The Stochastic Boundary-Layer in the Non-Newtonian Problem

Lei Hou, Hanling Li, Dezhi Lin, and Lin Qiu

***Abstract*—Mathematical researchers are interested in the properties of visco-elastic materials in multiple-scale impact conditions. However, the research is complex especially in the auto-crash safety engineering process. In this paper, we study the stochastic boundary layer for the non-Newtonian rate type impact hardening and share thinning phenomenon by considering the distribution of the contact angles. The numerical scheme yields the convergent finite element analysis (FEA) solution and stable semi-discrete Galerkin-Runge-Kutta (G-RK) iteration. The high performance computing (HPC) tool has also been used for the passive safety analysis with engineering application.**

***Key Words*—boundary layer, stochastic analysis, auto-crash safety, non-Newtonian**

I. INTRODUCTION

Auto-crash safety is of great interest in engineering, upon which the research of the non-Newtonian visco-elastic/plastic material is complex, especially dealing with the boundary layer problem in the contact interface.

Reference [1]-[2] gave the standard mathematic description of the relaxing-recovery material property. Furthermore, [3]-[4] use the Cauchy conservation equation, coupled with P-T/T equation, to describe the velocity and the stress during the impact. They also introduced some of the FEA simulations and the related analysis of the mathematic models.

Apart from the FEA simulations, the stochastic post-processing is also presented in this model. The boundary data is from the coupled PDE equations mentioned above. After briefly introducing the mathematic model, the calculation results of stress and strain on each node is given. This paper then focuses mainly on the angle effects of the distribution functions based on the stochastic data analysis.

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II. MATHEMATIC MODEL AND NODE RESULT

The coupled PDE equations of the standard P-T/T equation are the best estimates for the stress over-shoot. Cauchy conservation equation may be used to calculate the large e/p deformation resulting from stress (shear thinning). It can be used to describe the velocity and the stress distribution in the auto-crash impact problem [6].

The P-T/T equation is

$$\rho\dot{\underline{u}} = \left[\nabla \cdot \underline{\underline{\tau}} - \rho \underline{u} \cdot \nabla \underline{u}\right] \;\; in\ \Omega - \Gamma_0 \tag{1}$$

and the Cauchy conservation equation is

$$\begin{aligned} \lambda\dot{\underline{\underline{\tau}}} = \left[2\eta\underline{\underline{D}} - \exp\left\{\frac{\epsilon\lambda}{\eta_0}\left(\tau_{xx} + \tau_{yy}\right)\right\}\tau\right] - \\ \lambda\ \left[\underline{u} \cdot \nabla\underline{\underline{\tau}} - \nabla u \cdot \underline{\underline{\tau}} - \left(\nabla u \cdot \underline{\underline{\tau}}\right)^T \right. \\ \left. +\xi\left(\underline{\underline{D}} \cdot \underline{\underline{\tau}} + \left(\underline{\underline{D}} \cdot \underline{\underline{\tau}}\right)^T\right]\right) \;\; in\ \Omega \end{aligned} \tag{2}$$

where τ is the stress field, D is strain, u is the deformation velocity field in e/p material, η is the viscosity, λ the relaxation constant, ε is the elongation rate, ξ is the shear rate, ρ is the density.

The complex boundary condition of stress is decided by static tests $\tau(0)$ in the impact experiment. The impact speed is set to $u(0)$. That is: $\tau_0 = \tau(static),\ u_0 = u_{x0}$ on $\Gamma_0 \subset \Gamma$.

The semi-discrete finite element method is used to solve the above PDE equation system. A nonlinear Riccati differential system can keep the LBB positive definite condition in time domain [7], [8]; therefore provide us with great freedom of choosing an approximation space. Still, we need to choose such approximation space that it preserves the positivism of interpolation scheme.

Fig. 1 shows the calculation result using different time scheme, among which the 2rd order Runge-Kutta (R-K2) is of best economical and accurate while comparing with R-K3 and Euler time scheme.

