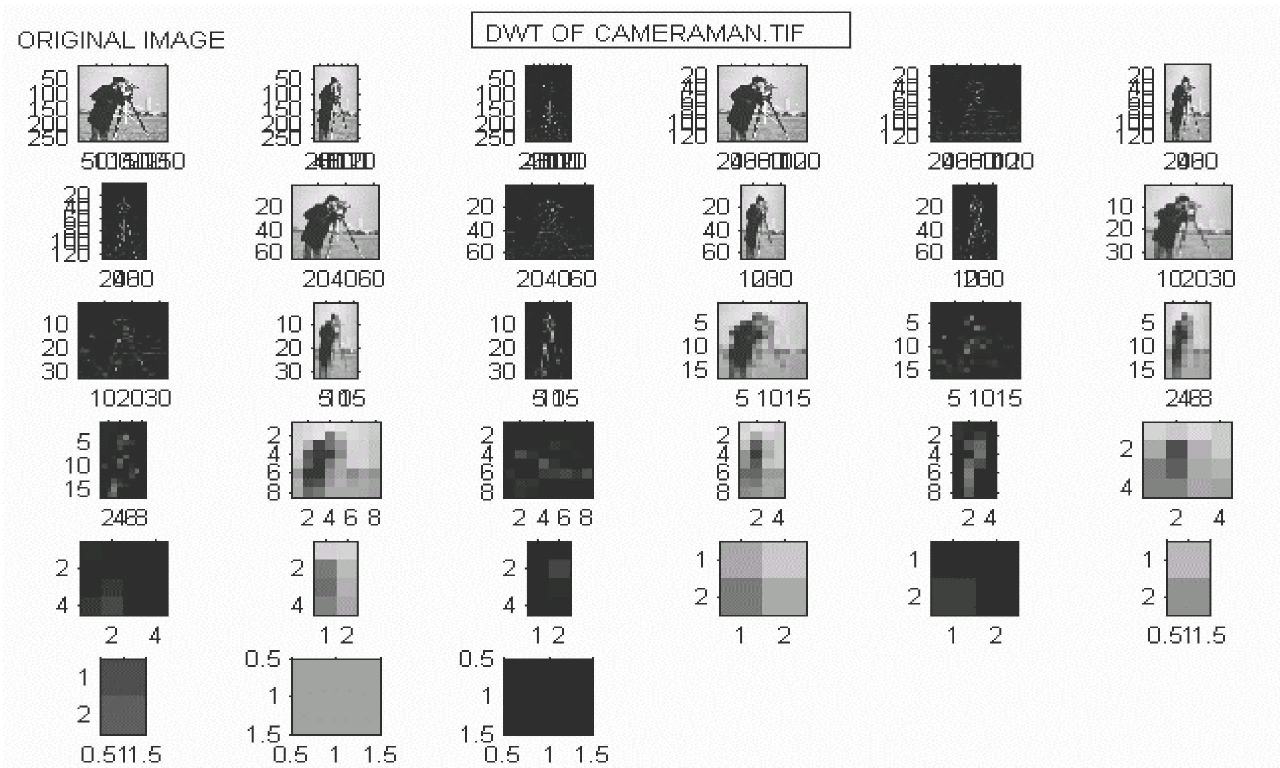


Fig 8. Lifting Scheme DWT applied to Cameraman to achieve size reduction till 2 pixels.

| Sr. no. | Image | Size (rxc) | MSE | | | PSNR | | | MSE | PSNR |
|---|---|---|---|---|---|---|---|---|---|---|
| | | | Red | Green | Blue | Red | Green | Blue | | |
| 1 | Autumn.tif | 345x256 | 1.5154 | 1.6524 | 1.9377 | 46.3256 | 45.9495 | 45.2579 | 1.7018 | 45.8443 |
| 2 | Board.tif | 306x648 | 2.1255 | 1.8530 | 1.8103 | 44.8643 | 46.2192 | 45.5534 | 1.9296 | 45.5456 |
| 3 | Onion.png | 198x135 | 1.1758 | 1.2008 | 1.2529 | 47.4276 | 47.3360 | 47.1516 | 1.2098 | 47.3050 |
| 4 | Fabric.png | 640x480 | 1.3901 | 1.8431 | 1.8915 | 46.7002 | 45.4752 | 46.1127 | 1.7082 | 49.0960 |
| 5 | Football.png | 320x256 | 2.5441 | 1.1646 | 1.2229 | 44.0754 | 47.4692 | 47.2567 | 1.6438 | 46.2671 |
| 6 | Green.png | 560x300 | 1.8396 | 1.5091 | 1.6008 | 45.4836 | 46.3535 | 46.0873 | 1.6498 | 45.9748 |
| 7 | Gantrycrane.pn | 400x264 | 1.6675 | 2.2853 | 1.0741 | 45.9102 | 44.5413 | 47.8203 | 1.6756 | 46.0906 |

