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Mine Valuation in the Presence of a Stochastic Ore-Grade Uncertainty

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Abstract—**Mining companies world-wide are faced with the problem of how to accurately value and plan extraction projects subject to uncertainty in both future price and ore grade. Whilst the methodology of modelling price uncertainty is reasonably well understood, modelling ore-grade uncertainty is a much harder problem to formulate, and when attempts have been made the solutions take unfeasibly long times to compute. By treating the grade uncertainty as a stochastic variable in the amount extracted from the resource, this paper provides a new approach to the problem. We show that this method is well-posed, since it can realistically reflect the geology of the situation, and in addition it enables solutions to be derived in the order of a few seconds. A comparison is made between a real mine valuation where the prior estimate of ore grade variation is taken as fact, and our approach, where we treat it as an uncertain estimate.**

Keywords: Real-Options, Stochastic Control, Reserve Valuations.

1 Introduction

The planning of an extraction project and its associated valuation is exposed to many uncertainties. A project can last for many years, and is therefore subject to wide variations in the underlying commodity price. It is also subject to variation in the grade or quality of ore, whose expectations are estimated by interpolating between pre-extraction bore-holes. This interpolation, known as Kriging, produces estimation errors, even though subsequent mine planning treats the Kriging estimates as fact. The traditional, and most widely-used application of Kriging within numerical-based mine planning, is in the Lerchs-Grossmann algorithm [8] which is an extremely useful method for devising an optimal mine design. This involves taking the estimated orebody model, which consists of a large number of constituent blocks, and then designating which blocks should be extracted so as to maximise the mines value. However, the Lerchs-Grossman algorithm does not take account of price uncertainty, grade uncertainty, or discount rates [13], and therefore current mine valuations and planning do not properly address these important uncertainties. As such a new approach is required to incorporate grade and price uncertainty into mine valuations and planning.

Whilst there is general agreement that the commodity price can be modelled as a exogenous stochastic process [1], [12], there is no agreed method for modelling grade uncertainty. Recent attempts have focused upon planning for plausible simulated ore-bodies using mixed integer programming (MIP), where multiple ore-bodies are generated in a Monte-Carlo-type fashion [7], [9], [11]. Once a particular ore-body has been generated, numerous paths through the mine are calculated, where each path carries its own valuation, which in turn is calculated from numerous price simulations. It is clear that this approach involves an extremely large number of simulations, and finding an approximately optimal valuation and schedule can involve computational times of nearly twenty-four hours [3], even for mines with fewer than the 10^6 blocks which are common, making MIPs unfeasible for large mines. In addition, the possible inconsistency between a price process in time and an ore grade process in space is not usually considered.

This paper addresses the computational limitations of existing approaches and provides an alternative, more transparent, valuation methodology. In Section 2 we propose to model the grade quality as a stochastic uncertainty, where the stochastic behaviour is realised as one extracts ore from the mine; a higher extraction rate implies the grade uncertainty will fluctuate faster. As shown in Section 3, this method can model a mine in a continuum manner using partial differential equations (PDEs). This formulation allows for model behaviour to be investigated and, as shown in Section 4, in some instances it allows for exact solutions to be derived. We present results for a particular mine in Section 5. This use of PDEs not only gives far faster computational speeds, allowing rapid calculation of model sensitivities but also allows several classes of optimal decisions to be incorporated. This form of modelling extends existing PDE mine valuation methodologies which have not previously

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considered grade uncertainty [2], [6].

2 Data Interpretation

Our sample grade data for the weight of resource per unit of estimated ore extracted, G (grams per tonne) is from a sample mine composed of 30,000 blocks, and the order of extraction has been specified[1]. By viewing this data in order of extraction, Figure 1 (top), one can see how G varies through the cumulative amount extracted, $\bar{Q}$. However, given that measurement error exists at each data point, we interpret this sample of data as one random simulation from a whole range of possible Kriging samples, where each simulation will lie closely around a specified mean-path. This mean-path could be regarded as an interpolation between the known data-points (which themselves are functions of measurement error). To model this large range of random simulations, we treat the grade variation as a stochastic random process through $\bar{Q}$. A suitable model is a CIR process [5] of the form,

$$dG = k(\alpha(\bar{Q}) - G)d\bar{Q} + \sigma_G \sqrt{G} dX_G, \qquad (1)$$

where $\alpha(\bar{Q})$ is the mean-path of G through the ore-body (its spatial pattern being given), and dX_G, is normally distributed as $N(0, \sqrt{d\bar{Q}})$. This process allows the realised ore grade to vary either side of the estimated mean-path, but without the grade becoming less than zero. This model is easy to generalise, for example to include Levy processes, which we leave for future work.

