Finite Element Modeling of Residual Thermal Stresses in Fiber-Reinforced Composites Using Different Representative Volume Elements

M. M. Shokrieh and A. R. Ghanei Mohammadi

***Abstract*—In this paper, finite element analysis of residual thermal stresses in fiber-reinforced composites has been carried out. For a more realistic simulation of the microstructure of these materials subjected to different loadings, a representative volume element (RVE) may be used. In this paper, three different types of RVE configurations, circular, square and hexagonal are modeled and the effects of each type of fiber packing are studied. A mono fiber circular unit cell is considered using Finite Element (FE) method. Extending the mono fiber model, FE models with different arrays of fibers have been created to investigate the effects of neighboring fibers on the results. The results obtained are used to introduce new boundary conditions for the mono fiber model to make it able to predict the macro behavior in an efficient way. In all steps, the results are also compared with theoretical results presented in the literature. The boundary conditions presented in this research are proved to model the overall behavior efficiently.**

***Index Terms*— fibrous composite material, finite element, representative volume element, inhomogeneous interphase, boundary conditions.**

I. Introduction

Composite materials are becoming an essential part of present engineered materials because they offer advantages such as higher specific stiffness and strength, better fatigue strength and improved corrosion resistance compared to conventional materials. These high performance composites consist of different constituents. Subjected to thermal or thermo-mechanical loads, different deformations occur in different constituents leading to large differences of deformations and stresses between these constituents, which are known as residual deformations and stresses. However, the use of composite materials is limited by the lack of efficient tools to predict their degradation and lifetime under service loads, environment and the process induced residual stresses. For a more realistic simulation of the microstructure of these materials subjected to mechanical, thermal or thermo-mechanical loadings, a representative volume element (RVE) may be used. In the investigations applying FE method, one common procedure is the numerical generation of a RVE or a unit cell (UC) of the material being studied. A RVE or a UC is a statistical representation of the material [1]. Two major groups of these models are the composite cylinder models (CCM) [2] in which the RVE consist of two or more concentric cylinders and unit cell models (UCM) [3]-[5]. Because of its ability in reproducing the real stress and strain evolution, the simulation through a RVE may provide the understanding of the composite thermal behavior. This understanding is a need for the proposal of macroscopical results. In the analysis of the microstructure, the periodicity hypothesis of the fiber within the composite has been traditionally employed. This hypothesis reduces the analysis of the microstructure to the analysis of a single unit cell (the simpler RVE) and may lead to analytical solutions [6]. Three representative volume elements (Fig. 1) based on the 3-D elasticity theory have been proposed in (Liu and Chen, 2002) for the study of fiber reinforced composites. They are the cylindrical RVE (Fig. 1(a)), square RVE (Fig. 1(b)) and hexagonal RVE (Fig. 1(c)). The cylindrical RVE can be applied to model the models including different diameters (Hyer, 1998). Under axisymmetric as well as antisymmetric loading, a 2-D axisymmetric model can be applied for the cylindrical RVE, which can significantly reduce the computational work (Liu and Chen, 2002). The square RVE models can be applied when the conventional fiber-reinforced composites are arranged evenly in a square array, while the hexagonal RVE models can be applied when they are in a hexagonal array, in the transverse direction. These RVEs can be used to study the interactions with the matrix, such as the load transfer mechanism and stress distributions along the interfaces (Liu and Chen, 2002) or to evaluate the effective material properties of the composites [7].

