

Analytical and Experimental Investigation of a Tuned Undamped Dynamic Vibration Absorber in Torsion

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Abstract— subject of mechanical vibration at the under graduate level is introduced in recent years in many leading universities of the world. This necessitates the development of good vibration laboratory course work, using the resources available in the institute workshop with little operating cost. In this paper design and development of experimental setup for determining the response characteristics of torsional, tuned, undamped, dynamic vibration absorber is presented. A mathematical model for the absorber with base excitation is developed and the theoretical values of the torsional amplitude are calculated for different values of excitation frequency. The theoretical and experimental results are correlated.

Key words — Absorber system, natural frequency, excitation.

I. INTRODUCTION

During the last few years many leading universities in the world have introduced the subject of mechanical vibration at the under graduate level. It is matter of common experience that the subject matter is taught only on the black board and power point presentation without any exposure to the laboratory work. The laboratory equipments are procured either from within the country of origin or are imported, paying a fabulous sum of money. These are mostly electronic equipments requiring a good deal of knowledge and experience for handling them and such equipments are generally used by research workers and post graduate students. However, the students at lower level do not get ample opportunity to handle these sophisticated equipments. This necessitates the development of a good vibration laboratory course work, using the resources available in the institute workshop with very little operating cost. In this paper an experimental set up for determining the response characteristics of a torsional, tuned, undamped-dynamic vibration absorber is presented, though no originality is claimed by the authors.

The principle on which this set up works is that the governing equation for a system with forcing functions are the same as those of an undamped system with base excitation. The frequency of excitation is varied through a variable speed drive which mainly consists of two exactly identical wooden tapered rollers driven by means of flexible belt, the position of which could easily be altered through a pulley which can be slid on a bridge.

The main system consists of a torsional pendulum with base excitation. The absorber mass and its spring are tuned to

work at the same natural frequency as that of the main system. The torsional amplitude of vibration at a given excitation frequency is obtained by the tangent formula with the help of optical system as explained in the article under the heading of instrumentation. The torsional response due to variation in the excitation frequency is obtained by changing the excitation frequency from its lowest value to the maximum value.

A mathematical model for the absorber with base excitation is developed and the theoretical values of the torsional amplitudes are calculated for different values of excitation frequency. The theoretical and experimental results are correlated.

II. NOTATION

- ω_m Natural circular frequency of the main system,
- ω_a Natural circular frequency of the absorber system,
- ω Circular frequency of excitation,
- K_m Stiffness of main shaft,
- K_a Stiffness of absorber spring,
- I_m Moment of Inertia of the main system,
- I_a Moment of Inertia of the absorber system,
- μ I_a/I_m ,
- θ Angular displacement of base at anytime, t ,
- θ_1 Angular displacement of main system at anytime, t ,
- θ_2 Angular displacement of absorber system at anytime, t ,
- θ_o Amplitude of vibration of base.

III. THEORETICAL ANALYSIS

The theoretical analysis for a single degree of freedom system as well as the absorber are fully described in almost all the textbooks of mechanical vibrations and as such the detailed derivations for the amplitude ratios are not presented here. However, the expressions for the amplitude ratios are given.

A. Single Degree of Freedom System

The amplitude ratios of a single degree of freedom system with no damping can be written as [3].

$$\frac{\theta_1}{\theta_2} = \frac{1}{1 - \left(\frac{\omega}{\omega_m}\right)^2} \quad (3.1)$$

The value of $\left(\frac{\omega}{\omega_m}\right)^2$ are varied from 0 to 2 and the corresponding values of θ_1/θ_o are calculated using equation 3.1. The theoretical results are plotted by full line as shown in Fig. 1.