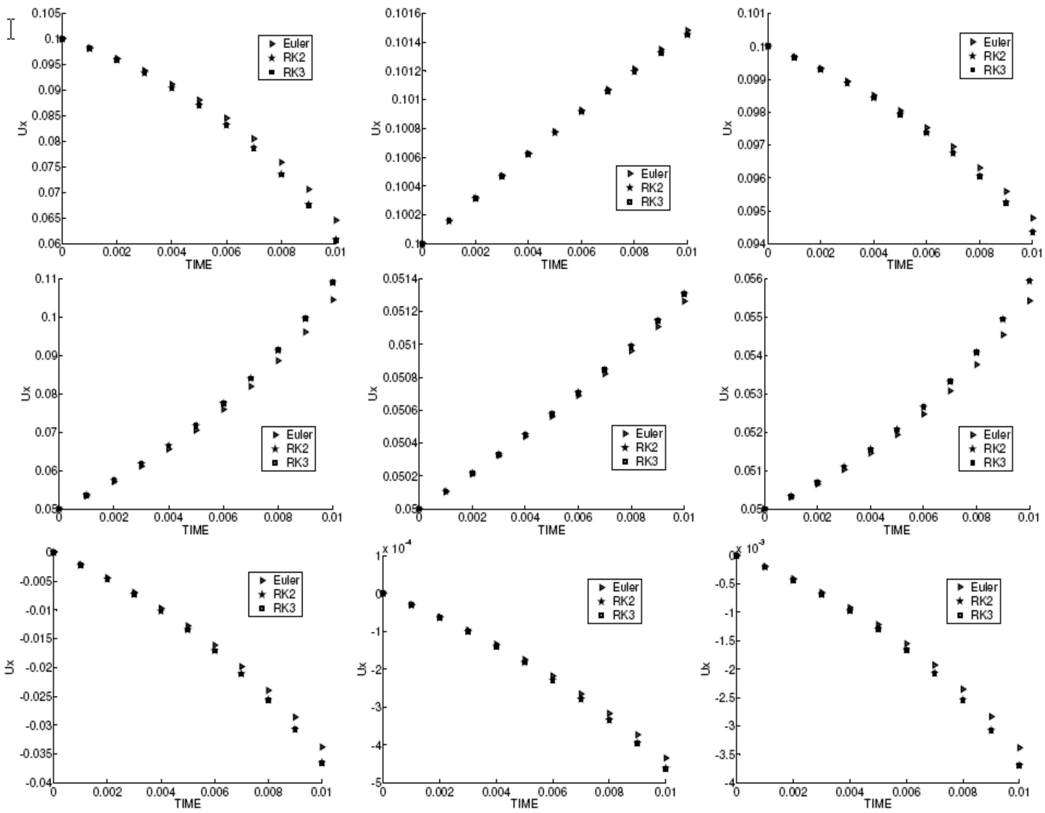
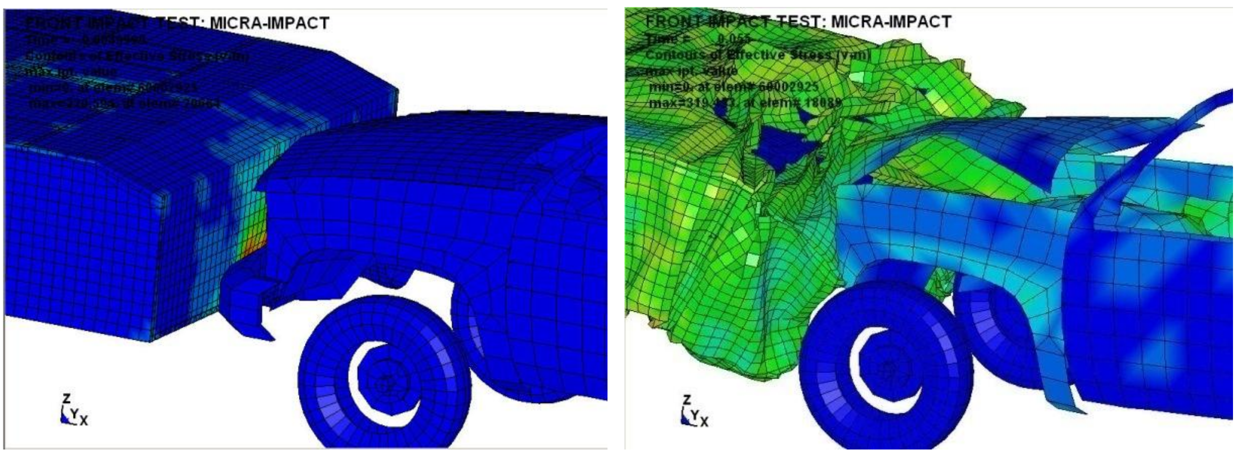


