

Non-Newtonian Effects on Lubricant Hydrodynamic Thin Film Flows in the Initial Engine Start Up

M Afzaal Malik, S Adnan Qasim, Usman Farooq., Riaz Mufti.

Abstract—A 2-D hydrodynamic lubrication model and analysis of piston skirts by incorporating non-Newtonian effects at the time of engine start up is presented based on the upper convected Maxwell viscoelastic model having a constant viscosity and relaxation time. A perturbation method is used to analyze the flow of a non-Newtonian lubricant between piston and cylinder liner by using characteristic lubricant relaxation times in all order of magnitude analysis. The effect of viscoelasticity on lubricant velocity and pressure fields is examined and the influence of film thickness on lubricant characteristics is investigated. Numerical simulations show reduced piston eccentricities, significantly improved film thickness profiles and hydrodynamic pressure fields at low engine speed. This study suggests that viscoelasticity produces a beneficial effect on piston skirt lubrication in initial engine start up conditions.

Index Terms— Engine lubrication, Maxwell model, Non-Newtonian, Viscoelasticity.

I. INTRODUCTION

Piston's small oscillatory motion in a reciprocating engine significantly affects its lubrication during initial engine start up cycles. Multi grade engine lubricants show viscoelastic non-Newtonian behavior due to the presence of polymer additives [1]. Maxwell lubricant is the first viscoelastic rate type model exhibiting energy storage by characterizing lubricant's elastic response and energy dissipation by its viscous nature [2]. Compared to Newtonian fluids, kinetic energy dissipation and power due to shear stress at the wall decrease in Maxwell model [3]. Polymer additives as viscosity index (VI) improvers in viscoelastic lubricants significantly improve base stock performance over a wide range of automotive engine operating conditions [4]. Initial

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engine start up is a critical operating condition when engine lubricant must be mobile enough to assist 'cold starting' and yet viscous enough to prevent wear.

Earlier efforts included studies on flow rheology of non-Newtonian fluids on the time dependant film thickness history between non-parallel sliding surfaces on the basis of Power Law fluid model [21]. Use of poly alpha Olefins in synthetic engine oils to cater for severe engine operating conditions highlights significance of viscoelasticity aspects of non-Newtonian lubricants [7]. Many researchers modeled non-Newtonian lubricants as Power Law fluids to characterize shear thinning, although such models do not exhibit any elastic normal stress effects [19]. Most of the researchers employed 'Sinh law' to describe shear thinning effects and primarily referred to Ree-Eyring model for the purpose. In reality, 'Sinh law' was not intended to characterize shear thinning lubricants [16]. When describing shear thinning aspects using Ree-Eyring model, viscoelastic effects in compression and in shear were neglected [15]. Despite this fact, researchers of late, while modeling thermal non-Newtonian lubrication ignored viscoelastic effects and instead described shear stress and shear strain rate by following Eyring fluid model [14]. Incidentally, such studies also did not model engine start up conditions with emphasis on viscoelasticity.

This study covers viscoelastic aspects of non-Newtonian lubricants at engine start up, when, in reality, the absence of elasto-hydrodynamic lubrication (EHL) may result in early engine wear. Multigrade, viscoelastic, non-Newtonian engine lubricant is modeled as upper convected Maxwell fluid in simulated engine start up conditions. This model calculates piston eccentricities, their effect on modified hydrodynamic contact pressures, lubricant properties, piston contact geometry and shape of lubricant film. To do all this, it is imperative to take the following logical assumptions:

1. Lubricant is incompressible and side leakage is neglected.
2. Pressure at inlet is zero and thermal effects are neglected.
3. Surfaces are oil-flooded at the time of engine start up.
4. Surfaces are assumed to have fine finish and roughness effects are neglected.