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B. The Absorber System

The amplitude ratios for the main system and absorber system can be written as [3].

$$\frac{\theta_1}{\theta_o} = \frac{1 - \left(\frac{\omega}{\omega_m}\right)^2}{\left[1 - \left(\frac{\omega}{\omega_m}\right)^2\right] \left[1 + \mu - \left(\frac{\omega}{\omega_m}\right)^2\right] - \mu} \quad (3.2)$$

$$\frac{\theta_2}{\theta_o} = \frac{1}{\left[1 - \left(\frac{\omega}{\omega_m}\right)^2\right] \left[1 + \mu - \left(\frac{\omega}{\omega_m}\right)^2\right] - \mu} \quad (3.3)$$

The ratio μ of the inertias of the absorber and the main system is found to be 0.0963. Using this value of μ and varying $\left(\frac{\omega}{\omega_m}\right)$ from 0 to 2, the values of θ_1/θ_o and θ_2/θ_o are calculated using equations (3.2) and (3.3) respectively and the results are plotted as shown by full line in Fig. 2 and 3.

IV. EXPERIMENTAL SET UP

A. Descriptions

Fig. 4 shows the isometric view of the whole experimental set up and Fig. 5 and 6 show the plan and elevation of the assembly of the tuned torsional dynamic vibration absorber. The whole set up mainly consist of a single phase induction motor, variable speed drive with excitation system, the absorber system and an optical device for the measurement of dynamic torsional amplitude are also shown in the above figures.

B. Induction Motor

A single phase 230V, 50 C/S, 1440 R.P.M. induction motor is used to drive the variable speed drive. This motor and the variable speed drive are mounted on the same table so that shocks due to any type of unbalance and mechanical defects in the manufacture of variable speed drive are not transmitted to the main system.

C. Variable Speed Drive

The variable speed drive mainly consists of two exactly identical wooden tapered rollers, fixed on shaft supported in ball bearings. A leather belt can be adjusted and fixed in any desired position by a sliding bracket which in turn can be fixed anywhere on the bridge depending upon the speed requirement. On the output end of the device, an eccentric is mounted to excite the fixed end of the shaft of the main system through a system of levers. This system of levers imparts a sort of base excitation to the shaft which is mounted in six- perfectly aligned bearings. The base end of the shaft of the main system is fixed rigidly to the end of the lever. Thus the rotation of the output end of the variable speed drive results in harmonic oscillation defined by $\theta = (\theta_0 \sin \omega t)$ of the end of the shaft of the main system.

D. Main System

As pointed out in the earlier paragraph the main system consist of a shaft of 3/16 inch diameter, supported on six bearings as shown in Fig. 4. At the other end of the shaft a

rotor is fixed which is of 82 mm diameter and 5.5 mm thickness.

A square boss is machined centrally on the other face of the rotor in order to accommodate the absorber system whenever required. This gives a little flexibility in the experiment so that the set up can be used to study the response in forced vibration of a single degree of freedom system as well.

E. Absorber System

The absorber system consists of a helical spring and rotor of 45 mm diameter and 5.75 mm thick. This rotor is free to rotate on the shaft of the main system and free end of the helical spring is attached to the boss having a square tapered hole in order to attach the absorber system to the main system whenever required. The absorber system is tuned to the same natural frequency as that of the main system.

V. INSTRUMENTATION

It is known from the theory of reflection of light that the reflected beam of light will deviate through an angle which is two times that of rotation of the mirror when the incident ray on this mirror is kept fixed in direction. This principle is used for finding the amplitude of torsional vibration.

A. Arrangement

The main system as well as absorber system is kept inside a wooden box. At the back of box, a hole is provided through which a source of light enters to provide incident beam. The incident ray from this source of light, focused on the mirror fixed on the boss of the main system, is received on a screen which is a vertical wooden strip. The screen is marked with zero at a height equal to height of centre of mirror and a calibrated scale is fixed to the screen with the centre zero coinciding with zero on the screen. Thus when the rotor of the main system vibrates with amplitude, the reflected ray of light is deflected through dynamic amplitude $2\theta_1$ and when the main system vibrates, this gives strip of light extending equally on both sides of the zero mark on the calibrated scale. The horizontal distance between mirror and screen is measured as $L = 10.8$ cm. The length X , in cm of strip of light measured from the calibrated scale and amplitude is calculated from the relation,

$$\tan 2\theta_1 = \frac{X}{2L}$$

Similar procedure is adopted for the measurement of amplitude of absorber system. The part list with materials of various components is given in table IV.

