Dynamic Answer and Experimental Research concerning the Mechanisms of Mowers Machine

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Abstract—In this paper we present the dynamic answer modelling, with the dynamic models method and after that using the finite element method, for an experimental mechanism used to mowers machine. The proposed mechanism is RTR-TRT type. The paper is structured in three parts. In the first part we present the stage to day of the various type of mechanism used as design solution to the cut-off systems of the mowers machines. We present the kinematic scheme of the proposed mechanism as a structural equivalent mechanism, following the structural and geometric synthesis. In the second part we present the mechanism’s kinematic model and we perform a dynamic calculus. With this we obtain the kinematic parameters variation laws in dynamic regime, and also other dynamic parameters. In the last part of the paper is presented the finite element analysis in dynamic regime, using as input law for the load, the motor torque obtained by experimental analysis. It is presented the finite element analysis results: stress, strain and displacement distribution for the 3D model.

Index Terms—modelling, kinematics, dynamics.

I. INTRODUCTION

Aspects concerning the dynamic answer analysis of the mobile mechanical systems are presented in the researches of many authors. The dynamic analysis is presented in [1], in two variants, respectively with the dynamic models method and with Newton-Euler method, completed with the Lagrange multipliers. In Fig. 1 we present some mechanism models kinematic schemes used in the mowers cut-off systems structures, (a-crank – rod, b and c – balancing mechanism, and d - oscillatory washer mechanism) [2, 4].

II. STRUCTURAL ANALYSIS

In Fig. 2 we present the proposed mechanism kinematic scheme, for the mowers machine cut-off system.

As it is observed from the kinematic scheme the mechanism has 5 kinematic elements and 7 kinematic joints. So we have the degree of mobility of the mechanism: M=3·5-2·7=1.

That means that we have a motor element that is the rod 1. Analyzing the structural decomposition we observe that we have 2 dyads, the BBC dyad of RTR type, and the DEF dyad by TRT type.

III. THE DYNAMIC ANSWER ANALYSIS OF THE MECHANISM USING THE DYNAMICS MODELS METHOD

The kinematic scheme of the mechanism is presented in Fig. 2.

We know that the dynamic analysis straight on the mechanism with the Lagrange or Hamilton method is difficult, that why we appeal to dynamic study based on the dynamics models [1].

It is necessary to respect two conditions:
1. The power of the forces and moments which acts upon the mechanism elements, to be equal with the power of the forces and moments which acts upon the model.
2. The kinetic energy of the mechanism must be equal in any moment of the movement with the kinetic energy of the
model.

A. Positions

We write the relations for the positions (according to Fig. 4). The point B coordinates are determined with the relations:

\[
\begin{align*}
\mathbf{x}_B &= x_A + l_{AB} \cdot \cos \phi_1 \\
\mathbf{y}_B &= y_A + l_{AB} \cdot \sin \phi_1
\end{align*}
\]

The point C coordinates are known, and they are gibel by the relations:

\[
\begin{align*}
\mathbf{x}_C &= x_B + S_1 \cdot \cos \phi_3 = a \\
\mathbf{y}_C &= y_B + S_1 \cdot \sin \phi_3 = b
\end{align*}
\]

From (2) we could determine the \( S_1 \) movement.

\[
S_1 = \frac{a - x_B}{\cos \phi_3}
\]

\[\text{Fig.4. The calculus scheme for the mechanism kinematic model}\]

The \( \phi_3 \) angle is determined with (4):

\[
\phi_3 = 2 \arccos \left( \frac{A_1 \pm \sqrt{A_1^2 + A_2^2 - A_1}}{A_1 - A_1} \right)
\]

(4)

The point D coordinates are obtained with relation (5):

\[
\begin{align*}
\mathbf{x}_D &= x_C + S_1 \cdot \cos \phi_3 \\
\mathbf{y}_D &= y_C + S_1 \cdot \sin \phi_3
\end{align*}
\]

(5)

The point E coordinates are determined with relation (6):

\[
\begin{align*}
\mathbf{x}_E &= x_C + S_2^2 \cdot \cos \phi_3 + l_{DE} \cdot \cos \phi_3 = x_E + S_3 \cdot \cos \phi_3 \\
\mathbf{y}_E &= y_C + S_2^2 \cdot \sin \phi_3 + l_{DE} \cdot \sin \phi_3 = y_E + S_3 \cdot \sin \phi_3
\end{align*}
\]

(6)

We know: \( x_E = a; \ y_E = b; \phi_3; \phi_3 = 2\pi - \alpha; \alpha = \alpha \).

\[
\begin{align*}
\mathbf{x}_F &= \text{ and } \mathbf{y}_F = \text{- The point upon the slide (which is stationary);}
\end{align*}
\]

B. Speeds

We derive in report with the time the relations (1), (2), (5) and (6).