We next need to estimate the associated parameter values of k, σ_G and $\alpha(\bar{Q})$. The mean value α is relatively straight-forward to generate; we create this using a cubic spline interpolated from Kriged data and is shown in Figure 1 (centre). This gives us a smooth mean-path for ore grade variation. For the remaining two parameters we compute infill maximum likelihood estimates for a CIR process (eq. 71 and 72, [10]). Using these equations on our data estimates the parameter values to be

$$k = 52 \text{ kg}^{-1} \quad \text{and} \quad \sigma_{\mathrm{G}} = 9.6 \text{ G}^{1/2} \text{ kg}^{-1/2}. \qquad (2)$$

To show how these parameter estimates behave, Figure 1 (bottom) shows one particular simulation using these parameter values, SDE and mean-path. As it demonstrates, the simulation is qualitatively consistent and representative of the data, consequently we view this methodology well-posed. In Section 5 we compare a valuation assuming this level of grade uncertainty with a valuation which treats the input Kriging data as fact.

3 Model Construction

To create a finite-reserve valuation, V, we first prescribe four state-space variables. These are the price S per

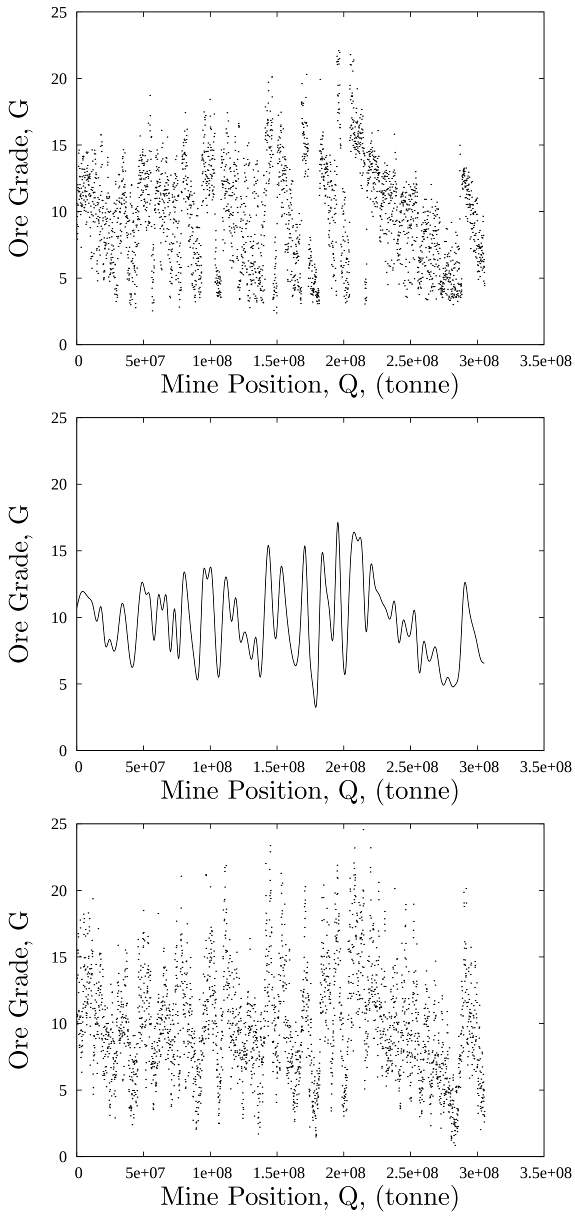

Figure 1: The top figure shows ore-grade data from a sample mine (supplied by Gemcom Software International), the middle graph shows its mean path and the third graph shows a grade simulation generated from equation (1) using the mean-path and the parameter values from (2).

unit of the underlying resource in the ore, the cumulative weight of ore extracted from the mine $\bar{Q}$, time t and resource ore grade G. However, to aid notational consistency, we use the variable for remaining resource, Q, defined by,

$$Q = Q_{max} - \bar{Q}, \qquad (3)$$

where Q_{max} is the maximum ore quantity extractable from the reserve. An obvious consequence of this is that $dQ = -d\bar{Q}$. With this, the rate of extraction of ore-

[1] The data has been supplied by Gemcom Software International, a large mining industry solutions provider.