VI. EXPERIMENTAL PROCEDURES

The torsional stiffness of the shaft and the moment of inertia of the rotor of the main system were determined experimentally and the results obtained are also checked by theoretical formulae. The response of the system was determined separately both for a single degree of freedom system (torsional pendulum) and the system with the absorber fitted.

A. Response of a Single Degree of Freedom System

In order to obtain the response characteristic of a single degree of freedom system, the absorber was disconnected from the main system. The belt position on the variable speed drive was adjusted so as to get lowest excitation frequency. The driving motor was then switched on and the rotational

speed of the exciting cam shaft was measured with the help of tachometer and corresponding length X of the strip of the light on the main scale was noted. The speed of rotation was changed from this lowest value up to the end of the traverse of the sliding bracket on the bridge. The results obtained were tabulated as shown in table I. The speed of rotation of the cam shaft corresponding to the resonance condition was noted. This amplitude of excitation θ_0 was found by slowly rotating the exciting cam and noting the extreme position of the reflected ray of light on the main scale. The dimensionless responses θ_1/θ_0 versus ω/ω_m was then plotted as shown in Fig. 1 by dots.

Table-I: Main System (Single Degree of Freedom)

$$\theta_0 = 0.0124 \text{ rad.}$$

Excitation Frequency ω r.p.m.	Strip length 3cm	$\theta_1 = \frac{1}{2} \tan^{-1} \frac{X}{2L_i}$	ω/ω_n	θ_1/θ_0
1040	1.0	0.023	0.675	1.846
1320	2.0	0.046	0.857	3.744
1400	3.0	0.069	0.909	5.624
1440	4.0	0.093	0.935	7.500
1460	5.0	0.116	0.948	9.374
1480	6.0	0.139	0.961	11.247
1500	7.0	0.162	0.974	13.126
1520	8.0	0.185	0.987	15.006
1540	16.00	0.370	1.000	30.000
1580	12.0	0.277	1.026	22.448
1600	9.0	0.208	1.039	16.845
1620	5.0	0.116	1.052	9.374
1660	4.0	0.093	1.078	7.500
1700	3.0	0.069	1.104	5.616
1770	2.0	0.046	1.149	3.725
1980	1.0	0.023	1.286	1.862

B Response of Tuned Undamped Dynamic Vibration Absorber

To obtain the response characteristics of the absorber, the absorber mass was connected to the main system and exactly the same procedure as described in article VI A was followed and the result for both the main mass and absorber mass were tabulated as shown in table II and table III. The results were plotted as shown in Figs. 2 and 3 by dots.

Table-II: Main System (Two Degree of Freedom)

$$\theta_0 = 0.0124 \text{ rad.}$$

Excitation Frequency ω r.p.m.	Strip length 3cm	$\theta_1 = \frac{1}{2} \tan^{-1} \frac{X}{2L_i}$	ω/ω_m	θ_1/θ_0
1000	1.0	0.023	0.649	1.684
1240	2.0	0.046	0.805	3.747
1260	4.0	0.093	.0818	7.500
1280	8.0	0.185	0.831	15.000
1290	10.0	0.232	0.838	18.768
1300	9.0	0.208	0.844	16.845
1310	8.0	0.185	0.851	15.006
1320	6.0	0.139	0.857	11.247
1340	4.0	0.093	0.870	7.500
1360	3.0	0.069	0.883	5.615
1420	1.0	0.023	0.922	1.862
1520	0.0	0.000	0.987	0.000
1650	1.0	0.023	1.071	1.862
1700	2.0	0.046	1.104	3.747
1720	4.0	0.093	1.117	8.500
1760	9.0	0.208	1.143	16.844
1800	6.0	0.139	1.165	11.844
1820	5.0	0.116	1.182	9.373
1830	4.0	0.093	1.188	7.500
1860	3.0	0.069	1.208	5.616
1900	2.0	0.046	1.234	3.747
2140	1.0	0.023	1.389	1.862