II. MATHEMATICAL MODEL

A. Governing Equations of Piston Motion

Piston equations of motion must be defined by incorporating small secondary displacements in the form of piston eccentricities along the direction perpendicular to the axis of cylinder. Significant piston skirt eccentricities at the top and bottom are calculated by considering piston inertia, hydrodynamic force, hydrodynamic friction force and moments in the form of equations similar to that defined by Zhu et al [20]:

$$\begin{bmatrix} m_{\text{pis}} \left(1 - \frac{a}{L}\right) + m_{\text{pis}} \left(1 - \frac{b}{L}\right) & m_{\text{pis}} \frac{a}{L} + m_{\text{pis}} \frac{b}{L} \\ \frac{I_{\text{pis}}}{L} + m_{\text{pis}}(a - b) \left(1 - \frac{b}{L}\right) & m_{\text{pis}}(a - b) \frac{b}{L} - \frac{I_{\text{pis}}}{L} \end{bmatrix} \begin{bmatrix} \ddot{e}_t \\ \ddot{e}_b \end{bmatrix} = \begin{bmatrix} F + F_s + F_f \tan \phi \\ M + M_s + M_f \end{bmatrix} \quad (1)$$

B Reynolds Equation and Hydrodynamic Pressure

The following 2-D Reynolds equation in non-dimensional form is used to calculate hydrodynamic pressures and forces as suggested by G.W. Stachowiak et al [22]:

$$\frac{\partial}{\partial x^*} \left(h^{*3} \frac{\partial p^*}{\partial x^*} \right) + \left(\frac{R}{L} \right)^2 \frac{\partial}{\partial y^*} \left(h^{*3} \frac{\partial p^*}{\partial y^*} \right) = \frac{\partial h^*}{\partial x^*} \quad (2)$$

Boundary conditions for Reynolds equation are given by [1]:

$$\frac{\partial p}{\partial x_{x=0}} = \frac{\partial p}{\partial x_{x=\pi}} = 0 ;$$

$$p = 0 \text{ when } x_1 \leq x \leq x_2$$

$$p(x,0) = p(x,L) = 0 \quad (3)$$

C. Hydrodynamic Forces and Shear Stress

Hydrodynamic pressure and hydrodynamic friction forces are given by [22]:

$$F_h = R \iint p(x,y) \cos x \, dx dy \quad (4)$$

$$F_{fh} = R \iint \tau(x,y) \, dx dy \quad (5)$$

$$F_{fh} = \iint \left(\eta \frac{u}{h} + \frac{h}{2} \frac{dp}{dy} \right) dx dy \quad (6)$$

The total normal force acting on piston skirt is given by [20]:

$$F_s = \tan \phi (F_G + F_{IP} + F_{IC}) \quad (7)$$

D. Lubricant Film Thickness

Considering piston eccentricities, lubricant film thickness can be approximated by [22]:

$$h = C + e_t(t) \cos x + [e_b(t) - e_t(t)] y \cos x \quad (8)$$

E. Viscoelastic Fluid Model

Upper convected Maxwell model is used by incorporating constant density, viscosity and a small fluid relaxation time. The continuity, momentum and constitutive equations for the incompressible non-Newtonian fluid flow are solved by using a regular perturbation method. Specifically, we assume an asymptotic solution in the form of a double perturbation expansion in powers of “ ε ” and “De”. The leading term in the conventional lubrication solution is denoted by $[\ell]$ while the two perturbation corrections are denoted by $[\varepsilon]$ and $[De]$.

The leading term of partial derivative of pressure is obtained from the simple Reynolds equation [1]:

$$\frac{\partial}{\partial x} \left(\bar{h}^3 \frac{\partial p_h}{\partial x} \right) + \frac{\partial}{\partial y} \left(\bar{h}^3 \frac{\partial p_h}{\partial y} \right) = 6\eta U \frac{d\bar{h}}{dy} \quad (9)$$

The correction term of the pressure is obtained by taking the double derivative of leading term of velocities [1].

$$\frac{\partial P^{(\varepsilon)}}{\partial x} = 2 \frac{\partial^2 U^{(\ell)}}{\partial x^2} + \frac{\partial^2 U^{(\ell)}}{\partial y^2} \quad (10)$$

which gives

$$\frac{\partial P^{(\varepsilon)}}{\partial x} = 12 \left(\frac{h^3 - h_m}{h^3} \right) + 2 \left(\frac{3h_m}{h^3} + \frac{2}{h^2} \right) \frac{dh}{dx} \quad (11)$$