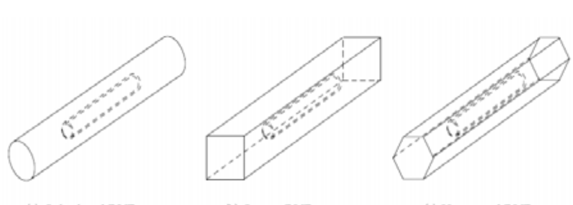

Fig. 1 three representative volume elements [7]

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Although these unit cells can be useful for some purposes and can be employed successfully in two-scale methods to reproduce macroscopical behavior [8, 9], they do not reflect the reality of composite materials, in which the fiber is randomly distributed or it is placed among many other neighboring fibers, and consequently, they are not usable to simulate some of the complex mechanisms which take place in long fiber reinforced polymers and which may cause microscopic failure [10]. The selection of appropriate boundary conditions has been a challenge for many researchers. Since the RVEs must be modeled in a way that results obtained can efficiently describe the overall the composite material behavior, these boundary conditions have an important role in modeling the effect of the neighboring fibers, which are not considered in a mono fiber RVE cell. Many researchers have used micromechanical method to provide overall behavior of the composites from known properties of their constituents (fiber and matrix) through an analysis of a periodic representative volume element (RVE) or a unit-cell model [11,12]. In the macro-mechanical approach, on the other hand, the heterogeneous structure of the composite is replaced by a homogeneous medium with anisotropic properties. The advantage of the micromechanical approach is not only the global properties of the composites but also various mechanisms such as damage initiation and propagation, can be studied through the analysis [13, 14]. In the previous works, square and hexagonal arrays for RVEs have been studied extensively, but RVEs with curved boundaries, such as circular, have not been studied very much.

In this paper, a new unit cell finite element model has been presented to investigate fiber-reinforced composites subjected to thermal loading. An inhomogeneous interphase region has been assumed in the model. A mono fiber circular unit cell model is presented and then the model is generalized to include different geometrical configurations, such as a square unit cell, square cell arrays and hexagonal arrays. In order to present a mono fiber model which is efficiently able to model the macro thermal behavior of a fibrous composite material, the mono fiber model is extended and FE models with different arrays of fibers are created to investigate the effects of neighboring fibers on the results. The results obtained in these models are used to introduce new boundary conditions in the mono fiber model to make it able to predict the macro behavior in an efficient way. In all steps, the results are also compared with theoretical results available in the literature. The boundary conditions selected in the present work are proved to model the overall behavior of the mono fiber model efficiently.

II. FINITE ELEMENT MODELING

In this paper, a circular RVE, containing a fiber, interphase region and surrounding matrix, is presented and considered in the FE model. In order to reduce the FE problem size and the runtime, one fourth of the circular model is considered here. The interphase region is taken to be inhomogeneous. If the material properties of the interphase keep unchanged, the interphase is called homogeneous. Otherwise, it is called inhomogeneous. In the inhomogeneous interphase considered in this research, the mechanical properties of this region undergo an exponential variation with the radial coordinate. The mathematical representation of this property variation can be written as [15]:

$$t = t_m + \frac{1-\bar{r}e^{1-\bar{r}}}{1-\bar{r}_f e^{1-\bar{r}_f}}\left(t_f - t_m\right) \quad where \quad \bar{r} = \frac{r}{r_i}, \bar{r}_f = \frac{r_f}{r_i} \tag{1}$$

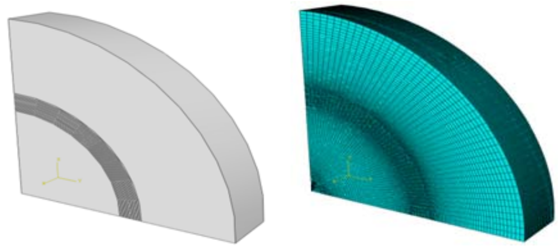
where

$$t = (E_L, E_T, \nu_{12}, \nu_{13}, \alpha_L, \alpha_T)$$

which it represents the transverse and longitudinal Young's moduli E_T and E_L, Poisson's ratios ν_{12} and ν_{13}, and thermal expansion coefficients in the transverse and longitudinal directions α_T and α_L, respectively. Also, r is the radial distance from the fiber center and r_f and r_i are the fiber and interphase outer radii.

