
B. Gurudatt, S. Seetharamu, P. S. Sampathkumaran and Vikram Krishna

Abstract—Resonance is a common thread that runs through almost every branch of engineering. Yet, this phenomenon often goes unobserved, silently resulting in inconveniences, such as causing a bridge to collapse or a helicopter to fly apart, to name a few. It is therefore of utmost importance to avert resonance, for which determining the frequency of the system becomes indispensable. In complex rotating structures as one considered in this paper, theoretical determination of frequency is as difficult and laborious as a task can be. Inspite of being an effective tool to reveal the natural frequency of a system, modal analysis falls short of providing any information about the amplitude and phase angle of vibration of the system. This paper presents an alternative procedure called harmonic analysis to identify frequency of a system through amplitude and phase angle plots. The unbalance that exists in any rotor due to eccentricity has been used as excitation to perform such an analysis. ANSYS parametric design language has been implemented to achieve the results.

Index Terms—ANSYS Parametric Design Language, Critical Speed, Nelson rotor, Out-of-balance response

I. INTRODUCTION

All rotors, unless under ideal conditions, possess an amount of unbalance due to material inhomogeneities, manufacturing processes, keyways or slots and will get into resonance when they rotate at speeds equal to the bending natural frequency of the system. These speeds, called critical speeds, should be avoided as far as possible. Such an unbalance has been used in the methodology presented in this paper as excitation force to produce forced vibrations in the system to obtain the response of the considered system in the form of amplitude and phase angle plots from which the critical speeds can be determined.

Harmonic response analysis is a technique used to determine the steady-state response of a linear structure to loads that vary sinusoidally (harmonically) with time and is used to predict the sustained dynamic behavior of structures to consistent cyclic loading. Thus, it can be verified whether or not a machine design will successfully overcome resonance, fatigue, and other harmful effects of forced vibrations.

ANSYS Parametric Design Language (APDL) is a scripting language that has been used here to build the model and automate tasks by using parameters (variables). A sequence of ANSYS commands can be recorded in a macro file which enables the user to create a customized ANSYS command that executes all of the commands required for a particular analysis. APDL is the foundation for sophisticated features such as design optimization and adaptive meshing.

Model considered in this paper is the same model that was analyzed by Nelson and McVaugh[1]. The complexity of this model lies in the fact that it carries a rotor on a multi-sectioned shaft which is supported on fluid film bearings. Theoretical determination of frequency of such a system is a protracted procedure at best and often approximate. Nevertheless, modal analysis using FEA packages has proven to be an effective tool in determining frequency of such a system. This paper provides validation for the frequency found by modal analysis through harmonic analysis.

II. METHODOLOGY

A. Model

The model considered is a Nelson rotor [1] which is a 0.355(m) long overhanging steel shaft of 14 different cross-sections (Table I). The shaft carries a rotor of mass 1.401(kg) and eccentricity 0.635(cm) at 0.0889(m) from left end and is supported by two identical fluid film bearings at a
distance of 0.1651(m) and 0.287(m) from the left end respectively. Stiffness components of the bearing are k_{zz}=k_{yy}=3.503x10^6(N/m) and k_{yy}=k_{zz}=8.756x10^6(N/m) while damping components are C_{zz}=C_{yy}=1752(Ns/m). Six stations are considered during harmonic analysis as shown in Fig.1, where station numbers denote different nodes in the model-(1) Left extreme of shaft, (2) Disc, (3) First bearing node, (4) Between the two bearings, (5) Second bearing node and (6) Right extreme of shaft.

B. Geometric Modeling and Finite Element Modeling Using APDL

The multisection shaft has been modeled in ANSYS using Beam 188 [2] which is a linear/quadratic two-node beam element in three dimensions with six degrees of freedom at each node. This element facilitates the meticulous definition of all the cross-sections of the shaft. The rotor and the bearings have been modeled using Mass21 and Matrix27 elements respectively. The nodes, elements, material properties, real constants, boundary conditions and other physical system-defining features that constitute the model have been created by exclusively using APDL commands such as ET, MAT, K, N, LSTR, R, RMORE, LATT, LESIZE and E.