The absolute speed of the point B is determined with the relation:

\[
\begin{align*}
\dot{x}_B &= -l_{AB} \dot{\phi}_1 \cdot \sin \phi_1 \\
\dot{y}_B &= l_{AB} \dot{\phi}_1 \cdot \cos \phi_1
\end{align*}
\]

(7)

The absolute speed of the point D is determined with the relation:

\[
\begin{align*}
\dot{x}_D &= S_1 \cdot \cos \phi_3 - S_1 \cdot \sin \phi_3 \cdot \dot{\phi}_3 \\
\dot{y}_D &= S_1 \cdot \sin \phi_3 + S_1 \cdot \cos \phi_3 \cdot \dot{\phi}_3
\end{align*}
\]

(8)

The absolute speed of the point E is determined with (9):

\[
\begin{align*}
\dot{x}_E &= S_2^2 \cdot \cos \phi_3 - S_2^2 \cdot \sin \phi_3 - l_{DE} \dot{\phi}_1 \cdot \sin \phi_3 = S_3 \cdot \cos \phi_3 \\
\dot{y}_E &= S_2^2 \cdot \sin \phi_3 + S_2^2 \cdot \cos \phi_3 + l_{DE} \dot{\phi}_1 \cdot \cos \phi_3 = S_3 \cdot \sin \phi_3
\end{align*}
\]

(9)

C. Accelerations

We derivate in report with the time the relations (7), (8) and (9).

\[
\begin{align*}
\ddot{x}_B &= -l_{AB} \ddot{\phi}_1 \cdot \sin \phi_1 \\
\ddot{y}_B &= l_{AB} \ddot{\phi}_1 \cdot \cos \phi_1
\end{align*}
\]

(10)

The components of the point B accelerations vector are computed by the relations:

\[
\begin{align*}
\ddot{x}_D &= S_1^2 \cdot \cos \phi_3 - S_1^2 \cdot \sin \phi_3 - S_3^2 \cdot \sin \phi_3 \cdot \ddot{\phi}_3 - \\
&- S_1^2 \cdot \cos \phi_3 \cdot \ddot{\phi}_3^2 - S_3^2 \cdot \sin \phi_3 \cdot \ddot{\phi}_3 \\
\ddot{y}_D &= \ddot{x}_D \cdot \cos \phi_3 + \ddot{x}_D \cdot \cos \phi_3 - S_3^2 \cdot \cos \phi_3 \cdot \ddot{\phi}_3 - \\
&- S_3^2 \cdot \sin \phi_3 \cdot \ddot{\phi}_3 + \ddot{x}_D \cdot \cos \phi_3
\end{align*}
\]

(11)

D. The reaction forces establish

TRT dyad is represented in Fig. 5.

Input dates (know):

- Vectors \( \mathbf{\ddot{F}}_{i4} \) and \( \mathbf{\ddot{F}}_{i5} \):

\[
\begin{align*}
\mathbf{\ddot{F}}_{i4}^x &= \ddot{F}_{i4}^x \\
\mathbf{\ddot{F}}_{i4}^y &= \ddot{F}_{i4}^y \\
\mathbf{M}_{i4} &= \mathbf{M}_{i4}
\end{align*}
\]

The D, E and F joints coordinates.
The translating joints D and F angles, that are $\varphi_4$ and $\varphi_5$.

We want to obtain:
- The reactions forces from the D, E and F kinematic joints, that is:
$$\vec{F}_{34} = \vec{F}_{43} \cdot \vec{F}_{45} = \vec{F}_{45}, \quad \vec{F}_{54} = \vec{F}_{45} \quad \text{and} \quad \vec{F}_{05}.$$

We proceed in the following way:
- We write the axes projection equations for the forces that act upon the dyad, that are:
$$\sum X^{(4)} = 0: F_{14} + F_{14} + F_{34} = 0 \quad \sum Y^{(4)} = 0: F_{34} + F_{34} - G_4 + F_{34} = 0 \quad \sum C^{(4)} = 0: F_{34} - F_{34} - F_{34} = 0$$

Resolving the system (14) we determine the forces $F_{34}$ and $F_{34}$.

