Investigation of Package Vibration During the Repetitive Shock Test

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Abstract- Packages are subjected to dynamic forces from vibration during transportation. Long term exposure to these forces can adversely affect the shipping container and the product inside. The package can be crushed and the product can break. Vibration testing of these packages has become routine to ensure package integrity. One standard test, ASTM D999, requires that the package and product be able to survive one hour of vertical vibration at a frequency that causes it to repeatedly lift off the vibration table high enough to insert a 1.5875 mm (0.0625 in.) shim underneath. In this paper, mathematical modeling and computer simulation of this situation is presented. It is shown that the shim requirement in the repetitive shock test can be satisfied using a range of table frequencies. This produces a range of impact forces as well as the number of impacts. The result is that the package and product can exhibit varying degrees of fatigue failure.

Index Terms– natural frequency, repetitive shock test, resonance.

I. Introduction

Vibration simulation is a cost-effective way to test for damage caused by the repetitive shocks generated during truck transportation. In the vibration test, ASTM D999 [1], a package is placed on a vertical vibration table in its normal shipping orientation (Fig. 1). The package is restrained from horizontal motion by vertical guides. Vibration is conducted using a slow "sine sweep" starting with a frequency of 2 Hz and gradually increasing this until the package starts to break contact with the table. The table acceleration is held at 0.5 g's peak during this sweep. The frequency that causes the package to become airborne is usually found to be within ± 3 Hz of the natural frequency of the package. To ensure that the package leaves the table, a thin metal "shim," 1.5875 mm (0.0625 in.) thick is inserted under the package while it vibrates. The test is continued for one hour after which both the package and the product are inspected for damage.

II. Mathematical Modeling

In Fig. 1, the package is modeled as a product of mass m supported by a cushion in a box, which is modeled as a material with stiffness k and dashpot constant c. Masses of the cushion and the box are assumed to be small compared to the mass m of the product. When the table and package are in contact, the separation s=0, and:

Geometry: $$x+y=z, \quad s=0 \tag{1}$$

Cushion Force:

$$F(t)=k(h-y)-c\dot{y} = k(h-z+x)-c(\dot{z}-\dot{x}) \tag{2}$$

Newton's Law: $$\ddot{z} = -g+\frac{F}{m} \tag{3}$$

where x(t) is the position of the vibration table, z(t) is the position of the product, and y(t) is the thickness of the cushion. The initial cushion thickness is h. The upward force exerted by the cushion on the product is the spring constant k of the cushion multiplied by the cushion compression plus the dashpot constant c multiplied by the compression rate.

When the package is airborne, F=0 and

Geometry: $$s=z-x-y \tag{4}$$

Cushion Thickness: $$\dot{y}=\frac{k}{c}(h-y) \tag{5}$$

Newton's Law: $$\ddot{z} = -g \tag{6}$$

Using the relationships for the natural frequency and the damping ratio in terms of m, c, and k during free vibration [2], [3], we have

$$\frac{k}{m} = \frac{(2\pi f_n)^2}{1-\xi^2} \tag{7}$$

$$\frac{c}{m} = \frac{4\pi f_n \xi}{\sqrt{1-\xi^2}} \tag{8}$$

where f_n is the natural frequency of the product on its cushion and ξ is the damping ratio.

The motion of the table is $x(t) = A\sin(2\pi f_t t)$, where A is the displacement amplitude and f_t is the table frequency. The

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amplitude of the table can be obtained by using the fact that in ASTM D999, the acceleration amplitude is held constant at G_t = 0.5 g's. Therefore,

$$A(2\pi f_i)^2 = G_i g \tag{9}$$

where g is the acceleration due to gravity = 9.81 m/s^2 (386.4 in/s^2). Therefore,

$$A = \frac{G_t g}{(2\pi f_t)^2} \tag{10}$$

Equations (1)-(6) can be solved for y, z, and s using the assumed initial conditions (motion less state):

$$z(0)=h-\frac{mg}{k}, \qquad \dot{z}(0)=0 \tag{11}$$

The acceleration of the product is determined by either equation (3) or (6). Given the initial conditions, equation (3) will control the motion initially. If and when the force F drops below zero, the package becomes airborne and equation (6) takes over. Equation (6) then continues to apply until the separation s becomes negative, in which case equation (3) takes over again.

