Numerical Investigation on Heat Transfer of Power Law Fluids in a Pipe with Constant Wall Temperature

Botong Li, Liancun Zheng, and Xinxin Zhang

Abstract—**This paper presented numerical studies on the problem of steady heat transfer of power law fluids in a circular duct with constant wall temperature. The thermal equation mathematical model was solved by assuming that the thermal diffusivity was a function of temperature gradient. The solutions were obtained by using the control volume based finite difference model coupled with the LU decomposition technique. It is shown that the results are strongly depending on the power law index. The associate transfer behavior is also discussed in detail.**

Index Terms—**power law fluids, convective heat transfer, numerical investigation, thermal diffusivity**

I. INTRODUCTION

Since 1960, considerable attention has been paid to the problem of how to predict the velocity field and heat transfer of non-Newtonian flow because these fluids (such as molten plastics, pulps, slurries, emulsions, etc.) are applied industrially in increasing quantities.

The theoretical analysis of steady incompressible non-Newtonian power law fluids was first performed by Schowalter [1] and Acrivos *et al.* [2]. The boundary-layer equations were formulated, and the conditions of the existence of similarity solutions were established. Campo *et al.* [3] discussed the algebraic solution to a 2-D partial differential energy equation under a robin boundary condition. Raithby [4] considered the laminar heat transfer in the thermal entrance region of circular tubes and two-dimensional rectangular ducts with wall suction and injection. Li *et al.* [5] made the numerical studies on flow and heat transfer in parallel-plate fin heat exchangers.

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The purpose of this paper is to present a numerical investigation on the problem of steady state conduction of heat and diffusion in a fluid flowing in a circular duct. The process of solving equations is more difficult than the case of Newtonian flows. Thus, special emphases are put on the formulation and the discretization of the problem.

II. FORMULATION OF THE PROBLEM AND DISCRETIZATION

Consider the case that the duct wall is maintained at a constant temperature different from the uniform temperature of the fluid at the entrance. We make the following assumptions: (1) fluid flow is hydrodynamically developed and laminar; (2) power law fluids are to be studied; (3) fluid axial conduction, viscous dissipation, and thermal energy sources are negligible. Take z and r to be co-ordinate axes parallel and perpendicular to walls. The axial component of the velocity field is given by [6]:

$$V_z = (3n+1)/(n+1)V_m(1-(r/a)^{(n+1)/n}) \tag{1}$$

where V_m and a are the mean velocity and the radius of the duct, respectively. n is the power law index while the case $n=1$ corresponds to a Newtonian fluid problem known as the Graetz problem. The thermal conductivity is chosen as $k(T)=k_0\left|\partial T/\partial r\right|^{n-1}$ with a positive constant k_0 [7]-[10]. Thus, the energy equation becomes:

$$\rho c_p V_z \frac{\partial T}{\partial z} = \frac{1}{r}\frac{\partial}{\partial r}(rk_0\left|\frac{\partial T}{\partial r}\right|^{n-1}\frac{\partial T}{\partial r}) \tag{2}$$

where ρ and c_p are the density and the specific heat of the fluid, respectively.

This paper introduces the following dimensionless quantities: dimensionless temperature Θ, dimensionless radial coordinate R, dimensionless velocity V_z^*, and dimensionless axial coordinate Z^*.

Applying these quantities to the energy equation and conditions, the problem under consideration is given as:

$$V_z^* \frac{\partial \Theta}{\partial Z^*} = \frac{1}{R}\frac{\partial}{\partial R}(R\frac{\partial \Theta}{\partial R}\left|\frac{\partial \Theta}{\partial R}\right|^{n-1}) \tag{3}$$

$$\Theta\big|_{Z^*=0} = 1 \tag{4}$$

$$\Theta\big|_{R=1} = 0 \tag{5}$$

$$\left.\frac{\partial \Theta}{\partial R}\right|_{R=0} = 0 \tag{6}$$

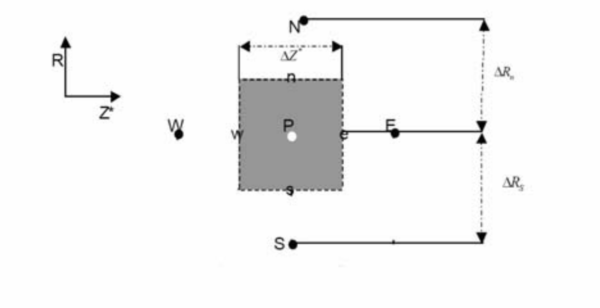

Fig.1. Control volume

In order to obtain numerical solutions, we transfer (3)-(6) to a system of algebraic equations. The calculation domain is divided into a few non-overlapping control volumes each of which surrounds a grid point (Fig.1). The energy equation (3) is integrated over each control volume. $\frac{\partial \Theta}{\partial R} < 0$, so we have:

$$V_z^*(\Theta_P - \Theta_W)\frac{R_n^{\,2} - R_s^2}{2} = [R_s(\frac{\Theta_S - \Theta_P}{\Delta R_s})^n - R_n(\frac{\Theta_P - \Theta_N}{\Delta R_n})^n]\Delta Z^* \tag{7}$$

where R_n is the dimensionless radial coordinate in north direction; R_s, in south direction. We find out a least-squares approximating polynomial $y = \tilde{a}x$ on $[0,1]$ to minimize the errors $\int_0^1[(\frac{\Theta_S - \Theta_P}{\Delta R_s})^n - \tilde{a}\frac{\Theta_S - \Theta_P}{\Delta R_s}]^2 dx$ and $\int_0^1[(\frac{\Theta_P - \Theta_N}{\Delta R_n})^n - \tilde{a}\frac{\Theta_P - \Theta_N}{\Delta R_n}]^2 dx$, so that $\tilde{a} = \langle f, \varphi \rangle / \langle \varphi, \varphi \rangle = 3/(n+2)$.