Fig. 1. The calculation result using Euler, 2nd order R-K, 3rd order R-K time scheme

Reference [3] presents the discrete form of the P-T/T equation, while [9] and [10] focus mainly the Cauchy equation. Both of the articles use the Lagrange interpolating space. Reference [4] discusses the Hermite-Runge-Kutta scheme, which yields 3rd order convergence, one order higher than the Lagrange- Runge-Kutta scheme [11].

III. FEA Simulation

We have compressed the information of crash safety analysis into the coupled PDE equations: (1) and (2). Therefore, the analysis of the 3-D FEA simulation is based on equations (1) and (2), with which each node uses Lagrange-Runge-Kutta scheme. The stochastic boundary condition is defined in Fig. 2 and Fig. 3. The numerical results of two crash impact model use two calculation schemes: the impact with a crash-barrier; and the impact between two cars. Fig. 2 presents both the deformation and the stress field, where the darker color indicates higher stress.



(a) The deformation result (b) The stress result

Fig. 2. The FEA results of the impact model: impact with a crash-barrier.

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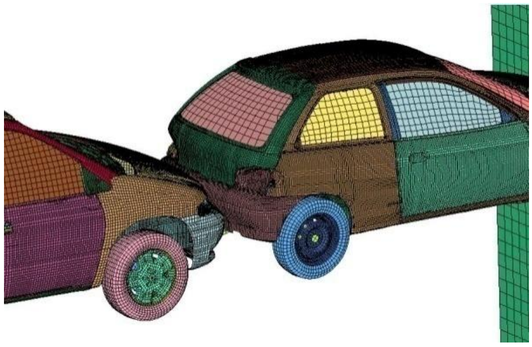

Fig. 3. The FEA results of the impact between two cars

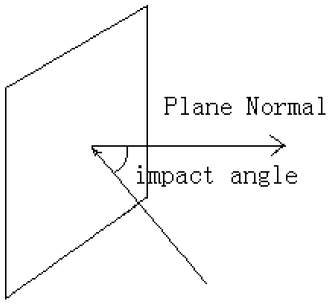

Fig. 4. The definition of impact angle

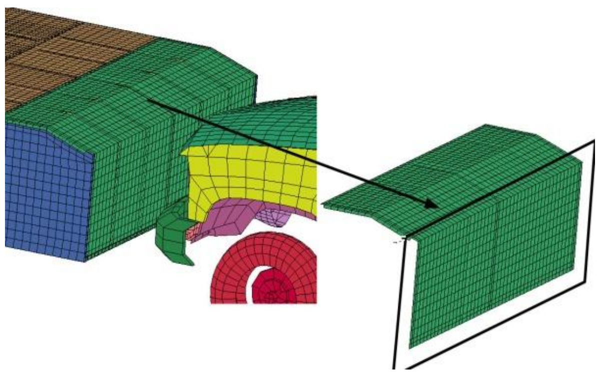

Fig. 5. The region analyzed in this paper

IV. Stochastic Analysis

Non-Newtonian material helps us to study the auto-crash safety. Scientists and engineers have been studying in depth how to use the soft solid concept for recoverable controlled fluid-structure interaction[3, 6]. Apart from solving FEA model, we analyze the data resulting from boundary layer. This is a new method to study the non-Newtonian materials as well as the contact interfaces in the impact model.