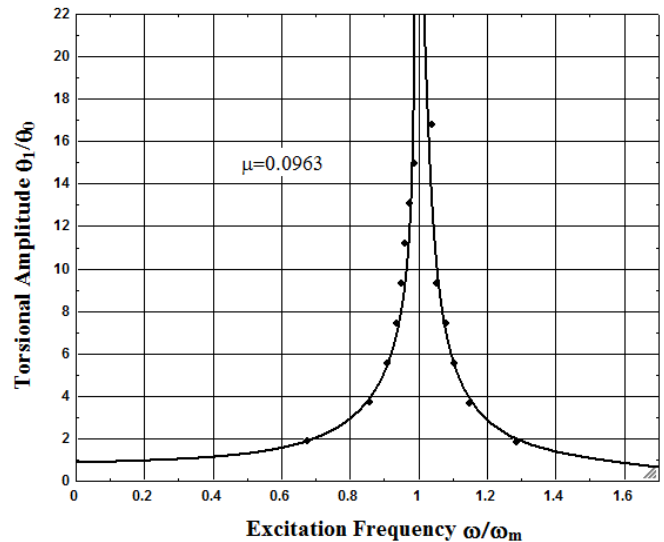


Fig. 1 Frequency response single degree of freedom

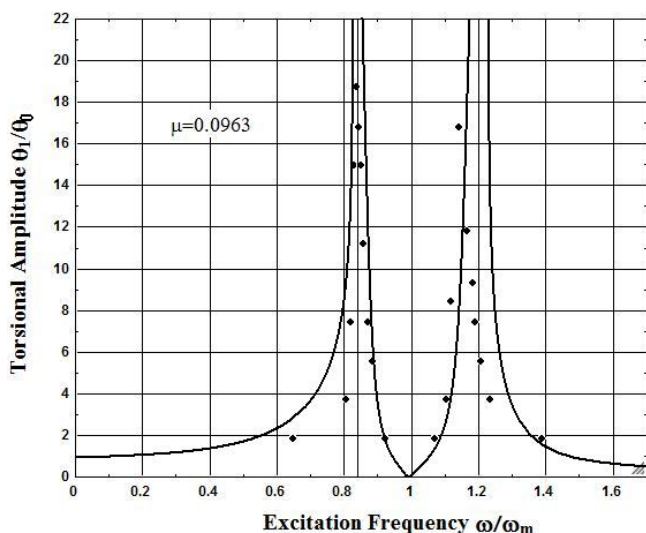


Fig. 2 Frequency response two degree of freedom system

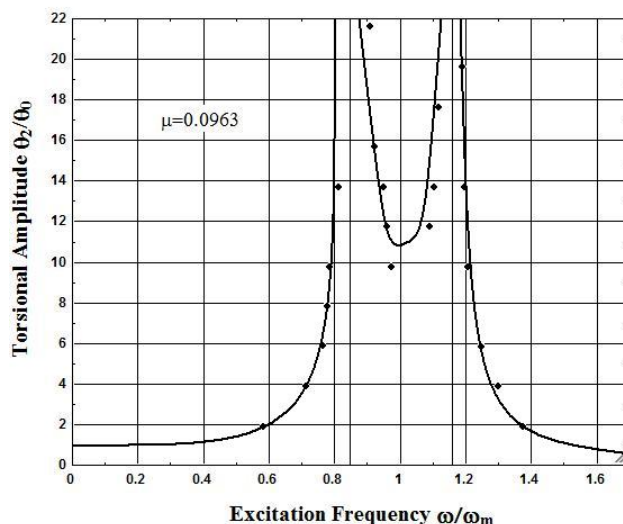


Fig. 3 Frequency response two degree of freedom system

Table-III: Absorber System (Two Degree of Freedom)
 $\theta_0 = 0.0124 \text{ rad.}$