C. Solution and Post-processing

Once the finite element model has been prepared, modal analysis is performed to extract the first mode frequency of vibration of the system. Then, harmonic analysis is performed by applying an unbalance force [3] at the rotor (assuming an eccentricity of 0.635(cm)). The range of excitation frequencies is decided by the APDL algorithm based on natural frequency that was obtained through modal analysis. The system is then solved using frontal solver to find response of the system in terms of amplitude and phase angle plots. Response is determined at 6 stations-(1) Left extreme of shaft, (2) Disc, (3) First bearing node, (4) Between the two bearings, (5) Second bearing node and (6) Right extreme of shaft. The resulting graphs are exported as jpeg files. The whole procedure explained above is carried out using an APDL macro, which is as explained below.

D. APDL Macro Algorithm

An APDL macro has been developed to generate all the required results, containing amplitude plots and phase angle plots at all the nodes of the model, with minimal effort on part of the user. The algorithm incorporated in the macro is as below-

1. Setup the model. Impose boundary conditions and apply excitation force.
2. Create scalar parameters ‘f’, to store frequency value, and ‘n’, to represent node number.
4. Set the range of excitation frequencies to increment from ‘f-150’ (Hz) to ‘f+150’ (Hz) in 300 steps.
5. Solve for unbalance response. Plot results to get unbalance response at node ‘n’. Increment parameter ‘n’ by 1. If n>18 (since the model contains 18 nodes), then go to next step. Otherwise, go back to step 4.
6. End of program.

III. DISCUSSION OF RESULTS

As explained earlier, detailed modal and harmonic analyses have been carried out on the considered model to study the unbalance response of the system at a range of excitation frequencies around the first mode frequency that was obtained from modal analysis and the corresponding amplitude and phase angle plots have been obtained. Forward whirl speeds are of utmost importance in the design of the rotor and the first mode of vibration is the most prominent mode of vibration of a system at which the system will vibrate, dominating all the higher frequency modes. We therefore present the results that correspond to 268.3Hz, which is the first forward whirl speed of the system as calculated by Nelson[1] and also the first mode of vibration as found from the methodology used in this paper. Fig. 2 through 7 display the variation of amplitude of vibration of the system at the six identified stations respectively. It can be observed that the amplitude reaches a peak value at one particular excitation frequency. Fig. 8 displays the typical variation of phase angle with excitation frequency at all the stations. These results were generated using the said APDL program whose algorithm has been described under the section “APDL Macro Algorithm”.

IV. INTERPRETATION OF RESULTS

For a disc of mass M and eccentricity 'a' mounted on an elastic shaft supported by radial bearings, the centrifugal force of eccentric disc rotating about the shaft acts as an exciting force. The equivalent values of stiffness, mass and damping of the system are K_{eq}, M_{eq} and C_{eq}, respectively. The equation of motion [4], considering excitation due to unbalance, is given by

\[ M_{eq} \ddot{r} + C_{eq} \dot{r} + K_{eq} r = M \omega^2 a e^{i \omega t} \]  

(1)

where \( \omega \) is excitation frequency and r is whirl radius. The steady state solution of Eq. 1 is

\[ r = Re^{i(\omega_0 - \varphi)} \]  

(2)

where amplitude \( R = \frac{a \Omega^2}{\sqrt{(1-\Omega^2)^2 + (2 \xi \Omega)^2}} \)  

(3)

and phase angle \( \varphi = \tan^{-1} \left( \frac{2 \xi \Omega}{1-\Omega^2} \right) \)  

(4)

where \( \Omega = \frac{\omega}{\omega_n} \), with \( \omega_n \) being frequency of the system, and

\[ \xi = \frac{c_{eq}}{2 \sqrt{K_{eq} M_{eq}}} \]

The maximum value of amplitude ‘R’ occurs when \( \frac{dB}{dt} = 0 \)

i.e. at \( \Omega = \frac{\omega_n}{\sqrt{1-2 \xi^2}} \)  