In this paper, we concern mainly about the trend of impact angle and its relationship with stress and strain.

The impact angle is defined in Fig. 4 in this paper, the stochastic analysis concerns mainly about the contact interface in the boundary layer. More than 26,000 data is extracted for the boundary layer analysis in the interface in Fig. 5.

A histogram graph of the frequency of the impact angle in several time-step is used to study its trend. Fig. 6 shows the frequency at beginning of the impact, which is the 2nd time-step. Apparently, the impact angle fits the normal distribution, which is consistent with the law of nature in philosophy [2]. It also shows that the impact angle agrees with a normal "Fisher's law" [12].

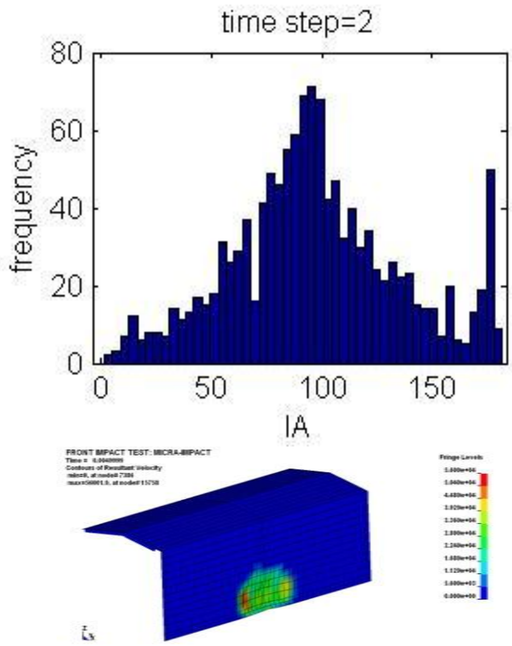

Fig. 6. A histogram graph of the frequency of the impact angle at the beginning of the impact,

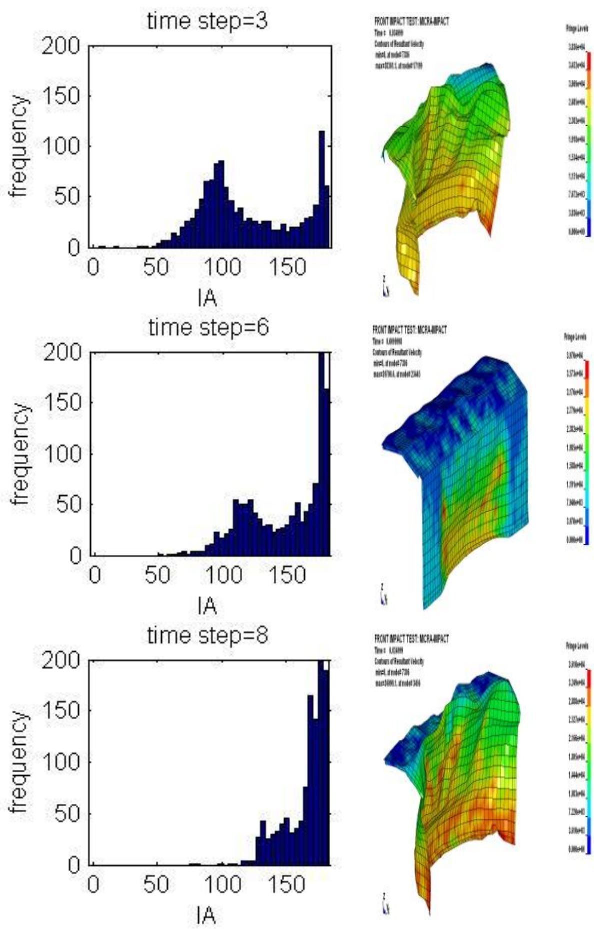

Fig. 7. The histogram graph of impact angle, as time goes by.

Fig. 7 shows the histogram graph of impact angle, as time goes by. A trend towards the angle of 180° (the front impact) is more likely. This indicates that a further study and more protection measure to be taken.