To apply the property variation above to the FE model, the interphase region is assumed to consist of some layers and the material properties are kept constant in each layer. To calculate the properties of each layer, a simple mathematical code is used to generate the properties as a function of the radial coordinate. The average value of two neighboring radial distances is calculated and taken to be the property value for the layer.

The Young's moduli, Poisson's ratios and thermal expansion coefficients of the fiber are taken to be E_T=364.49 GPa, E_L=488.45 GPa, ν_{12}=0.2508, ν_{13}=0.2, α_T=6.25e-6/°C and α_L=5.9e-6/°C. And those of the matrix are set to E_m=200 GPa, ν_m=0.3, and α_m=12.5e-6/°C. The outer radii of the fiber, interphase and matrix are taken as 5, 6 and 10 μm, respectively. The externally applied thermal load is a uniform temperature drop of -500 °C. ABAQUS 6.7-1 FE package has been used for modeling. Fig. 2 shows the circular mono fiber model used in this research.



(a) (b)

Fig. 2 a) One fourth mono fiber circular RVE model. b) finite element mesh

In order to investigate the neighboring effects on stress and displacement results around the fiber and interphase, extended RVEs are modeled. These models are the mono fiber square RVE (Fig. 3), multi fiber 2x2 square array RVE (Fig. 4), multi fiber 3x3 square array RVE (Fig. 5), multi fiber 4x4 square array RVE (Fig. 6) and multi fiber 5x5 square array RVE (Fig. 7).

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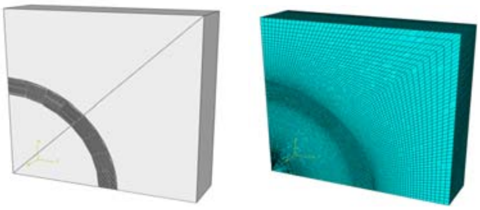
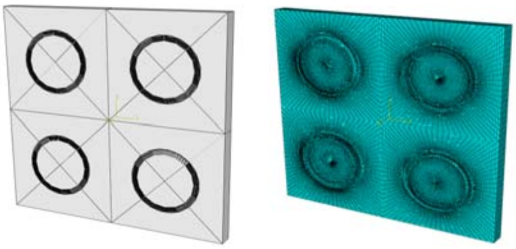


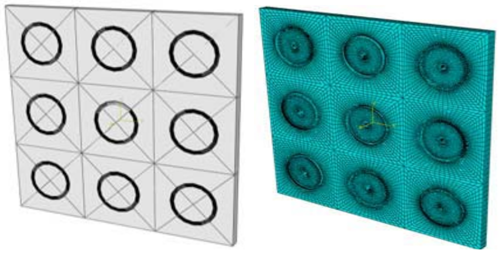
(a) (b)

Fig. 3 a) One fourth mono fiber square RVE model. b) finite element mesh



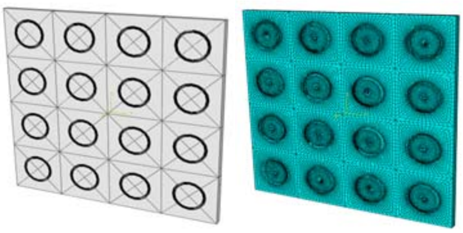
(a) (b)

Fig. 4 a) multi fiber 2x2 square array RVE model. b) finite element mesh



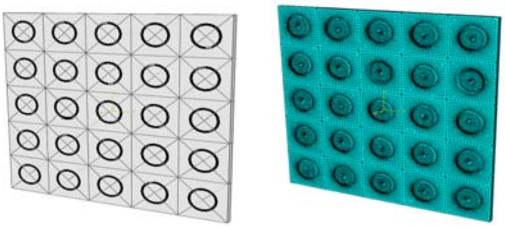
(a) (b)

Fig. 5 a) multi fiber 3x3 square array RVE model. b) finite element mesh



(a) (b)