(5)
From Eq. 4, it can be seen that only when excitation frequency and frequency of the system are equal i.e. when $\Omega=1$, phase angle attains a value of 90°. From Fig.8, it can be observed that a phase angle of 90° is caused by excitation frequencies of 268.3 Hz which happens to be the first mode frequency of the system as obtained from modal analysis. This validates the point that the frequency obtained from modal analysis is indeed the natural frequency of the system and therefore the critical speed of the system which, by definition, means that speed at which a system gets into resonance. Resonance [5] causes an increase in amplitude of oscillation of a mechanical system that is exposed to a periodic force whose frequency is equal to the natural frequency of the system. This phenomenon reveals itself in the amplitude plots, given in Fig.2 through Fig.7, which graphically illustrate the amplitude reaching a peak value only when Eq. 5 is satisfied i.e. only when excitation frequency lies near the natural frequency of the system. Thus, both phase angle and amplitude plots obtained through harmonic analysis serve as functional pointers towards critical speed of a rotor-bearing system.

V. SUMMARY

A novel way to reduce time and effort involved in performing modal analysis and multiple iterations of harmonic analysis to obtain unbalance response of the considered system has been suggested through adept use of ANSYS Parametric Design Language, an advanced feature of ANSYS. A method of identifying critical speeds with the help of amplitude and phase angle plots has been described. This method has been found to validate the critical speed results obtained from modal analysis.

REFERENCES


Table I: Position and diameters of various sections of shaft

<table>
<thead>
<tr>
<th>Sec No.</th>
<th>Origin (m)</th>
<th>Inner Diameter (m)</th>
<th>Outer Diameter (m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0</td>
<td></td>
<td>0.0102</td>
</tr>
<tr>
<td>2</td>
<td>0.0127</td>
<td></td>
<td>0.0204</td>
</tr>
<tr>
<td>3</td>
<td>0.0508</td>
<td></td>
<td>0.0152</td>
</tr>
<tr>
<td>4</td>
<td>0.0762</td>
<td></td>
<td>0.0406</td>
</tr>
<tr>
<td>5</td>
<td>0.1016</td>
<td></td>
<td>0.0660</td>
</tr>
<tr>
<td>6</td>
<td>0.1067</td>
<td>0.0304</td>
<td>0.0660</td>
</tr>
<tr>
<td>7</td>
<td>0.1143</td>
<td>0.0356</td>
<td>0.0508</td>
</tr>
<tr>
<td>8</td>
<td>0.1270</td>
<td></td>
<td>0.0508</td>
</tr>
<tr>
<td>9</td>
<td>0.1346</td>
<td></td>
<td>0.0254</td>
</tr>
<tr>
<td>10</td>
<td>0.1905</td>
<td></td>
<td>0.0304</td>
</tr>
<tr>
<td>11</td>
<td>0.2667</td>
<td></td>
<td>0.0254</td>
</tr>
<tr>
<td>12</td>
<td>0.3048</td>
<td></td>
<td>0.0762</td>
</tr>
<tr>
<td>13</td>
<td>0.3150</td>
<td></td>
<td>0.0406</td>
</tr>
<tr>
<td>14</td>
<td>0.3454</td>
<td>0.0304</td>
<td>0.0406</td>
</tr>
</tbody>
</table>

Figure 1: Model of Nelson rotor with various sections, disc and bearings. Numbers in red indicate station numbers.
Figure 2: Variation of amplitude of vibration (m) (on Y axis) at station 1 with excitation frequency (Hz) (on X axis)

Figure 3: Variation of amplitude of vibration (m) (on Y axis) at station 2 with excitation frequency (Hz) (on X axis)

Figure 4: Variation of amplitude of vibration (m) (on Y axis) at station 3 with excitation frequency (Hz) (on X axis)

Figure 5: Variation of amplitude of vibration (m) (on Y axis) at station 4 with excitation frequency (Hz) (on X axis)

Figure 6: Variation of amplitude of vibration (m) (on Y axis) at station 5 with excitation frequency (Hz) (on X axis)

Figure 7: Variation of amplitude of vibration (m) (on Y axis) at station 6 with excitation frequency (Hz) (on X axis)
Figure 8: Typical variation of phase angle with excitation frequency. $\omega_n$ denotes natural frequency of the system.