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bearing material, q, is introduced via the equation

$$dQ = -qdt, \tag{4}$$

and will be subject to physical and practical constraints on its maximum and minimum levels, requiring $q \in [q_{min}, q_{max}]$.

We maintain the use of a CIR process to describe the grade uncertainty as given by equation (1), and without loss of generality we assume the underlying price S to follow a geometric Brownian motion,

$$dS = \mu S dt + \sigma_s S dX_s, \tag{5}$$

where μ is the drift and σ_s the volatility of S. The random variable dX_s, is normally distributed as $N(0, \sqrt{dt})$.

Using this notation, we may apply Ito's lemma to write an incremental change in V as,

$$\begin{aligned} dV =& \sigma_s \frac{\partial V}{\partial S} dX_s + \sigma_G \sqrt{G} \frac{\partial V}{\partial G} dX_G \\ &+ \left(\frac{\partial V}{\partial t} + \frac{1}{2}{\sigma_s}^2 \frac{\partial^2 V}{\partial S^2} + \mu \frac{\partial V}{\partial S} \right) dt \\ &- \left(-\frac{\partial V}{\partial Q} + \frac{1}{2} G {\sigma_G}^2 \frac{\partial^2 V}{\partial G^2} + k(\alpha - G) \frac{\partial V}{\partial G} \right) dQ, \end{aligned} \tag{6}$$

where we have taken powers of $(dt)^2$ and $(dQ)^2$ to be negligible. We wish to remove the dQ term via equation (4), which means that equation (6) can be transformed into,

$$\begin{aligned} dV =& \sigma_1 \frac{\partial V}{\partial S} dX_s + \sigma_G \sqrt{G} \frac{\partial V}{\partial G} dX_G \\ &+ \left(\frac{\partial V}{\partial t} - q \frac{\partial V}{\partial Q} + \frac{1}{2}{\sigma_1}^2 \frac{\partial^2 V}{\partial S^2} + \mu \frac{\partial V}{\partial S} \right) dt \\ &+ \left(\frac{1}{2} q G {\sigma_G}^2 \frac{\partial^2 V}{\partial G^2} + qk(\alpha - G) \frac{\partial V}{\partial G} \right) dt. \end{aligned} \tag{7}$$

To follow the conventional approach in creating and valuing risk-free portfolios, we construct a portfolio, Π, in which we are instantaneously long in (owning) the mine and are short in (owing) γ_s amounts of commodity futures contracts and short in γ_G amounts of options on the mine C (this option could be a call or put option, just so long as it is an option on the same mine). This defines $\Pi = V - \gamma_s S - \gamma_G C$, such that,

$$d\Pi = dV - \gamma_s dS - \gamma_G dC. \tag{8}$$

This portfolio is designed to contain enough freedom in γ_s and γ_G to be able to continually hedge away the uncertainties of dX_s and dX_G, which is the standard approach in creating risk-free portfolios [12], [14]. It also means that within a small time increment, dt, the value of Π will increase by the risk-free rate of interest minus any associated economic value generated during the increment. This economic value is typically composed of two parts, the first, negative, being the cost per unit to extract ore, ϵ_M, and the second, positive, the cash generated by selling the resource content of the ore extracted. An extra stage of processing (e.g. milling) is usually required after ore extraction to isolate a saleable form of the resource. We model the case where the processing cost is variable and avoidable, so processing is done if $qSG > \epsilon_P$, where ϵ_P is the processing cost per unit of ore extracted. With this form of optimal decision the incremental change in portfolio value may be written as

$$d\Pi = r\Pi dt - \gamma_S \delta S dt - \max(0, qGS - \epsilon_P) dt - \epsilon_M dt. \tag{9}$$