For the Deborah term of pressure, modified Reynolds equation is solved [1], which gives,

$$\frac{\partial P^{(De)}}{\partial x} = -18 \frac{h_m^2}{h^5} \frac{dh}{dx} + 18 \frac{h_m}{h^4} \frac{dh}{dx} - \frac{4}{h^3} \frac{dh}{dx} - 12 \frac{h_d}{h^3} \quad (12)$$

where

$$h_d = \left\{ \frac{\frac{3}{8} \left(\frac{h_m^2}{h_1^4} - \frac{h_2^2}{h_0^4} \right) - \frac{1}{2} \left(\frac{h_m}{h_1^3} - \frac{h_m}{h_0^3} \right) + \frac{1}{6} \left(\frac{1}{h_1^2} - \frac{1}{h_0^2} \right)}{\int_0^1 \frac{1}{h^3(x)} dx} \right\} \quad (13)$$

Now total $\frac{dp}{dx}$ is calculated using asymptotic solutions.

$$\frac{dp}{dx} = \frac{\partial P^{(\ell)}}{\partial x} + \frac{\partial P^{(\varepsilon)}}{\partial x} + \frac{\partial P^{(De)}}{\partial x} \quad (14)$$

The resulting expression is used in the calculation of shear stress, which will further give hydrodynamic friction force.

III. NUMERICAL RESULTS AND DISCUSSION

There are three numerical processes that include solving Reynolds equation to find hydrodynamic pressure distribution on the piston skirt surface, getting the solution based on viscoelastic property of fluid (Deborah number, De) by computing corrections of the solution at order De for the pressure (Deborah number De is always less than unity [8]) and incorporation of small corrections from the effects of geometric parameter (ε). The asymptotic solution to the exact formulation of the Upper Convected Maxwell model is the sum of conventional lubrication solution plus solution based on viscoelastic property of fluid and incorporation of small stated corrections.

Inspection of the formulations of pressure given in the previous section shows that the pressure gradient $dp^{[I]}/dx$ depends only on the function $h(x)$, while, the viscoelastic pressure gradient $dp^{[D]}/dx$ depends not only on the surface slope dh/dx , but also on the surface curvature d^2h/dx^2 [6].

The simulation of basic model showing transient rigid hydrodynamic lubrication of piston skirts gave results indicating respective eccentricities of piston skirt (E_t & E_b), the maximum and the minimum oil film thickness profiles, hydrodynamic pressure fields over the skirt surface plane and all this corresponds to respective time steps/crank angles.

A. Piston Eccentricities

Figure 2 shows dimensionless eccentricities (E_t & E_b) and crank angle. There are three horizontal lines; the lower line is the touching line on the major thrust side, the upper line is the touching line on the minor thrust side and the mid-line (zero line) indicates piston eccentricity to be zero. $E_t(t)$ is the eccentricity of the piston skirt at the top position and $E_b(t)$ is the eccentricity of the piston skirt at the bottom. If the eccentricity curve goes beyond either the upper line or the lower line then solid-to-solid contact establishes. In our case, where we have assumed the model to be in hydrodynamic regime in the initial engine start up, it can be seen that 'E_t' curve goes close to the lower line during the piston expansion stroke but it does not touch the line. The E_t & E_b profile curves do not touch either upper or lower line in the complete 720 degree crank rotation, hence no solid-to-solid contact. This is the improved piston eccentricities profile as compared to Newtonian lubricant when modeling initial engine start up conditions [23].