We write the moment’s equations in report with the point E for the element 4 and 5 respectively, and in report with the point D and F for the whole dyad, that is:
$$\sum M(E)^{(4)} = 0: F_{14}(y_e - y_o) + F_{34}(x_e - x_o) + F_{34}(y_e - y_o) + F_{34}(x_e - x_o) - M_{4} = 0 \quad \sum M(D)^{(5)} = 0: F_{34}(x_d - x_o) + G_4(x_e - x_o) - M_{4} = 0 \quad \sum M(F)^{(6)} = 0: F_{34}(y_d - y_o) + F_{34}(x_e - x_o) + F_{34}(y_d - y_o) = 0 \quad \sum M(F)^{(7)} = 0: F_{34}(y_d - y_o) + F_{34}(x_e - x_o) + F_{34}(y_d - y_o) = 0$$

By solution of the system (15), we determine the application point coordinates of the reactions forces $\vec{F}_{34}$ and $\vec{F}_{05}$, which are orthogonal on the slide ways D and F, that is, $(x_d, y_d)$ and $(x_f, y_f)$.

RTR dyad is represented in Fig. 6.

Fig.6. The scheme of the forces and moment which act upon the dyad B, B, C
Input data (know):
- The point B coordinates, the elements mass centers coordinates.
- The vectors $\vec{r}_{2}$ and $\vec{r}_{3}$.

\[
\begin{bmatrix}
F_{1}^x \\
F_{1}^y \\
M_{1z}
\end{bmatrix}
\begin{bmatrix}
F_{2}^x \\
F_{2}^y \\
M_{2z}
\end{bmatrix}
\begin{bmatrix}
F_{3}^x \\
F_{3}^y \\
M_{3z}
\end{bmatrix}
\]

The joints C coordinate.

The B slide way angle, that is $\varphi_3$.

We want to determine:
- The reactions forces from the kinematic joints B and C, those are:
$$\vec{F}_{12} = \vec{F}_{21}, \quad \vec{F}_{23} = \vec{F}_{32} \quad \text{and} \quad \vec{F}_{03}$$

We proceed in the following way:
- We write the axis projection equations, for the forces which act upon the element 2, that is:
$$\sum X^{(2)} = 0: F_{12}^x + F_{12}^x + F_{32}^x = 0 \quad \sum Y^{(2)} = 0: F_{32}^x - G_2 + F_{32}^x + F_{32}^x = 0$$

We write the connecting relation between the components of the reaction $\vec{F}_{32}$, which is orthogonal on the slide way, that is:
$$\sum M(C)^{(2)} = 0: F_{12}^x(x_c - x_0) - M_{12} + F_{32}^x(y_c - y_0) + F_{32}^x(x_c - x_0) - M_{12} + F_{32}^x(x_c - x_0) + F_{32}^x(y_c - y_0) - G_2 = 0$$

From the equations (16), (17) and (18) we determine the forces: $F_{12}^x$, $F_{12}^x$, $F_{32}^x$, $F_{32}^x$.

We write the moment’s equation in report to the point C, for the entire dyad, that is:
$$\sum M(C)^{(2)} = 0: F_{12}^x(x_c - x_0) - M_{12} + F_{32}^x(y_c - y_0) + F_{32}^x(x_c - x_0) - M_{12} + F_{32}^x(x_c - x_0) + F_{32}^x(y_c - y_0) - G_2 = 0$$

By solution of the system (19), we establish the forces $F_{03}^y$ and $F_{03}^y$.

We write the moment’s equations reported to the point B for the element 2, and for the element 3 in report with the same point B, that is:
$$\sum M(B)^{(2)} = 0: F_{32}^x(y_a - y_1) - F_{32}^x(x_a - x_1) - M_{12} = 0 \quad \sum M(B)^{(3)} = 0: F_{32}^x(y_a - y_1) + F_{32}^x(x_a - x_1) - F_{32}^x(x_c - x_0) = 0 \quad \sum M(B)^{(4)} = 0: F_{32}^x(y_a - y_1) - F_{32}^x(x_a - x_1) + F_{32}^x(x_c - x_0) = 0$$