The trend of stress and strain is also interesting, which shows some properties of normal distribution in Fig. 8. and Fig. 9.

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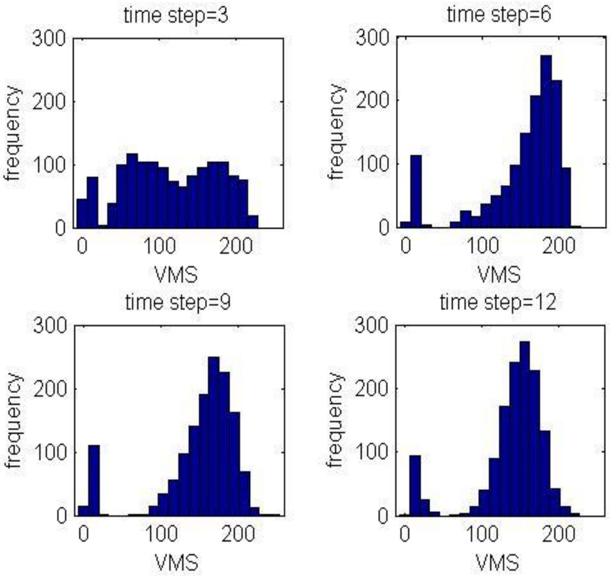

Fig. 8. The frequency trend of stress (VMS)

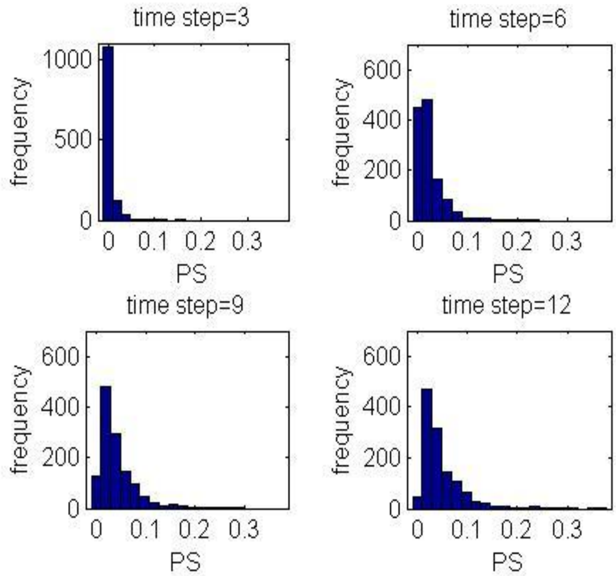

Fig. 9. The frequency trend of plastic strain (PS)