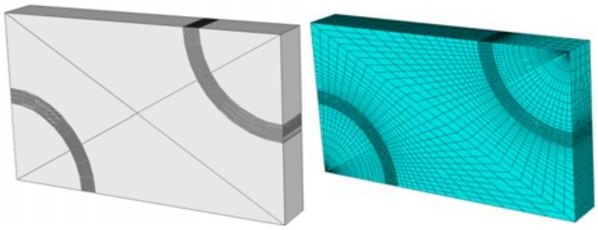
Fig. 6 a) multi fiber 4x4 square array RVE model. b) finite element mesh



(a) (b)

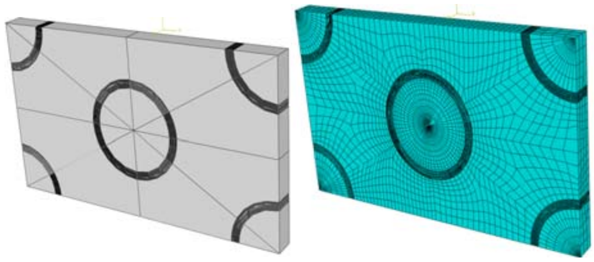
Fig. 7 a) multi fiber 5x5 square array RVE model. b) finite element mesh

As the third configuration possible, hexagonal packing of fibers in the matrix is modeled, too. Unlike the circular and square configuration, the simplest hexagonal RVE contains 2 fibers (see Fig.8). To investigate the effect of neighboring fibers and the convergence study of the results, an extended hexagonal unit cell (Fig.9) and a hexagonal array (Fig.10) are modeled.



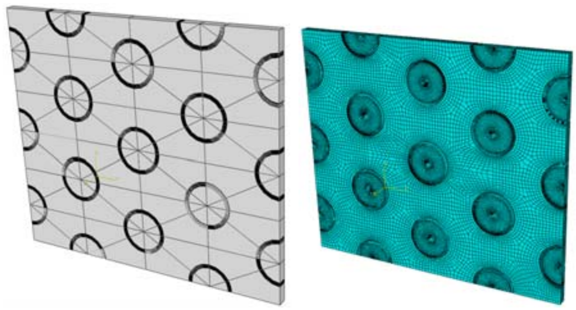
(a) (b)

Fig. 8 a) hexagonal unit cell. b) finite element mesh



(a) (b)

Fig. 9 a) extended hexagonal unit cell. b) finite element mesh



(a) (b)

Fig. 10 a) hexagonal array. b) finite element mesh

These extended RVEs can help us have a better and more realistic understanding of overall macroscopic behavior of composites subjected to thermal loadings. The results of the extended models are compared with the results obtained from the mono fiber circular RVE to investigate the effect of neighboring fibers existence on its stress and displacement results and these results are used to select appropriate boundary conditions for the mono fiber model to make it able to predict the macro behavior of the fibrous composite material efficiently. Such a model can be used to represent all the fibers in the composite material.

Considering different theoretical models available [14,15] and based on a repetitive procedure, a new boundary condition is obtained and introduced in which generalized plane strain assumption (with a strain equal to 4.5624e-3) is made and also mechanical symmetry conditions are applied to the one fourth model of the circular RVE.

III. RESULTS AND DISCUSSION

The results of residual radial and circumferential and axial stresses and radial displacement around the interphase region are presented in this section.