By using appropriate values of γ_s and γ_G to be,

$$\begin{aligned} \gamma_s &= \frac{\partial V}{\partial S} - \frac{\partial V}{\partial G} \left(\frac{\partial C}{\partial G} \right)^{-1} \frac{\partial C}{\partial S}, \\ \gamma_G &= \frac{\partial V}{\partial G} \left(\frac{\partial C}{\partial G} \right)^{-1}, \end{aligned} \tag{10}$$

and substituting equations (5), (7) and (8) into (9), we may write our two-factor valuation equation as,

$$\begin{aligned} &\frac{1}{2}{\sigma_s}^2 S^2 \frac{\partial^2 V}{\partial S^2} + \frac{1}{2} q G {\sigma_G}^2 \frac{\partial^2 V}{\partial G^2} + \frac{\partial V}{\partial t} - q \frac{\partial V}{\partial Q} \\ &\quad + (r - \delta) S \frac{\partial V}{\partial S} + qk(\hat{\alpha} - \delta) \frac{\partial V}{\partial G} \\ &\quad - rV + \max(0, qGS - \epsilon_P) - \epsilon_M = 0, \end{aligned} \tag{11}$$

where $\hat{\alpha} = \alpha - \sigma_G \lambda_G / \kappa$, and λ_G is the market price of risk for ore grade. If we wish to reduce this model to a one-factor model, with price as the only uncertainty, we can set the grade quality to be a constant, giving,

$$\begin{aligned} &\frac{\partial V}{\partial t} - q \frac{\partial V}{\partial Q} + (r - \delta) S \frac{\partial V}{\partial S} + \frac{1}{2}{\sigma_s}^2 S^2 \frac{\partial^2 V}{\partial S^2} \\ &\quad - rV + \max(0, qGS - \epsilon_P) - \epsilon_M = 0. \end{aligned} \tag{12}$$

This is the standard one-factor equation of Brennan and Schwartz (eq. 15, [2]), except that they added taxation terms.

We next need to prescribe boundary conditions for (11). The boundary condition that no more profit is possible occurs either when the reserve is exhausted $Q = 0$, or when a lease to operate the mine has reached its expiry date $t = T$, hence:

$$V = 0 \quad \text{on} \quad Q = 0, \quad \text{or} \quad t = T. \tag{13}$$

Since the extraction rate will have a physical upper bound, the extraction rate and cost will not vary with S when S is large. This permits a far field valuation of the form,

$$\frac{\partial V}{\partial S} \to A(G, Q, t) \quad \text{as} \quad S \to \infty. \tag{14}$$

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When the underlying resource price is zero we need only solve the reduced form of equation (11) with $S = 0$.

The boundary conditions on G are that its behaviour is convection-dominated as it moves far from above its mean or tends to zero, since diffusion effects are then negligible. In these cases we solve equation (11) without second derivatives of G. Hence G is unlikely to drift far from its mean, as is standard with mean-reverting processes. Specifically, the conditions become,

$$-\frac{\partial V}{\partial \tau} - q\frac{\partial V}{\partial Q} + q\kappa\alpha\frac{\partial V}{\partial G} + (r-d)S\frac{\partial V}{\partial S} + \frac{1}{2}\sigma_S^2 S^2\frac{\partial^2 V}{\partial S^2} - rV - \epsilon_M = 0, \tag{15}$$

as $G \to 0$, and as $G \to \infty$ we require,

$$-\frac{\partial V}{\partial \tau} - q\frac{\partial V}{\partial Q} + q\kappa(\alpha - G)\frac{\partial V}{\partial G} + (r-d)S\frac{\partial V}{\partial S} + \frac{1}{2}\sigma_S^2 S^2\frac{\partial^2 V}{\partial S^2} - rV + qGS - \epsilon_P - \epsilon_M = 0. \tag{16}$$

4 Reserve- and Time- Dependent Extraction Rate

In this section we simplify to assume that the rate of extraction and the decision to process are independent of both price and ore grade. This simplifies the extraction rate and the extraction-processing cost per unit of ore respectively to the forms $q = q(Q,t)$ and $\epsilon = \epsilon(Q,t)$. Hence the cash flows generated by the mine are of the form $qSG - \epsilon$, since there is no longer a decision whether or not to process. Using this and the fact that $dQ/dt = -q$, we may determine $Q(t)$ exactly, and calculate the moment when the reserve will be exhausted, T. There are two alternative ways of defining T; first as the lease contract expiry date, and the other as the date when the extractable amount of reserve is exhausted. If these dates differ, one would take T as the lesser of the two. These assumptions allow us to remove the Q variation from our model, and write $\bar{q}(t) = q(Q(t),t)$ and $\bar{\epsilon}(t) = \epsilon(Q(t),t)$.