Table1. MSE & PSNR values for color images

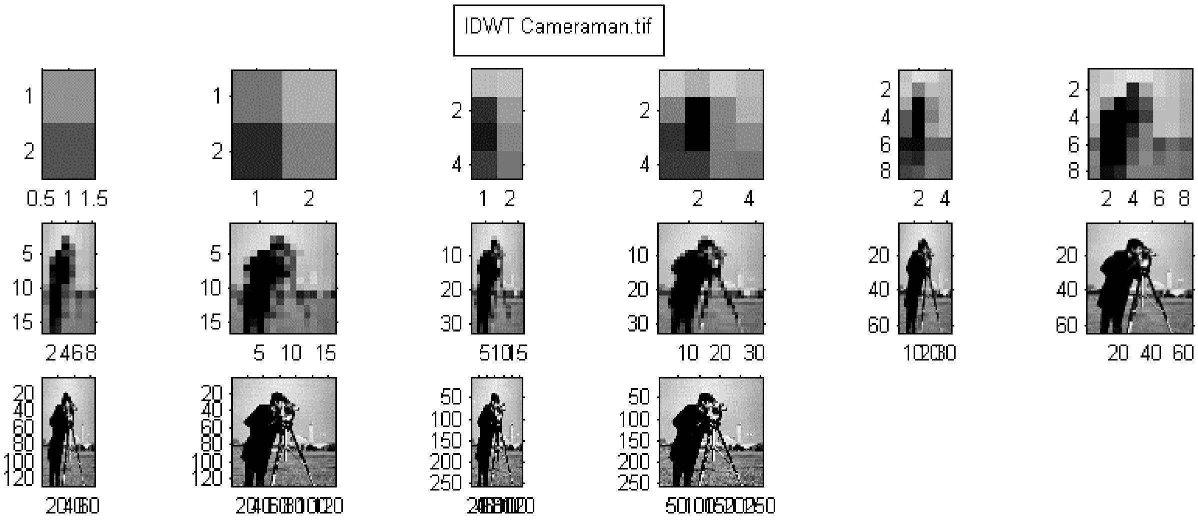


Fig. 9 Lifting Scheme IDWT to reconstruct Cameraman.tif

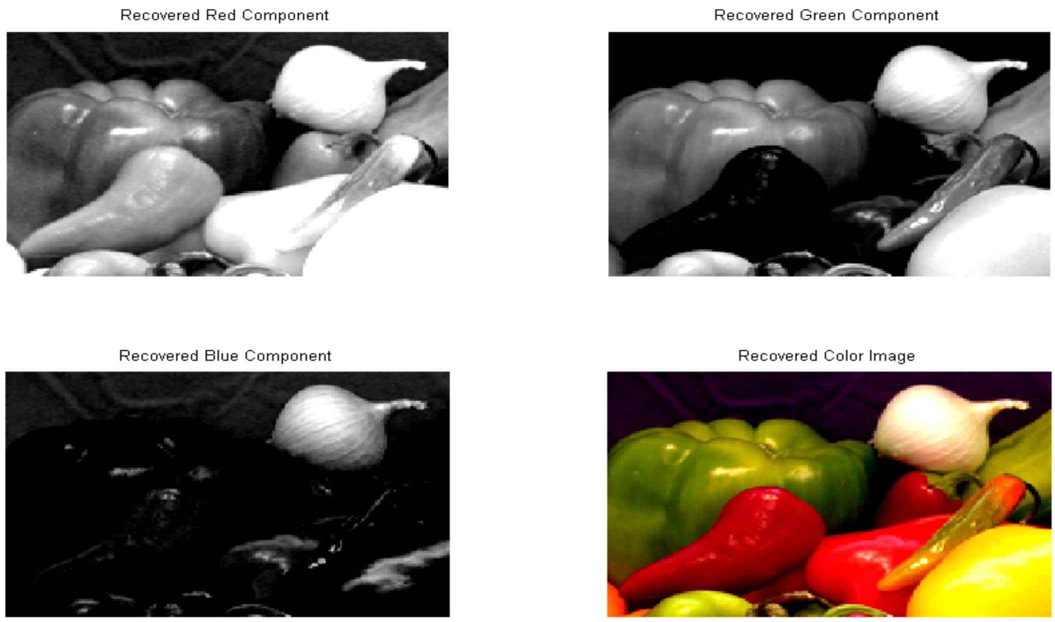


Fig.10 Lifting Scheme experimentation on colored image onion showing recovered red, green and blue component and reconstructed image

| Image | Original Size | Reduced Size | MSE | PSNR |
|---|---|---|---|---|
| Cameraman.tif | 256 x256 | 230x 230 | 112.53 | 27.61 |
| Cell.tif | 191x 159 | 172x 143 | 14.265 | 36.58 |
| Circle.png | 256x 256 | 230x 230 | 150.77 | 26.34 |
| Circuit.tif | 272x 280 | 244x 252 | 23.964 | 34.33 |
| Coins.png | 300x 246 | 270x 221 | 51.249 | 31.03 |

Table2. MSE&PSNR values for 10% reduction in image size.

| Image | Original Size | Reduced Size | MSE | PSNR |
|---|---|---|---|---|
| Cameraman.tif | 256 x 256 | 205 x 205 | 107.12 | 27.83 |
| Cell.tif | 191 x 159 | 153 x 127 | 12.74 | 37.07 |
| Circle.png | 256 x 256 | 205 x 205 | 147.52 | 26.44 |