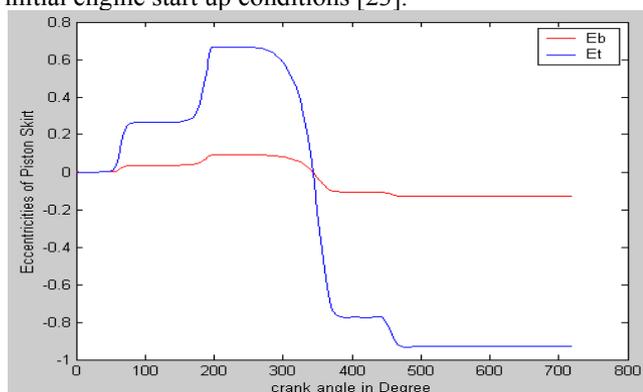


Fig. 2 Dimensionless Piston Skirt Eccentricities

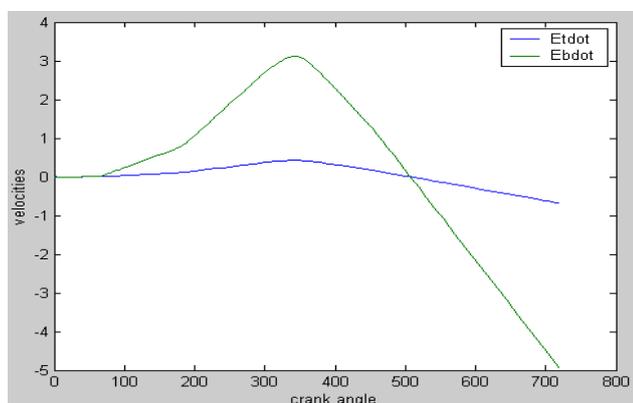


Fig. 3 Piston Skirt top & Bottom Velocities

B. Effect of Minimum Film Thickness

Time based maximum & minimum lubricant film thickness established between opposing surfaces of piston

skirts and cylinder wall is determined as shown in figure 4. The intake stroke has negligible film thickness. This time period becomes very critical. This corresponds to the piston eccentricities towards minor thrust side. In the compression stroke, oil film thickness rises and attains its vertex near the end of that stroke of piston. This means that more film thickness value increases, more are the chances that piston eccentricities will get reduced. Hence, even lesser chances for any solid-to-solid contacts to take place. In the piston expansion stroke, maximum value of combustion gas force due to ignition (firing) is at 372 degree crank rotation. There is corresponding drop in the values of maximum and minimum film thickness. These values reach the minimum at the end of expansion stroke, which means that by that time full impact of combustion gas force on hydrodynamic pressures as well as film thickness has fully materialized. In the exhaust stroke, oil film thickness starts increasing again until it attains significantly high values at the end of exhaust stroke which corresponds to the 720 degree crank angle.

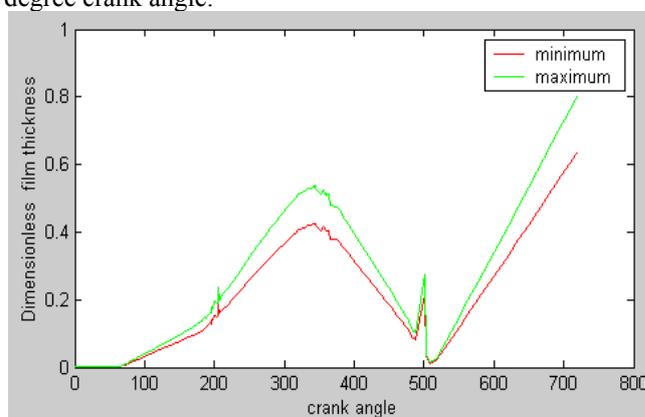


Fig. 4 Maximum & Minimum Film Profiles

C. Effects of Viscoelasticity on Pressure

Hydrodynamic pressure distribution after incorporating viscoelastic effects is uniform through the entire crank rotation angles, which indicates perfectly aligned piston skirts model, maintaining desirable clearances throughout the entire cycle. It is possible to generate instantaneous 3-D pressure profiles on the skirt surface plane corresponding to all 720° of crank rotation. In our analysis we generated eight such pressure fields each for every 90° crank rotation to evaluate changes in pressure profiles in the entire cycle. For our basic model, results of eight pressure fields at engine start up speed of 600 rpm are shown and discussed below.

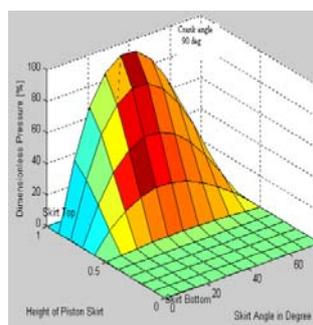


Fig. 5a Crank angle 90°

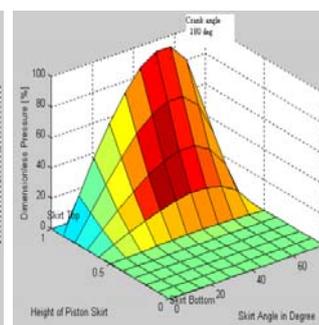


Fig. 5b Crank angle 180°

In the intake stroke, the peak hydrodynamic pressure values are closer to the piston skirt top surface showing gentle slope of instantaneous pressure fields. At 90° crank

