Probabilistic Analysis of a Desalination Unit with Nine Failure Categories

S M Rizwan, N Padmavathi, G Taneja, A G Mathew and Ali Mohammed Al-Balushi

***Abstract*— The paper presents a probabilistic analysis of a desalination unit. Multi stage flash desalination process is being used for water treatment/purification. The desalination plant operates round the clock and many evaporators are in operation for water production. For the purpose of analysis, evaporator number 7 has been identified and seven years maintenance data of this unit have been collected. Any major failure/annual maintenance brings the unit to a complete halt and stops the production. The unit fails due to any one of the nine types of failure as categorized in the data. The probabilistic analysis of the plant have been carried out and as a result, measures of unit effectiveness such as mean time to unit failure and unit availability are estimated numerically by using semi-Markov processes and regenerative point techniques.**

***Index Terms*— Desalination plant, failures, repairs, Semi – Markov, regenerative processes.**

I. Introduction

Standby systems are commonly used in industries and therefore, researchers have spent a great deal of efforts in analyzing such systems to get the optimized reliability results which are useful for effective equipment/plant maintenance. Gopalan and Muralidhar [3] wrote about a repairable system subject to online preventive maintenance and thereafter many have contributed further in this area due to the potential application to industries [1][4]&[5]. In all these papers, various situations have been considered for system analysis, such as stochastic analysis of a repairable system with three units and repair facilities, system analysis with perfect repair at partial or complete failure, warm standby system analysis with various types of repair, and recently Bhupender & Gulshan [2] analyzed a two unit PLC hot standby system based on master-slave concept and two types of repair facilities.

A potential application of the reliability concepts could also be explored in terms of developing a specific probabilistic model for a desalination unit and thereby achieving some reliability measures of the plant/unit effectiveness which in turn are meaningful in understanding the plant/unit performance. Thus, the present paper is an attempt to analyze a desalination unit probabilistically and some optimized reliability measures of interest are obtained. The innovative part of this research is the development of the robust model embedding the real failure situations as depicted in the data for analysis, and the real values of various failure rates and probabilities are being used for achieving the final results. For this purpose, a desalination plant in Oman which operates on Multi Stage Flash Desalination process is identified. The desalination plant consists of many evaporators; it operates round the clock for water purification and ensures the continuous production of water for domestic usage. Any major failure/annual maintenance brings the particular unit to a complete halt and the production stops during the fixing back period. Seven years maintenance data of a unit (i.e., evaporator number 7) have been collected. The data reveals that the unit fails due to any one of the nine types of failure viz., instrumental repairable, instrumental replaceable, instrumental serviceable, electrical repairable, electrical replaceable, electrical serviceable, mechanical repairable, mechanical replaceable and mechanical serviceable type. Using the data, the following values are estimated:

Estimated value of failure rate (λ) = 0.00002714 per hour

Estimated value of instrumental repair rate [α_1 (say)] = 0.09688 per hour

Estimated value of electrical repair rate [β_1 (say)] = 0.26498 per hour

Estimated value of mechanical repair rate [γ_1 (say)] = 0.02951 per hour

Estimated value of instrumental replacement rate [α_2 (say)] = 0.05186 per hour

Estimated value of electrical replacement rate [β_2 (say)] = 0.55165 per hour

Estimated value of mechanical replacement rate [γ_2 (say)] = 0.05607 per hour

Estimated value of instrumental service rate [α_3 (say)] = 0.1228 per hour

Estimated value of electrical service rate [β_3 (say)] = 0.2679 per hour

Estimated value of mechanical service rate [γ_3 (say)] = 0.5714 per hour

Probability that the unit suffers with Instrumental failure which is repairable (p_1) = 0.1936

Probability that the unit suffers with Instrumental failure which is replaceable (p_2) = 0.1613

Probability that the unit suffers with Instrumental failure which is serviceable (p_3) = 0.1290

Probability that the unit suffers with Electrical failure which is repairable (p_4) = 0.2258

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Probability that the unit suffers with Electrical failure which is replaceable (p_5) = 0.0323
Probability that the unit suffers with Electrical failure which is serviceable (p_6) = 0.0645
Probability that the unit suffers with Mechanical failure which is repairable (p_7) = 0.0967
Probability that the unit suffers with Mechanical failure which is replaceable (p_8) = 0.0645
Probability that the unit suffers with Mechanical failure which is serviceable (p_9) = 0.0323

The unit/evaporator number 7 is analyzed probabilistically by using semi-Markov processes and regenerative point techniques. The mean times to unit failure and unit availability are estimated numerically.

II. Model Description And Assumptions

1. The unit is initially operative at state 0 and transits probabilistically depending on the type of failure to any of the nine states 1 to 9 with probabilities p_1, p_2, p_3, p_4, p_5, p_6, p_7, p_8 and p_9 respectively as shown in Fig.1.
2. All failure times are assumed to have exponential distribution with failure rate (λ) whereas the repair times have general distributions.
3. After each repair/replacement/servicing at state's 1 to 9, the unit works as good as new.
4. Breakdowns are self announcing.
5. The unit is brought into operation as soon as possible.