The problem now becomes:

$$V_z^*(\Theta_P - \Theta_W)\frac{R_n^{\,2} - R_s^2}{2} = [R_s\frac{\tilde{a}\Theta_S - \tilde{a}\Theta_P}{\Delta R_s} - R_n\frac{\tilde{a}\Theta_P - \tilde{a}\Theta_N}{\Delta R_n}]\Delta Z^* \tag{8}$$

Equation (8) is solved by using LU decomposition method.

III. Results and Analysis

The problem will be discussed by dividing the power law index of fluids into two sections: the case $0 < n < 1$ is descriptive of pseudo-plastic non-Newtonian fluids and $n > 1$ describes the dilatant fluids. Fig.2-Fig.4 show the dimensionless temperature profiles of different dimensionless axial coordinate and power law index. It is founded that the behavior of fluids are strictly affected by the power law index.

Fig.2-3 display the dimensionless temperature profiles of different dimensionless axial coordinate. The curve labeled $n = 0.5$ corresponds to shear-thinning non-Newtonian flow; while $n = 1.5$, shear-thickening non-Newtonian flow. Duo to viscous heating, the effects of fluids become more dominant away from the wall. The temperature increases towards the center of the pipe. The thermal wave of the inlet temperature has less penetration with the increasing axial coordinate.

Fig. 4 shows the dimensionless temperature profiles of different power law index. The temperature profile is flatter as the power law index increases. The temperatures of shear-thickening fluids are higher, since they are affected much easier by the inlet temperature than the shear-thinning fluids. '+'presents the Graetz solution [4] in vivid contrast with the solution of n=1 obtained by using the method mentioned in this paper.

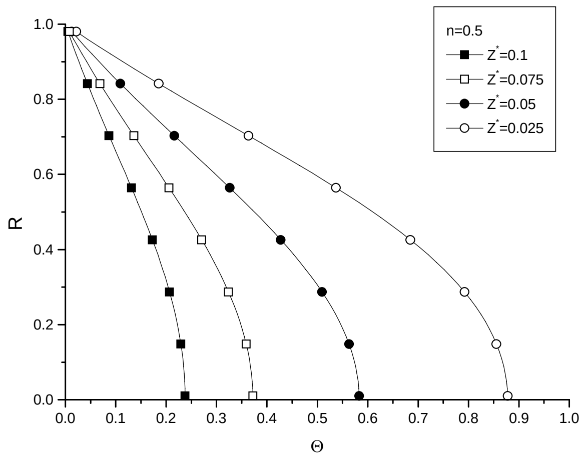

Fig.2. Temperature profiles of different dimensionless axial coordinate with $n = 0.5, \Theta_w = 0, \Theta_{in} = 1$

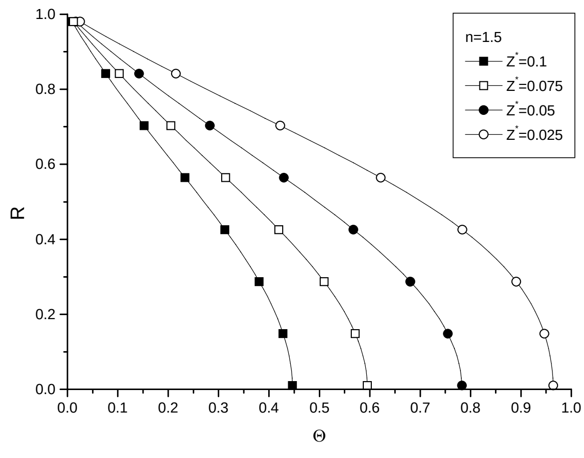

Fig.3. Temperature profiles of different dimensionless axial coordinate with $n = 1.5, \Theta_w = 0, \Theta_{in} = 1$

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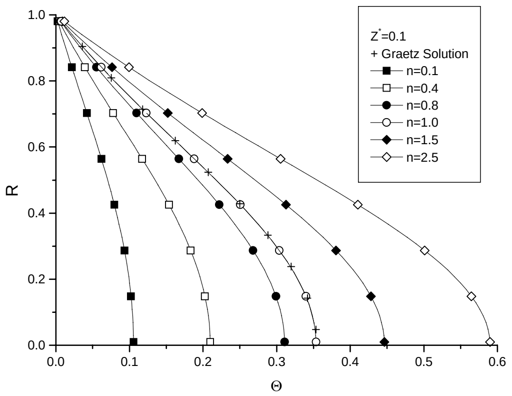

Fig.4. Temperature profiles of different power law index with $Z^* = 0.1$

IV. CONCLUSION

The problem of steady state conduction of heat and diffusion in a fluid flowing in a circular duct is studied. The numerical solutions of different power law index and dimensionless axial coordinate are obtained and the transfer characteristics of varying values of parameters are also revealed in figures.

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