III. Numerical Procedure

The finite difference method can be used to solve equations (1)-(6). This method is applied to the following example:

Package weight:	W=44.48 N (10 lbf)
Initial cushion thickness:	h = 50.8 mm (2 in.)
Package natural frequency:	f_n=5 Hz
Damping ratio:	ξ =0.20
Table frequency:	f_t = 5 Hz
Table acceleration in g':	G_t=0.5
Time increment:	Δt=0.005/f_t

The spring constant k and dashpot constant c are computed from equations (7) and (8). The result are k=4.6593 N/mm (26.6067 lbf/in.) and c=0.0581 N-s/mm (0.3319 lbf-sec/in.). After several cycles of motion it is seen that the response z(t) reaches steady state. Specifically, z(t) vs. time is the same for every table cycle. Table I shows the steady state results for the displacements and accelerations. Time t=0 corresponds to the beginning of a table cycle which lasts 1/f_t=0.2 sec. Note that when s=0, F is not, and vice-versa. Fig. 2 also shows this.

Now, consider what happens when the table frequency varies, such as during the frequency sweep that is done in ASTM D999. Table II shows the maximum rise height, s_{max}, and the maximum force, F_{max}, on the product vs. table frequency at steady state. Fig. 3 also shows this. At f_t=5 Hz, the packages rises a maximum of 5.13 mm (0.202 in.) above the table and experiences a maximum force of 109.11 N (24.53 lbf) when it re-contacts the table. Table I shows that s_{max} occurs somewhere in the middle of a table cycle, at which time F=0, and F_{max} occurs at the beginning and end, at which time s=0.

IV. Experimental Results

Lab experiments were conducted using the ASTM D999 protocol on old CRT style computer monitors. This test requires that the packaged product be able to withstand one hour of vibration at a table frequency that is sufficient to cause the package to repeatedly lift off the table high enough for a 1.5875 mm (0.0625 in.) shim to be inserted underneath it. Following 12 independent tests on 12 monitors, cracks on the bases of a few of these were found after only 30 min. of vibration. Those tested for the full hour showed wide variations in damage levels. Some showed no cracking at all and some showed minor scuffing.

V. Conclusions

The variation in fatigue damage was likely due to the different frequencies that different table operators used. As Fig. 3 shows, there is a range of frequencies that will cause a maximum separation sufficient to pass a 1.5875 mm (0.0625 in.) shim underneath. In Fig. 3, this range is about 4.1 to 5.5 Hz. Within this range, F_{max} can vary by as much as 30%. The number of impacts during the one hour test time can also vary. At 4.1 Hz, there are 4.1*3600=14,760 impacts and at 5.5 Hz, 19,800 impacts. So the fatigue life could vary substantially. The variation in fatigue life is especially sensitive to the damping ratio, which is controlled by the choice of cushion material. Closed cell plastic foams tend to have low dashpot constants c compared to crush-able materials like pulp and starch foam cushions. This is evident in the amount of rebound they give packaged products in drops. Fig. 4 shows effect of the damping ratio on the maximum separation between the package and vibration table, as well as the maximum force experienced by the product during repetitive shocks as the package bounces on the table. The results are for a table frequency of 5 Hz. For low damping ratios, the package bounces higher and the impact force is greater. For damping ratios greater than 0.3, the package doesn't bounce at all, in which case the variation in fatigue life caused by different operators doing the test differently is likely to be much smaller.

References

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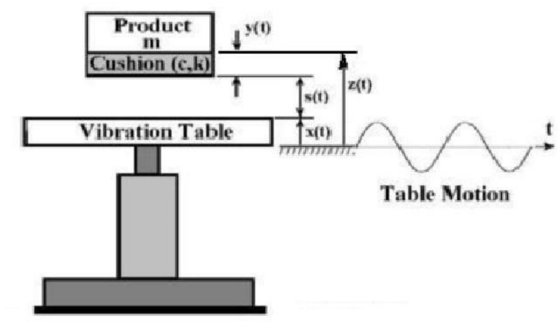

Fig. 1. Package Bouncing on Vibration Table.