COMPARISON OF RESULTS OF MONO FIBER CIRCULAR AND SQUARE AND HEXAGONAL MODELS

In Figs 11, 12, 13 and 14, the results of residual radial and circumferential and axial stresses and radial displacement around the interphase region are compared for the mono fiber square and circular and hexagonal models. (See Fig. 2,3 and 8)

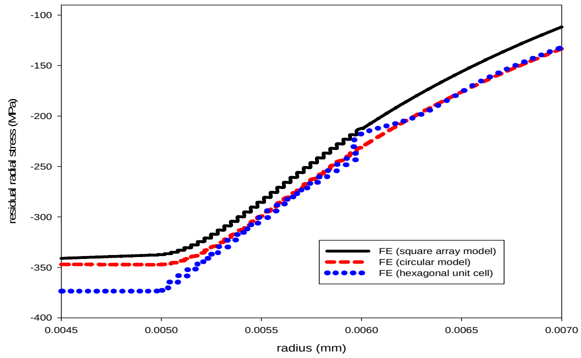


Fig. 11 Comparison of residual radial stresses for mono fiber square and circular and hexagonal models

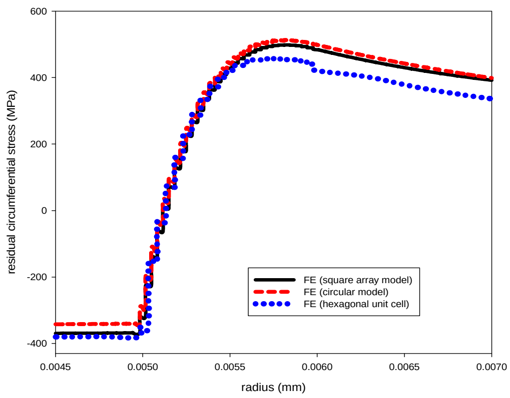


Fig. 12 Comparison of residual circumferential stresses for mono fiber square and circular and hexagonal models

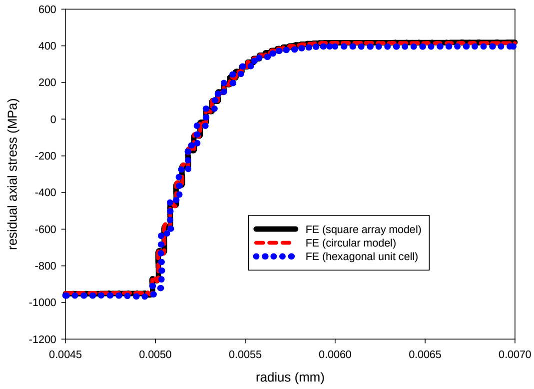


Fig. 13 Comparison of residual axial stresses for mono fiber square and circular and hexagonal models

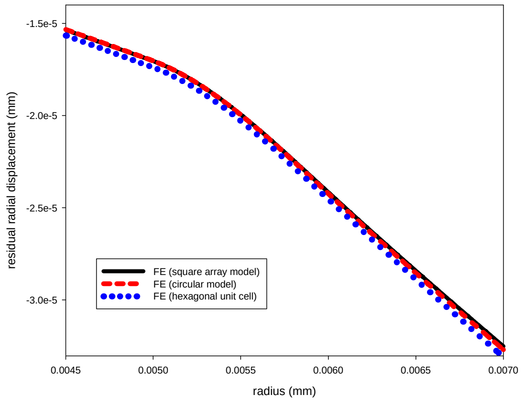


Fig. 14 Comparison of residual radial displacements for mono fiber square and circular and hexagonal models

As is shown, the results of residual axial stresses and radial displacements in all mono fiber models are quite similar. In the case of residual radial stresses, hexagonal unit cell shows less magnitude of these stresses and the residual radial stresses in square model are more than the other two models. In the interphase region, the results of hexagonal unit cell and the circular model are very close to each other. In the case of residual circumferential stresses in the fiber, the results of the square and hexagonal unit cell and in the matrix, the results of the circular and square models are very close. In all the regions, the circular model shows higher level of these stresses and in the interphase the results of all models are similar.

CONVERGENCE OF RESULTS FOR DIFFERENT FE ARRAYS

In Figs 15 to 22, the results of residual radial and circumferential and axial stresses and radial displacement around the interphase region are compared for the different square and hexagonal array FE models. (See Figs 4, 5, 6, 7, 9 and 10). As is shown, increasing the number of fibers in the model and creating a larger RVE, leads to more accurate results. Since the finite element mesh is chosen to be fine enough, the results converge very fast and increasing the number of fibers from 9 to 16 and 25 does not affect the results significantly.