Excitation Frequency ω r.p.m.	Strip length 3cm	$\theta_2 = \frac{1}{2} \tan^{-1} \frac{X}{2L_2}$	ω/ω_m	θ_2/θ_0
900	1.0	0.024	0.584	1.948
1100	2.0	0.049	0.714	3.931
1180	3.0	0.073	0.766	5.907
1200	4.0	0.097	0.779	7.861
1210	5.0	0.121	0.786	9.798
1240	7.0	0.170	0.805	13.768
1250	9.0	0.218	0.812	13.699
1260	13.0	0.316	0.818	25.565
1260	25.0	0.607	0.831	49.171
1360	19.0	0.461	0.888	37.368
1370	15.0	0.364	0.890	29.496
1400	11.0	0.267	0.909	21.635
1420	8.0	0.194	0.922	15.733
1460	7.0	0.170	0.942	13.768
1480	6.0	0.146	0.961	11.797
1500	5.0	0.121	0.974	9.826
1520	4.0	0.097	0.987	78.61
1680	6.0	0.146	1.091	11.797
1700	7.0	0.170	1.104	13.786
1720	9.0	0.218	1.117	17.699
1740	14.0	0.340	1.130	27.538
1750	23.0	0.558	1.136	45.218
1800	17.0	0.413	1.169	33.432
1810	15.0	0.364	1.175	29.496
1820	13.0	0.361	1.182	25.565
1830	10.0	0.243	1.188	19.670
1840	7.0	0.170	1.195	13.768
1860	5.0	0.121	1.208	9.826
1920	5.0	0.073	1.247	5.901
2000	2.0	0.049	1.299	3.931
2100	1.0	0.024	1.376	1.948

VII. COMPARISON OF THEORETICAL AND EXPERIMENTAL RESULTS

In Figs. 1, 2 and 3, the theoretical and experimental results are superimposed for both the single degree of freedom system and absorber respectively. From these graphs it can be observed that a very good correlation exists between theoretical and experimental results. In Fig 5, the resonance phenomenon occurs at frequency ratio equal to 1, for theoretical results. However in experimental results the maximum value of the amplitude ratio θ_1/θ_0 is found to be about 30 which has not been shown in the graph. This truly depicts the trend of the response curve at resonance.

In Fig 2, the two natural frequencies are observed to occur at ω/ω_m equal to 0.86 and 1.16 and the value of frequency ratio at which the absorption has taken place is 0.985. In this graph also, a satisfactory correlation between theoretical and experimental results are observed.

In Fig 3, the response characteristics of the absorber system are shown. The theoretical and experimental results are superimposed and it can be seen that at frequency ratio ω/ω_m equal to 1, the absorber amplitude is found to be minimum.