By resolving the system (20) we determine the application point of the reaction force $\vec{F}_{32}$. The motor element is represented in Fig. 7.
We proceed in the following way:
- We write the axis projection equations, for the forces that act upon the element 1, that is:
\[
\begin{align*}
\sum X^{(i)} &= 0: F_{x}^{A} - F_{x}^{B} + F_{x}^{C} = 0 \\
\sum Y^{(i)} &= 0: F_{y}^{A} - G_{i} + F_{y}^{C} = 0
\end{align*}
\]
(21)
- We write the moment’s equations for the element 1, reported to the point A:
\[
\sum M^{(i)} = 0: -F_{z}^{A}(y_{A} - y_{C}) + F_{z}^{B}(y_{B} - y_{C}) - M_{A} - F_{z}^{A}(x_{C} - x_{A}) + F_{z}^{B}(x_{C} - x_{B}) + G_{A}(x_{C} - x_{A}) - M_{A} = 0
\]
(22)
\[
M_{l_{1}} = -J\Delta C_{1} \cdot \phi_{1} \cdot \vec{k}; \\
M_{l_{2}} = 0 \cdot \vec{k}; \\
M_{l_{3}} = -J\Delta C_{3} \cdot \phi_{3} \cdot \vec{k}; \\
M_{l_{4}} = 0 \cdot \vec{k}; \\
M_{l_{5}} = \vec{0} \cdot \vec{k}.
\]
\[
\begin{align*}
J_{\Delta C_{1}} &= \frac{m_{1} \cdot l_{1}^2}{12}, & J_{\Delta C_{1}} &= \frac{m_{2} \cdot (a_{2} + b_{2})^2}{12}, & J_{\Delta C_{3}} &= \frac{m_{3} \cdot l_{2}^2}{12}, \\
J_{\Delta C_{3}} &= \frac{m_{4} \cdot (a_{3} + b_{3})^2}{12}, & J_{\Delta C_{3}} &= \frac{m_{5} \cdot (a_{3} + b_{3})^2}{12}.
\end{align*}
\]

The reduced moment calculus
Is made from the condition:
P model = P mechanism
\[
M_{red} \cdot \omega = \sum_{i=1}^{5} \left( F_i \cdot \vec{V}_{ci} + M_i \cdot \vec{\phi}_i \right)
\]
(23)
\[
M_{red} \cdot \omega = \left( F_{i_{1}} + G_{i_{1}} \right) \cdot \vec{v}_{c1} + \left( F_{i_{2}} + G_{i_{2}} \right) \cdot \vec{v}_{c2} + \left( F_{i_{3}} + G_{i_{3}} \right) \cdot \vec{v}_{c3} + \left( F_{i_{4}} + G_{i_{4}} \right) \cdot \vec{v}_{c4} + \left( F_{i_{5}} + G_{i_{5}} + \vec{Q} \right) \cdot \vec{v}_{c5}
\]
(24)
Where \( \vec{\phi}_i = \vec{\phi} \) is the angular speed of the element 1.
\[
M_{red} \cdot \omega = \left( F_{i_{1}} + G_{i_{1}} \right) \cdot v_{c1} + \left( F_{i_{2}} + G_{i_{2}} \right) \cdot v_{c2} + \left( F_{i_{3}} + G_{i_{3}} \right) \cdot v_{c3} + \left( F_{i_{4}} + G_{i_{4}} \right) \cdot v_{c4} + \left( F_{i_{5}} + G_{i_{5}} + \vec{Q} \right) \cdot v_{c5}
\]
If we neglect the inertia moments we have:
\[
M_{red} \cdot \omega = \tilde{G}_{i_{1}} \cdot \vec{v}_{c1} + \tilde{G}_{i_{2}} \cdot \vec{v}_{c2} + \tilde{G}_{i_{3}} \cdot \vec{v}_{c3} + \tilde{G}_{i_{4}} \cdot \vec{v}_{c4} + \tilde{G}_{i_{5}} \cdot \vec{v}_{c5} + \tilde{M}_{i_{1}} \cdot \vec{\phi}_i
\]
(25)
The calculus of the reduced inertia moment is made from the condition:
T model = T mechanism
\[
\frac{1}{2} J_{\omega} \cdot \omega^2 = \sum_{i=1}^{3} \left( \frac{1}{2} m_{i} \cdot v_{ci}^2 + \frac{1}{2} J_{\Delta C_{i}} \cdot \omega_{i}^2 \right)
\]
(26)
Where \( \omega = \vec{\phi} \) is the angular speed of the element 1.
\[
J_{red} = m_{1} \left( \frac{v_{c1}^2}{\omega} \right) + m_{2} \left( \frac{v_{c2}^2}{\omega} \right) + m_{3} \left( \frac{v_{c3}^2}{\omega} \right) + m_{4} \left( \frac{v_{c4}^2}{\omega} \right) + m_{5} \left( \frac{v_{c5}^2}{\omega} \right)
\]
Or
\[
J_{red} = J_{\Delta C_{1}} + m_{1} \left( \frac{v_{c1}^2}{\omega} \right) + m_{2} \left( \frac{v_{c2}^2}{\omega} \right) + m_{3} \left( \frac{v_{c3}^2}{\omega} \right) + m_{4} \left( \frac{v_{c4}^2}{\omega} \right) + m_{5} \left( \frac{v_{c5}^2}{\omega} \right)
\]
We apply the kinetic energy theorem:
\[
dT = \partial L
\]
(27)
sau:
\[
\frac{1}{2} J_{\omega} \cdot \omega^2 - \frac{1}{2} J_{\delta} \cdot \delta_{\omega}^2 = \int_{\omega_{0}}^{\omega} M_{red} \cdot d\phi.
\]
The angular speed for the motor element is given by the relation:
\[
\omega = \sqrt{\frac{2}{J_{\omega}} \left[ \phi_{M} \cdot M_{red} \cdot d\phi + \frac{1}{2} J_{\delta} \cdot \delta_{\omega}^2 \right]}
\]
(28)
F. Graphical results
The force are represented in newton, angle are in radian. Graphics’ for the kinematics parameters calculated in dynamic regime:
IV. EXPERIMENTAL RESULTS