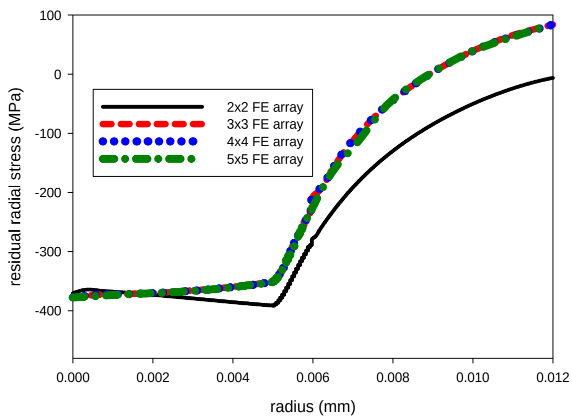


Fig. 15 Convergence of residual radial stresses for different square arrays

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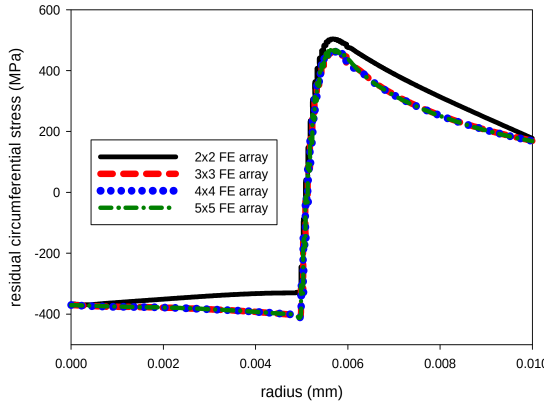

Fig. 16 Convergence of residual circumferential stresses for different square arrays