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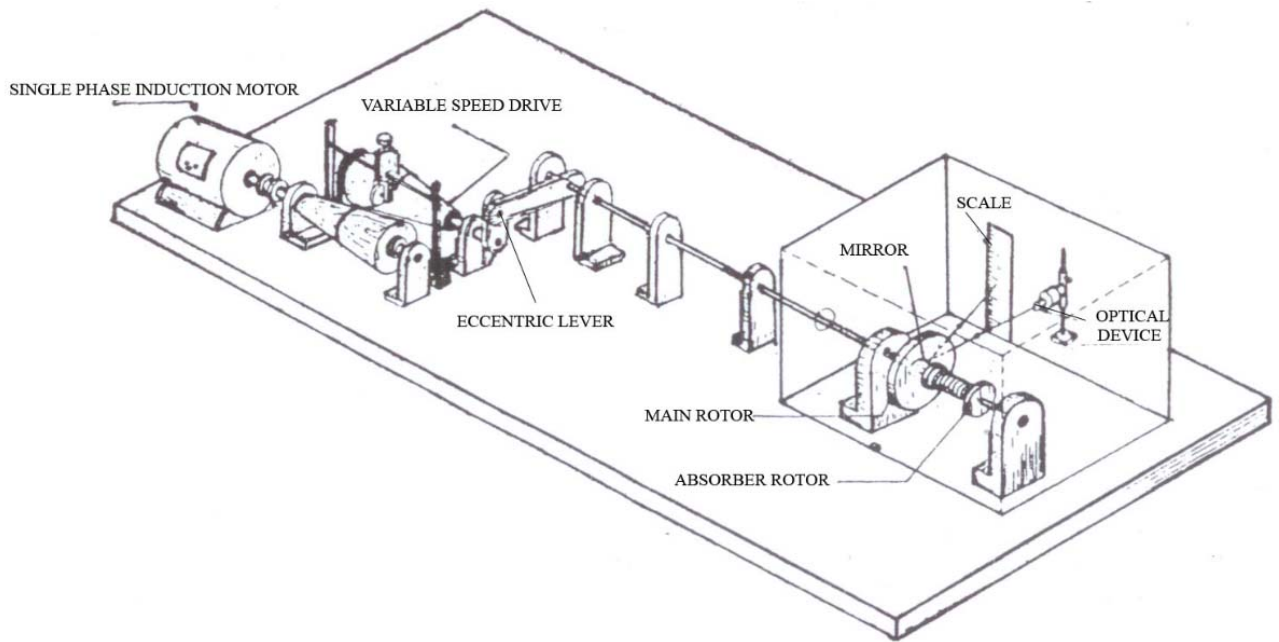


Fig. 4 Isometric view of experimental set up.

Table-IV: Part List

Sr.No.	Part Name	Material	Qty
1	Electric motor, ½ HP 1420 rpm		1
2	Pulley three stepped	Wood	2
3	Bearing bracket	M.S.	4
4	Bridge bracket assembly	M.S.	2
5	Slider for belt shifting	M.S.	1
6	Cones	Wood	2
7	Bearing for exciting lever	M.S.	2
8	Bearing supporting the shaft	M.S.	6
9	Shaft	M.S.	1
10	Wooden box		1
11	Main system for absorber	M.S.	1
12	Scale	Alluminium	2
13	Incident light source		1
14	Mirror	Glass	2
15	Main bearing for the exciting lever	M.S.	2
16	Leather belt	Leather	1