We begin by searching for a solution to (11), as suggested by (14), with the Q derivative no longer necessary, of the form,

$$V = SV_1(G,t) + V_2(G,t). \tag{17}$$

By substituting this into equation (11) we obtain the two equations,

$$\frac{\partial V_1}{\partial \tau} - \bar{q}\kappa(\hat{\alpha} - G)\frac{\partial V_1}{\partial G} - \bar{q}\frac{1}{2}\sigma_G^2 G\frac{\partial^2 V_1}{\partial G^2} + dV_1 = G\bar{q},$$
$$\frac{\partial V_2}{\partial \tau} - \bar{q}\kappa(\hat{\alpha} - G)\frac{\partial V_2}{\partial G} - \bar{q}\frac{1}{2}\sigma_G^2 G\frac{\partial^2 V_2}{\partial G^2} + rV_2 + \bar{\epsilon} = 0. \tag{18}$$

We may split equation (18) up further to seek a solution of the form $V_1(G,\tau) = \phi(\tau)G + \psi(\tau)$, such that,

$$\phi' + (\bar{q}\kappa + \delta)\phi = \bar{q},$$
$$-\psi' + q\kappa\alpha\phi - \delta\psi = 0. \tag{19}$$

By introducing the variable $\xi(\tau) = \int_0^\tau q(x)dx$, we may write our solution for ϕ, ψ and V_2 as,

$$\phi = e^{-(\kappa\xi + \delta\tau)}\int_0^\tau q(\eta)e^{\kappa\xi(\eta)+\delta\eta}d\eta,$$
$$\psi = e^{-d\tau}\int_0^\tau \kappa\alpha q(\eta)\phi(\eta)e^{\delta\eta}d\eta,$$
$$V_2 = -e^{-r\tau}\int_0^\tau \epsilon(\eta)e^{r\eta}d\eta. \tag{20}$$

In the particular case of a constant extraction regime, these integrals can be calculated to be,

$$\bar{\phi} = \frac{\bar{q}}{\bar{q}\kappa + d}\left(1 - e^{-(\bar{q}\kappa + d)\tau}\right),$$
$$\bar{\psi} = \frac{\bar{q}^2\kappa\alpha}{\bar{q}\kappa + \delta}\left\{\frac{1}{d}\left(1 - e^{-\delta\tau}\right) + \frac{1}{\bar{q}\kappa}\left(e^{-(\bar{q}\kappa + d)\tau} - e^{-\delta\tau}\right)\right\},$$
$$V_2 = \frac{\bar{\epsilon}}{r}(1 - e^{-r\tau}), \tag{21}$$

which determines our exact solution to a mine valuation in the presence of price and ore-grade uncertainty, when extraction and processing are independent of price and grade.

5 Price Dependent Extraction Rates

Let us return to the more general case where we operate a processing constraint, equation (11). Since we can no longer predict a date T when the mine's value will be exhausted, we must retain all derivatives within the model and turn to numerical techniques for solution. As detailed in [6] and [4], we choose to solve (11) using a semi-Lagrangian scheme, in which the solution is evaluated on the characteristics $dQ = -qdt$ via an interpolation between the adjacent nodes. This scheme can be second-order convergent in time and thus allows for accurate solutions to be quickly derived.

5.1 Example Valuation

We compare valuations of a sample mine (as detailed in Section 2) where one valuation assumes error of (and around) the Kriging estimate or ore grade, and the other takes that estimate as fact. This is equivalent to comparing a valuation made with all possible ore-grade simulations to a valuation made with just ore-grade simulation. For the grade uncertainty we use the inferred parameter values of (2) and the mean-path of the grade as shown in Figure 1 (middle), and take our other price parameter values to be,

$$\sigma_S = 0.5 \text{ yr}^{-1/2}, \quad r = 0.1 \text{ yr}^{-1}, \quad \delta = 0.1 \text{ yr}^{-1}. \tag{22}$$

For the mine extraction cost parameters we use $\epsilon_M = \$1 \text{ tonne}^{-1}$ and $\epsilon_P = \$4 \text{ tonne}^{-1}$, and the processing capacity constraint of ore-bearing tonnage is $q_{max} = 20,000,000 \text{ tonne yr}^{-1}$. With these we are able to construct our price and grade uncertainty valuation as shown by Figure 2. As expected the valuation becomes linear in

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S for higher prices, and losses are limited for low prices (roughly $S < 30$ \$ gr^{-1}). This is expected, as we are operating a processing decision rule where we only process cost-effective ore.