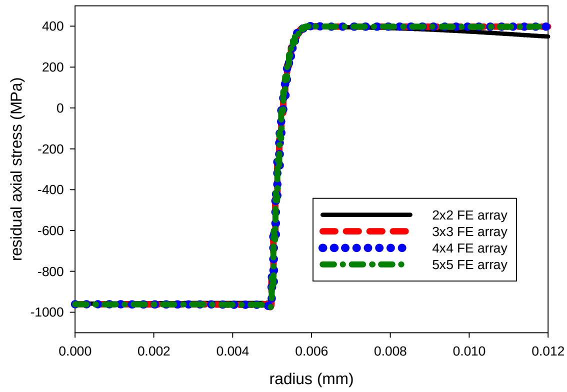

Fig. 17 Convergence of residual axial stresses for different square arrays

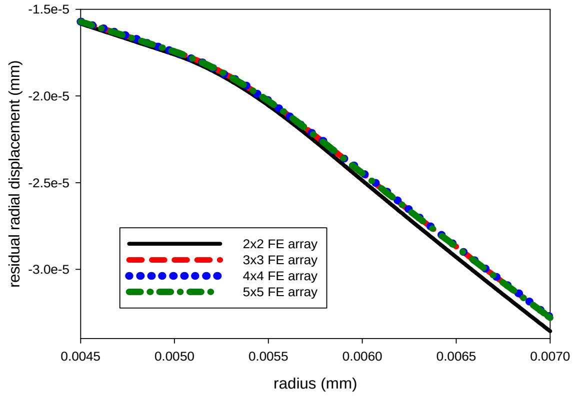

Fig. 18 Convergence of residual radial displacements for different square arrays

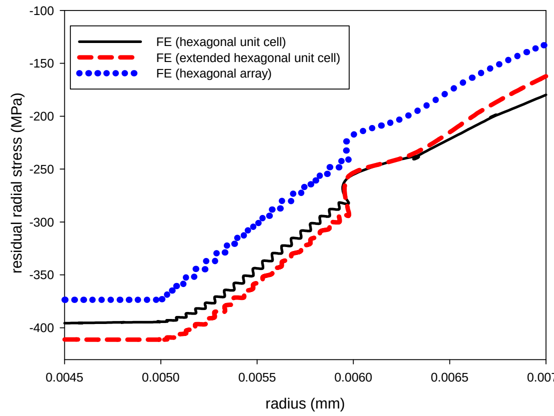

Fig. 19 Convergence of residual radial stresses for different hexagonal models

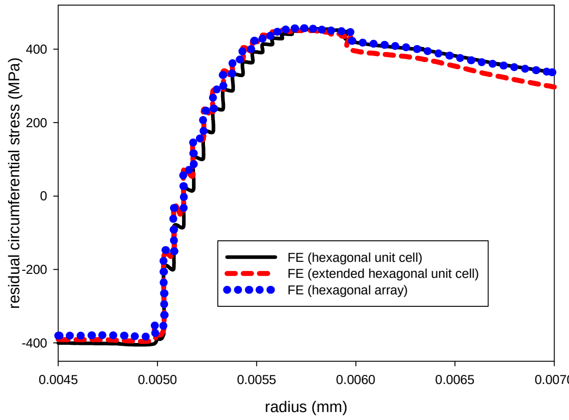

Fig. 20 Convergence of residual circumferential stresses for different hexagonal models

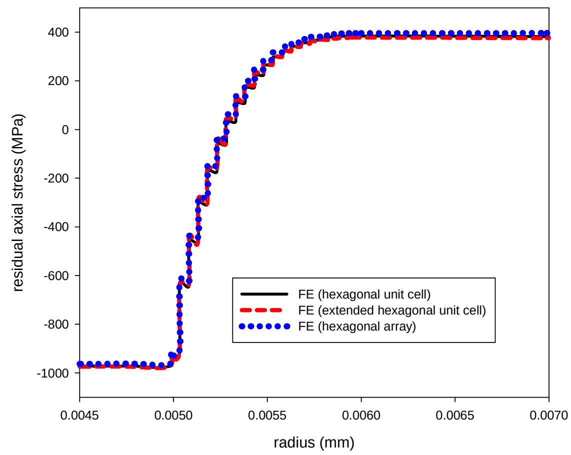

Fig. 21 Convergence of residual axial stresses for different hexagonal models

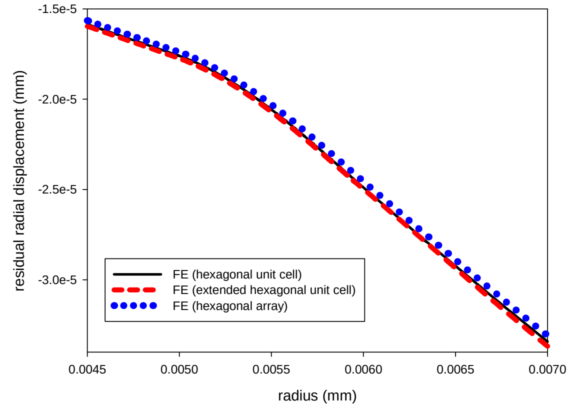

Fig. 22 Convergence of residual radial displacements for different hexagonal models

COMPARISON OF RESULTS OF MONO AND MULTI FIBER FE MODELS AND A THEORETICAL MODEL

In Figs 23, 24, 25 and 26, the residual radial and circumferential and axial stresses and radial displacement results obtained from the mono fiber circular RVE model and the multi fiber 5x5 array model and the hexagonal array are compared with You's [15] theoretical results. A good agreement is observed between the results, which proves that the presented mono fiber circular RVE model with the applied boundary conditions is capable of modeling the macro stress and displacement behavior of the composite material with an acceptable accuracy.