angle, piston is at the mid of intake stroke, positive hydrodynamic pressures rise from mid point to the skirt top surface but the slope is gentle and peak pressure value is witnessed at almost centre point of skirt top surface (Refer to Fig:5a). At 180° crank angle, when piston is at the end of intake stroke, positive hydrodynamic pressures rise from mid point to the skirt top surface but the slope gets steep and peak pressure values are seen to shift away from centre point of skirt top surface and towards right side (Refer to Fig:5b).

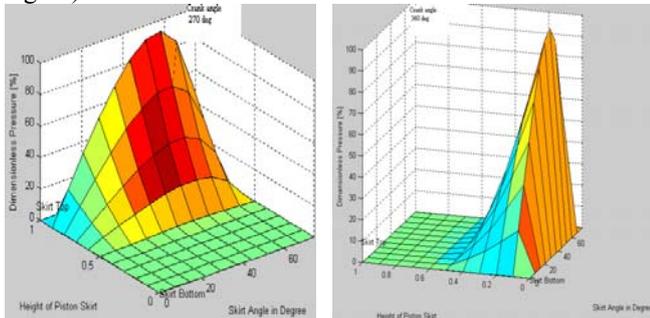


Fig: 5c Crank angle 270° Fig: 5d Crank angle 360°

In the compression stroke, the peak hydrodynamic pressures initially generate at the piston skirt top surface but later shift towards skirt bottom surface showing less gentle slope of instantaneous pressure fields. At 270° crank angle when piston is at the mid of compression stroke, positive hydrodynamic pressures rise from mid point to the skirt top surface but the slope remains steep and peak pressure value is found at the mid of centre and right corner of skirt top surface (Refer to Fig:5c). At 360° crank angle when piston is at the end of compression stroke, there is clear shift of positive hydrodynamic pressures to the skirt bottom surface such that these rise from mid point to the skirt bottom surface. The slope is steep but higher peak pressure values are close to the centre point of skirt bottom surface (Refer to Fig: 5d).

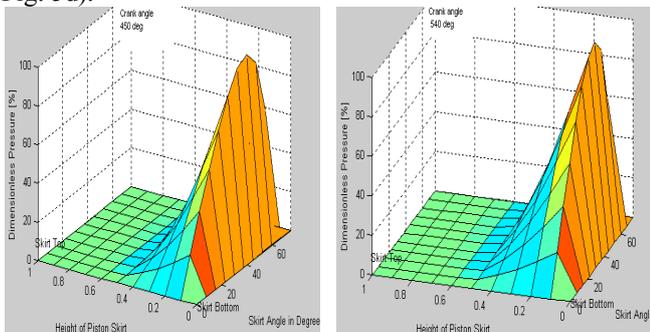


Fig: 5e Crank angle 450° Fig: 5f Crank angle 540°

In the power stroke, the peak hydrodynamic pressures are generated at the piston skirt bottom surface throughout the entire length of stroke showing less gentle slopes of instantaneous pressure fields. At 450° crank angle when piston is at the mid of power (expansion) stroke, positive hydrodynamic pressures rise from mid point to the skirt bottom surface but the slope gets gentle and higher peak pressure values are found close to the centre point of skirt bottom surface (Refer to Fig:5e). At 540° crank angle when piston is at the end of power/expansion stroke, positive hydrodynamic pressures profiles at the skirt bottom surface have their slope gets gentler but higher peak pressure values again shift from the centre point towards the right of skirt bottom surface (Refer to Fig:5f).