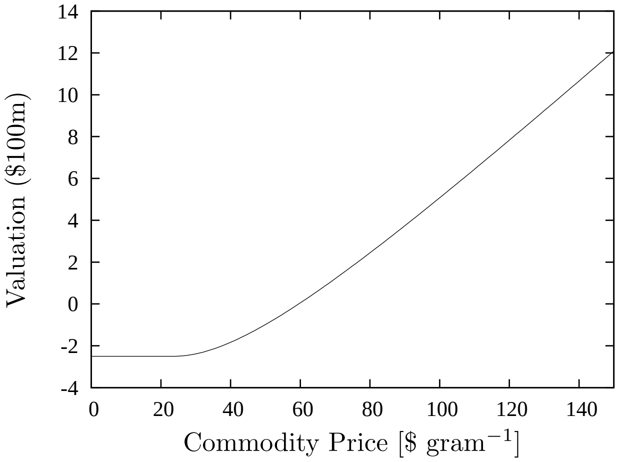

Figure 2: Valuation of a mine reserve for differing levels of current commodity price made in the presence of stochastic grade uncertainty.

To compare this valuation under both price and grade uncertainty with one where there is only price uncertainty, Figure 3 shows the difference between these valuations for a range of prices. At higher prices, grade uncertainty has only a small impact on the valuation. This might be expected, as the expected benefits of unexpectedly high ore grade are symmetrical with the losses of its unexpectedly low grade. The only region where these opposing effects do not cancel each other out, is when the expected cash from sales is similar to the cost of processing. Here the mine has a valuable option to process all or none of the current flow of extracted material, avoiding processing where it is unprofitable. This explains the existence of a maximum point in Figure 3.

6 Conclusions

This paper has presented a method for how to incorporate ore grade uncertainty, and a simple optimising decision in response to ore grade, into a finite resource valuation. The method of treating the grade as a CIR stochastic process (along a preset trajectory in space), not only allows plausible ore-body grade simulations to be produced, but also allows the model to be constructed as a single PDE. This can be solved in the order of 10 seconds on a modern laptop computer, giving values theoretically equivalent to infinitely large set of simulation runs of the resource price, the ore grade and the prescribed rule for processing ore. As such, the methodology can rapidly provide robust and defensible mine valuations under many alternative stochastic structures. Whilst possible enhancements to the CIR process could be made, such as the inclusion of

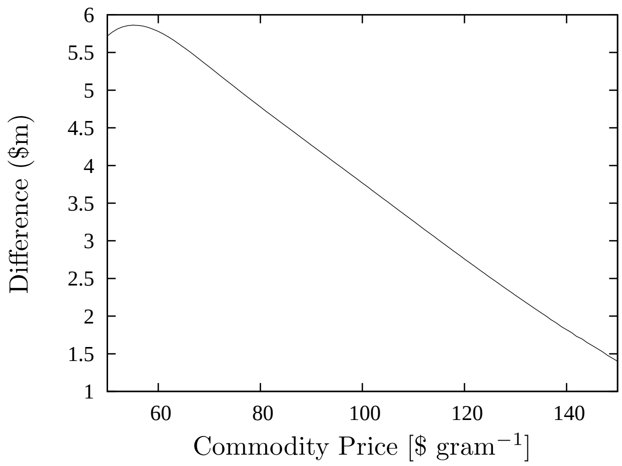

Figure 3: The difference between valuations where one treats the ore-grade as an uncertainty and the other where one treats it as fact, for a range of underlying prices. This is equivalent to viewing the difference between a valuation made with all possible ore-grade simulations, to a valuation made with just one simulation.

jumps, the underlying methodology of the paper would remain the same.

We find for our example mine, that including price and ore grade uncertainty adds up to five million dollars to a valuation assuming only price uncertainty. At high levels of the commodity price, the option not to process poorer grades of ore is seldom used and adds relatively little value, but at lower commodity prices, and higher levels of ore grade volatility, this option appreciably raises the mine's value.

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