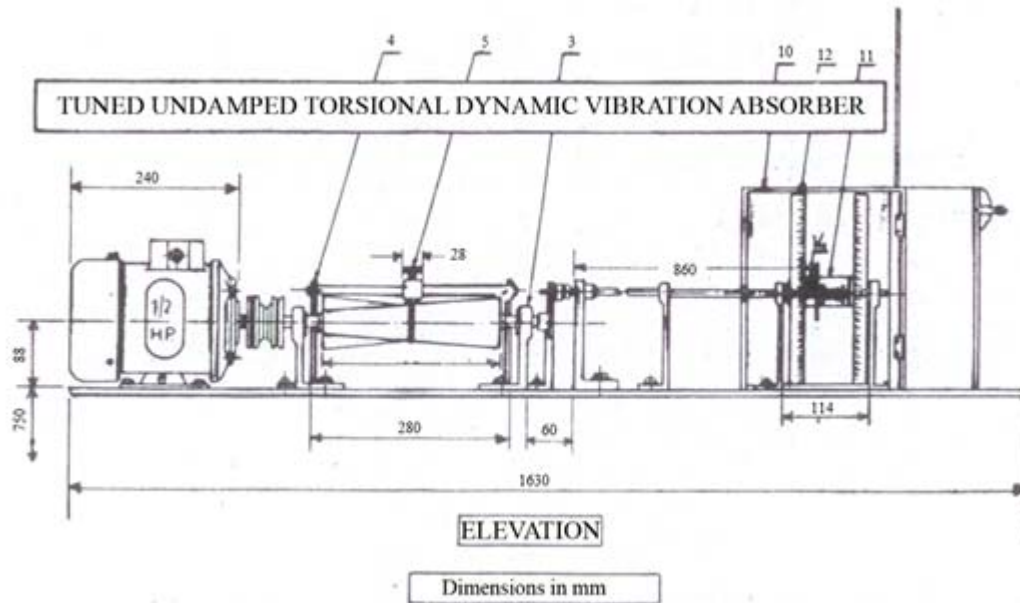


Fig. 5 Elevation of experimental set up.

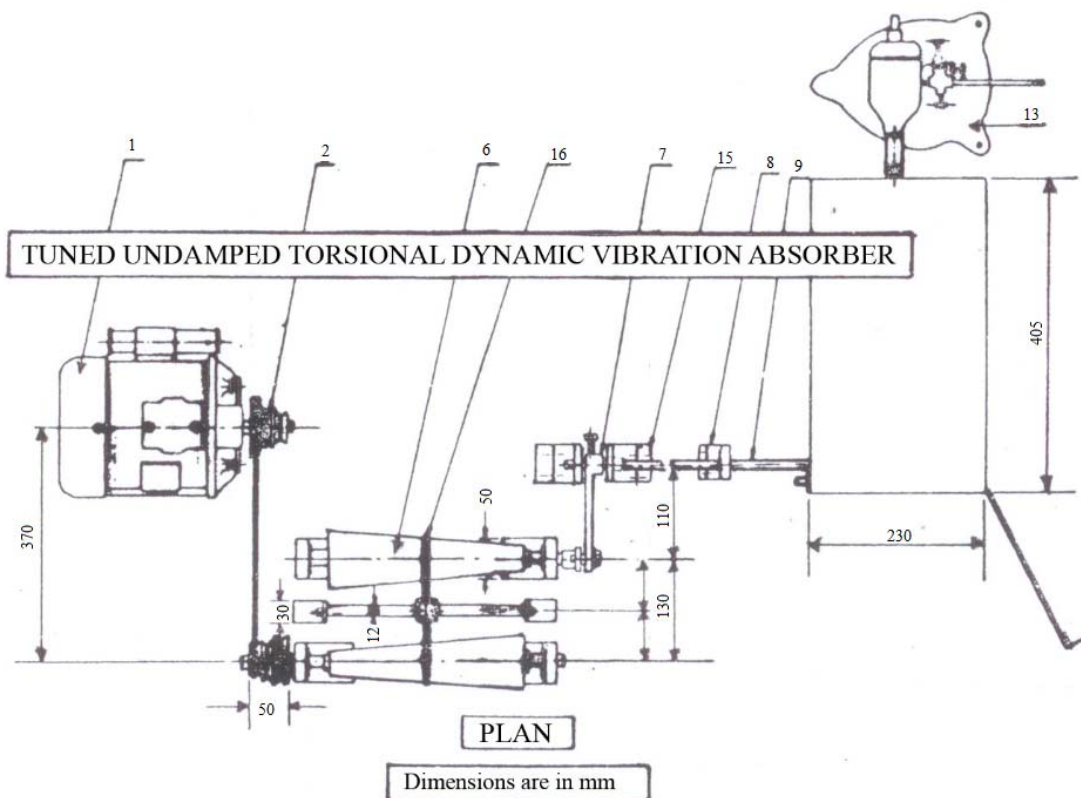


Fig. 6 Plan of experimental set up.