| Circuit.tif | 272 x 280 | 217 x 224 | 22.06 | 34.69 |
| Coins.png | 300 x 246 | 240 x 197 | 47.59 | 31.35 |

Table3. MSE & PSNR values for 20% reduction in image size

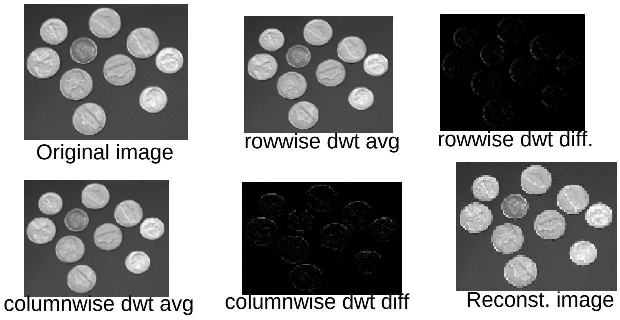


Fig.11 cell image & its10% reduction using lifting

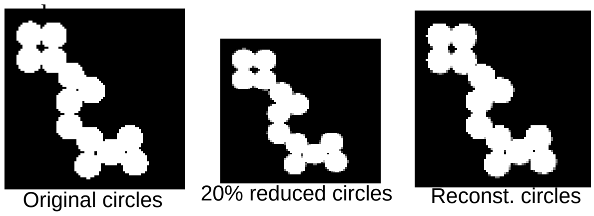


Fig.12 circles image & its 20% reduction in lifting

## V. Conclusion

Lifting Scheme is used as a tool to resize an image to any extent. Image can even be reduced to 2 pixels and its reconstruction using inverse Lifting scheme is achieved. Image reduction for an image with size not equal to $2^n$ x$2^n$ is also achieved using zero padding technique. Any odd scale reduction/magnification using piecewise Lifting Scheme technique is presented here with the help of various results in result section. MSE and PSNR values obtained for various images after reconstruction are found to be better when compared with earlier presented work [11],[13].

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