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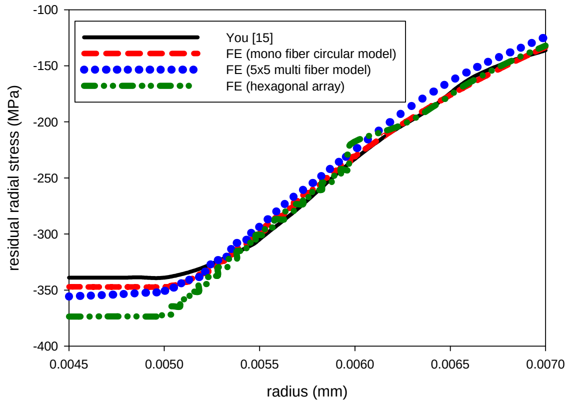

Fig. 23 Comparison of residual radial stresses for different models

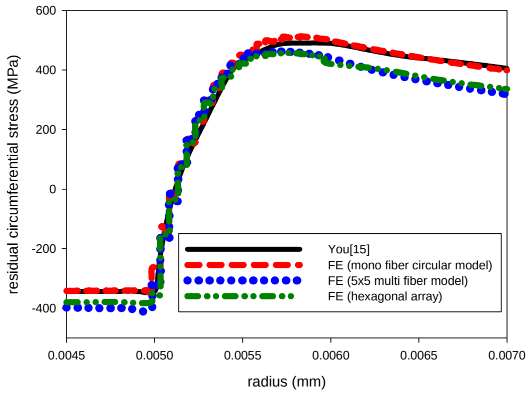

Fig. 24 Comparison of residual circumferential stresses for different models

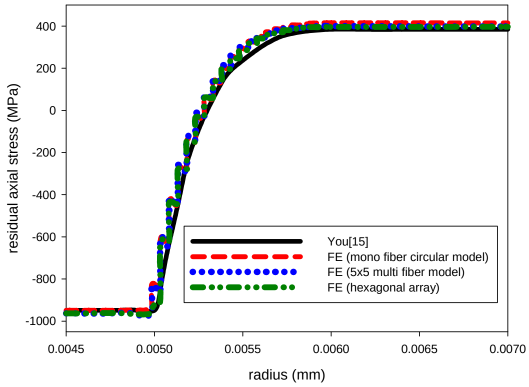

Fig. 25 Comparison of residual axial stresses for different models

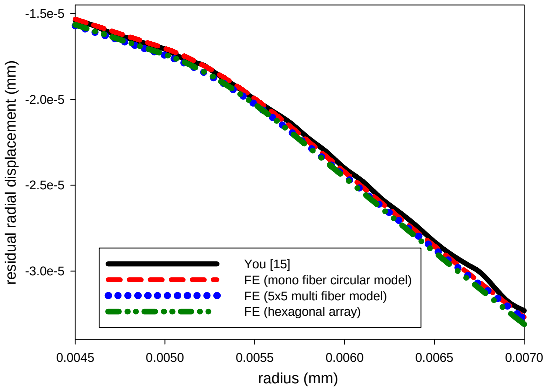

Fig. 26 Comparison of residual radial displacements for different models

IV. CONCLUSIONS

Following an extensive study, appropriate boundary conditions are selected for the mono fiber unit cell model and the results of the unit cell model show good agreement with You's theoretical model [15]. As the next step, multi-fiber square arrays of fibers, such as 2x2, 3x3, 4x4 and 5x5 arrays and also extended hexagonal arrays are modeled using FEM. Comparing the results of the mentioned arrays, a convergence is observed in the results. Also, the results of array modeling are compared with the theoretical results and a good agreement is observed between the results in this part. These comparisons offered the chance to investigate the behavior of each possible configuration of RVEs and fiber packing and its effect on the residual stress and displacement distribution around the interface region of the fiber and matrix, incorporating an inhomogeneous interphase. The results show that each configuration gives slightly different results in different analyses and regions and one single model cannot be chosen as the most appropriate and it was shown that all these models do not have very much different behavior.

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