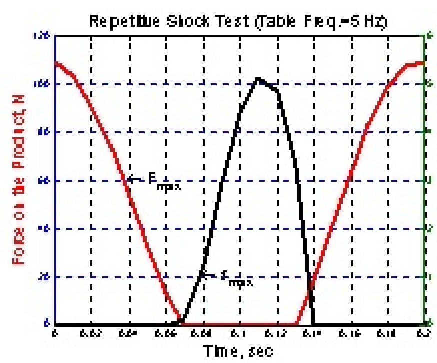

Fig. 2. Force and Separation vs. Time at Steady State for one Table Cycle.

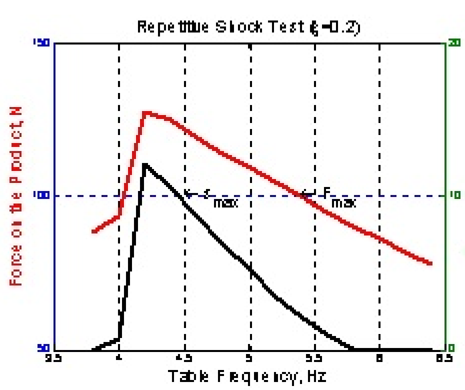

Fig. 3. Maximum Force and Separation at Steady State vs. Table Frequency.

Table I- Steady State Results for Displacements (mm) and Cushion Force (N) for ft=5 Hz and ξ=0.2

t	x	y	z	s	F
0	0	29.01	29.01	0	109.11
0.01	1.52	28.42	29.97	0	103.30
0.02	2.92	29.31	32.23	0	91.11
0.03	4.01	31.50	35.51	0	73.82
0.04	4.72	34.75	39.47	0	53.51
0.05	4.98	38.68	43.64	0	32.43
0.06	4.72	42.85	47.57	0	12.94
0.07	4.01	46.71	50.85	0.11	0
0.08	2.92	49.02	53.16	1.22	0
0.09	1.52	50.04	54.50	2.96	0
0.10	0	50.47	54.86	4.39	0
0.11	-1.52	50.65	54.23	5.13	0
0.12	-2.92	50.75	52.63	4.83	0
0.13	-4.01	50.77	50.04	3.30	0
0.14	-4.72	51.23	46.51	0	17.93
0.15	-4.98	47.35	42.37	0	40.92
0.16	-4.72	42.85	38.13	0	63.80
0.17	-4.01	38.30	34.29	0	83.83
0.18	-2.92	34.21	31.29	0	98.92
0.19	-1.52	30.99	29.46	0	107.75
0.20	0	29.01	29.01	0	109.13

Table II- Maximum Airborne Separation (mm) and Cushion Force (N) vs. Table Frequency (ξ=0.2).

f_t	s_{max}	F_{max}
3.80	0	88.12
4.00	0.69	93.36
4.20	12.05	127.79
4.40	10.64	124.89
4.60	8.76	119.47
4.80	6.85	113.56
5.00	5.13	109.11
5.20	3.48	104.57
5.40	2.11	99.95
5.60	0.95	95.23
5.80	0.02	90.38
6.00	0	86.11
6.20	0	81.71
6.40	0	77.75

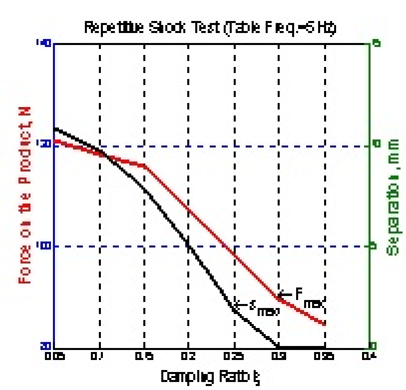

Fig. 4 Maximum Force and Separation vs. Damping Ratio at 5 Hz Table Frequency.

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