In Fig. 14 is presented the mechanism experimental model mounted on the essay stand.

For the experimental research the mechanism was mounted on a test stand, equipped with an electric motor. Also the stand offers the possibility to modify the angular speed by means of a conical variable speed drive. In Fig. 14 we present the acquisition system connected with the displacement transducers W50, W100 and W300. The force of technological resistance appear because of the adjust screw, as is presented in the Fig. 15, which push a plate upon the knife, resulting an friction force, which can be adjusted and experimentally measure.

We made tests for 3 technological forces, which have been determined whith the force transducer. Also have been determined the displacements S1 – displacement of the slide 1, S2 – displacement of the slide 2, and S3 – displacement of...
the knife, the motor moment and the resistance force. In Fig. 16 is presented the time variation of the slide displacement $S_1$, $S_2$ and $S_3$, for the first technological force, and the motor moment.

![Fig. 16. Original registrations. Test 2, $\omega_1$ angular speed, $F_2$ technological force](image)

We processed on to computer the dynamic model, and we obtained the graphics’ for the dynamic parameters, for a complete rotation of the motor element. After that, with the help of the Nastran finite element software package we made the dynamic modeling, with finite elements, and we obtained the stress, strain and displacement distribution, considering the fiction from the mechanisms’ kinematics joints. The finite element dynamic analyze results are presented in Fig. 17, 18 and 19.

![Fig. 17. The stress distribution for the mechanism assembly](image)

![Fig. 18. The displacements distribution for the mechanism assembly](image)

![Fig. 19. The strain distribution for the mechanism assembly](image)

V. CONCLUSIONS

After the dynamic analyze and of the dynamic model computer processing we made the following observations:
- The variations law of the motor element angular speed has been graphically represented, in the Fig. 8, for a complete rotation of the motor element, and he vary between the limits 10,5 and 8 rad/sec;
- The angular acceleration represented in Fig. 9, vary between the limits 0,6 to - 0,8 rad/sec$^2$;
- The angle $\phi_3$ vary between -180 and 165 degree;
- The displacement $S_{31}$ varies between 95 and -150 mm;
- The displacement $S_{32}$ varies between -94 and -106 mm;
- The displacement $S_{5}$ varies between the limits 175 to 125;
- The total displacement is 50 mm.
- That displacements are represented in Fig. 16, by experimental way;
- The bound force $F_{05}$ varies between 0 and 50 N;
- The reaction $F_{34}$ is greater upon the y axis; he varies between 101.7 and 101.2N, at a complete rotation of the element 1;
- The reaction $F_{44}$ is greater upon the y axis, varying between -99N and -102N;
- The great value has the reaction $F_{12}$, the component upon the x axis varying between -150N and 150N, and the component upon the y axis varying between -220 and 70N, the component upon the y axis being much greater that that upon the x axis.

REFERENCES