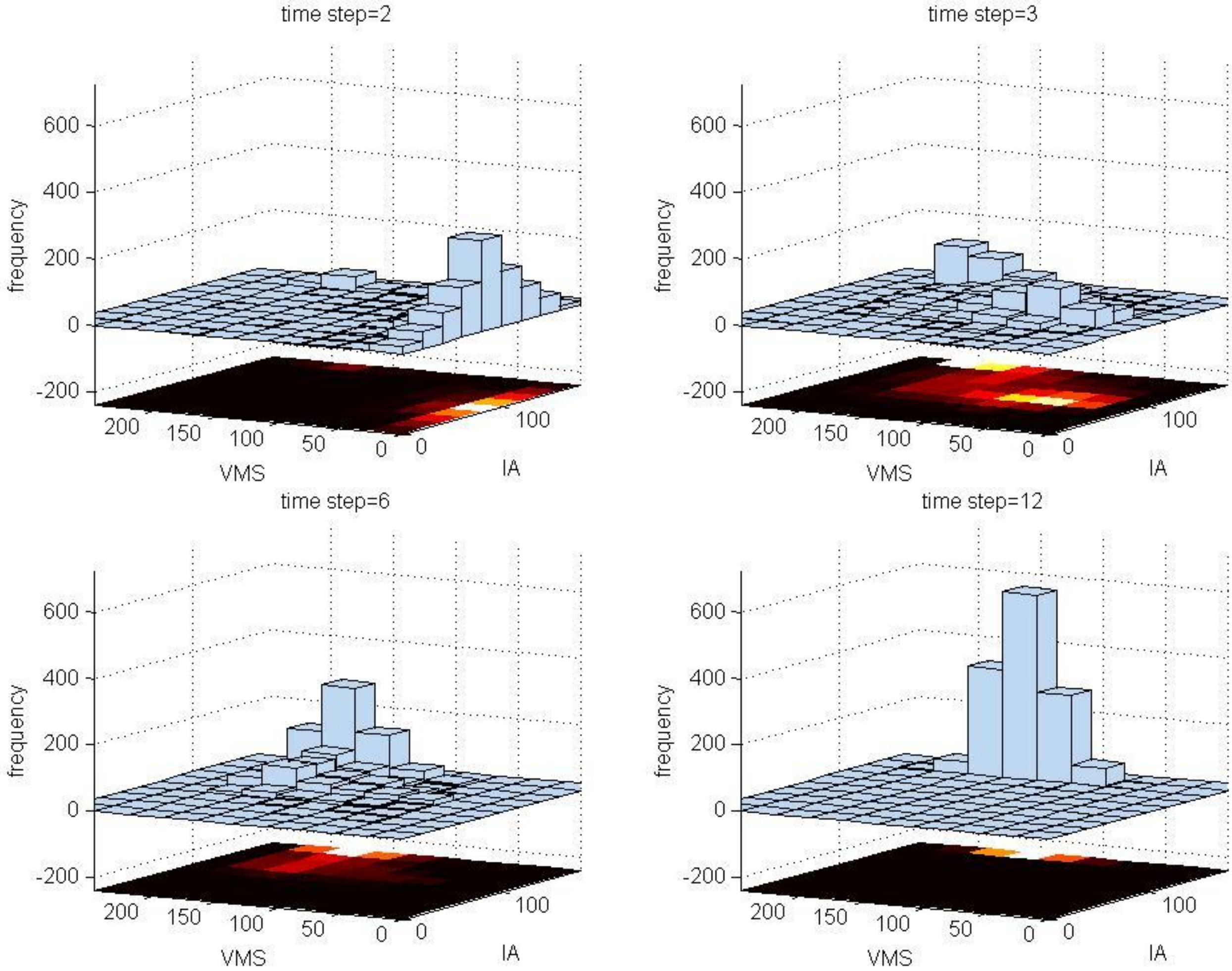

Fig. 10. The 3-D histogram graph of the frequency of strain (VMS) at certain impact angle. Below each histogram is the pseudo-color map of the frequency.

The plastic strain (PS) is influenced by multiple impact conditions, such as impact angle, material property, impact proportion, etc. The explicit relation of PS to the influencing factor is useful while studying the properties of non-Newtonian materials. Stochastic analysis is used to find out such relationships. Fig. 10 shows the 3-D histogram graph of the frequency of strain (VMS) at certain impact angle, which indicates that they fit two-dimensional normal distribution. Further mathematical equations are discussed in [13].

All the figures presented in the paper are result from LS-DYNA (V.971) and Matlab (R 2008a)

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V. Conclusion

The coupled Cauchy conservation equation (1) and P-T/T equation (2) is used to describe the impact problem of auto-crash safety. Finite element method is used to solve such PDE system, where the Lagrange-Runge-Kutta scheme is adopted on each node computing.

The stochastic analysis with standard deviation from the normal impact condition is given. The presumed sufficiently smoothed condition is in the frame of law of nature. Histogram graph of frequency is used to study the tendency of impact angle, stress, and strain. One impact condition, the impact angle, is analyzed to find out its influence on plastic strain and stress. Properties of normal distribution have been discovered in this paper.

Apart from the hard protection (solid crash barrier) and skin contact protection, we introduced the soft-solid non-Newtonian protection concept with the simulated statistical boundary condition. The mathematical simulation of the accurate prediction shows the impact defense scheme. The passive safety analysis is in a more realistic testing environment by use of the stochastic analysis.

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