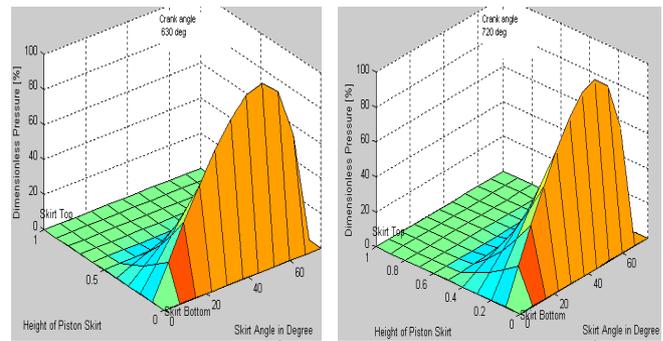


Fig: 5g Crank angle 630° Fig: 5h Crank angle 720°

In the exhaust stroke, the peak hydrodynamic pressures are generated at the piston skirt bottom surface throughout the entire length of stroke showing gentle slopes of reduced instantaneous peak pressures. At 630° crank angle when piston is at the mid of exhaust stroke, positive hydrodynamic pressures rise from midpoint of skirt to the skirt bottom surface but the slope becomes gentle, peak pressure values are found to have reduced values and shift close to the centre point of skirt bottom surface (Refer to Fig:5g). At 720° crank angle when piston reaches at the end of exhaust stroke, positive hydrodynamic pressure profiles at the skirt bottom surface maintain their gentle slope but at visibly reduced peak pressure values, which again shift slightly from the centre point towards the right of skirt bottom surface (Refer to Fig:5h).

In our case, it is the maximum hydrodynamic pressures that are generated near the top portion of piston skirt during the intake stroke and up to the mid of compression stroke of piston motion with the assumption of flooded regime. In reality, it is the boundary or thin film followed by partial (mixed) lubrication that actually takes place during the first 1-2 minutes of engine start up. This means that the hydrodynamic pressures are replaced by Hertzian pressures at possible solid-to-solid contacts. If this occurs then the piston skirt surface near the top is likely to get damaged. Significant shift of hydrodynamic pressure fields with higher values of peak pressures generating over the skirt bottom surface and simultaneous increased piston eccentricities result as combustion gas force acts over the piston crown and its impact travels down through the skirt. All this means that in reality there are more chances of damage to the skirt bottom surface, if starvation of piston skirt occurs during this period. Here, the role of viscoelastic non-Newtonian engine lubricant assists decreased piston eccentricities and reduces the chances of piston skirt surface establishing any physical contact with cylinder liner surface and subsequent adhesive wear.

IV. CONCLUSIONS

In this paper, we have investigated the effects of viscoelastic properties of a non-Newtonian fluid in improving hydrodynamic lubrication of piston skirt surface during engine initial startup conditions. The calculations in this paper are based on a Upper Convected Maxwell fluid model with constant viscosity and relaxation time. Numerical results show that incorporation of viscoelastic effects visibly modified transient hydrodynamic pressure

profiles and clearly reduced piston eccentricities under engine start up conditions. It is a very significant finding as the reduced eccentricities may proportionally reduce magnitude of piston skirt wear at engine start up accordingly. Furthermore, we have found that the effect of viscoelasticity on the pressure is dominated by minimum film thickness. Numerical simulations indicate that there is a significant enhancement of the viscoelastic pressure when the minimum film thickness is sufficiently small.

In conclusion, we have addressed and assessed the effects of viscoelasticity on piston lubrication during engine startup conditions. Further extensive work is needed to explore other non-Newtonian fluid-flow aspects related to the lubrication of piston skirts under specified engine operating conditions.

Nomenclature

C = Piston radial clearance = 0.00001m

$D = 2R$ = Diameter of Piston = 83.00 mm

F = Normal force acting on piston skirts

F_f = Friction force acting on skirt surface

F_G = Gas force

F_{IC} = Inertia force due to piston mass

F_{IP} = Inertia force due to piston pin mass

h = Film thickness

h_d = Deborah film thickness

h_m = Mean film thickness

I_{pis} = Piston inertia about its centre of mass

L = Piston skirt length = 33.8mm

l = Connecting rod length = 133 mm

M = Moment acting on piston skirts

M_f = Friction moment acting on skirt surface

m_{pis} = Mass of piston = 0.295 kg

m_{pin} = Mass of piston pin = 0.09 kg

P_h = Hydrodynamic pressure

r = Crank radius

(E_1, E_2) = Young's Moduli = 200GPa

μ = Lubricant viscosity = 0.016 Pa.s

ω = Engine speed = 600 rpm

τ = Shear stress

η = Kinematic viscosity

ν = Poisson's ratio = 0.3

Φ = Connecting rod angle

ϕ = Crank angle

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