III. Notations Used

O Operative unit
λ Constant failure rate of the unit
p_1 Probability of failure that is instrumental repairable
p_2 Probability of failure that is instrumental replaceable
p_3 Probability of failure that is instrumental serviceable
p_4 Probability of failure that is electrical repairable
p_5 Probability of failure that is electrical replaceable
p_6 Probability of failure that is electrical serviceable
p_7 Probability of failure that is mechanical repairable
p_8 Probability of failure that is mechanical replaceable
p_9 Probability of failure that is mechanical Serviceable
IFr Instrumental failure mode of repairable type
IFrp Instrumental failure mode of replaceable type
IFs Instrumental failure mode of serviceable type
EFr Electrical failure mode of repairable type
EFrp Electrical failure mode of replaceable type
EFs Electrical failure mode of serviceable type
MFr Mechanical failure mode of repairable type
MFrp Mechanical failure mode of replaceable type
MFs Mechanical failure mode of serviceable type
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$p_{ij}, Q_{ij}(t)$ Probability density function (p.d.f.), cumulative distribution function (c.d.f.) of first passage time from a regenerative state i to j or to a failed state j in (0, t]
$\varphi_i(t)$ c.d.f. of first passage time from a regenerative state i to a failed state j
* Laplace Transforms (LT), i.e., for any f(t) and g(t); $f(t) * g(t) = \int_0^t f(t-u)g(u)du$
f(t) p.d.f. of the failure time
$g_{1i}(t), g_{2i}(t), g_{3i}(t)$ p.d.f. of the time to repair/replace/service when the failure is instrumental
$g_{1e}(t), g_{2e}(t), g_{3e}(t)$ p.d.f. of the time to repair/replace/service when the failure is electrical
$g_{1m}(t), g_{2m}(t), g_{3m}(t)$ p.d.f. of the time to repair/replace/service when the failure is mechanical
$G_{1i}(t), G_{2i}(t), G_{3i}(t)$ c.d.f. of the time to repair/replace/service when the failure is instrumental
$G_{1e}(t), G_{2e}(t), G_{3e}(t)$ c.d.f. of the time to repair/replace/service when the failure is electrical
$G_{1m}(t), G_{2m}(t), G_{3m}(t)$ c.d.f. of the time to repair/replace/service when the failure is mechanical

IV. Transition Probabilities And Mean Sojourn Times

A state transition diagram showing the possible states of transition of the plant is shown in Fig. 1. The epochs of entry into states 0, 1, 2, 3, 4, 6, 7, 8 and 9 are regenerative points and hence these states are regenerative states. The transition probabilities are given by:

$$dQ_{01} = p_1\lambda e^{-\lambda t}dt\ ,\ dQ_{02} = p_2\lambda e^{-\lambda t}dt\ ,$$
$$dQ_{03} = p_3\lambda e^{-\lambda t}dt\ ,\ dQ_{04} = p_4\lambda e^{-\lambda t}dt\ ,$$
$$dQ_{05} = p_5\lambda e^{-\lambda t}dt\ , dQ_{06} = p_6\lambda e^{-\lambda t}dt\ ,$$
$$dQ_{07} = p_7\lambda e^{-\lambda t}dt,\quad dQ_{08} = p_8\lambda e^{-\lambda t}dt,$$
$$dQ_{09} = p_9\lambda e^{-\lambda t}dt\ , dQ_{10} = g_{1i}(t)dt\ ,$$
$$dQ_{20} = g_{2i}(t)dt\ ,\ dQ_{30} = g_{3i}(t)dt\ ,$$
$$dQ_{40} = g_{1e}(t)dt\ ,\ dQ_{50} = g_{2e}(t)dt\ ,$$
$$dQ_{60} = g_{3e}(t)dt\ ,\ dQ_{70} = g_{1m}(t)dt$$
$$dQ_{80} = g_{2m}(t)dt,\ dQ_{90} = g_{3m}(t)dt \qquad (1\text{-}18)$$

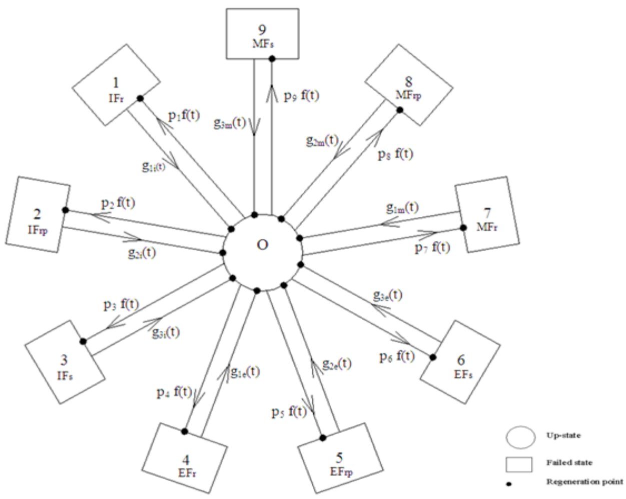

Fig. 1. State transition diagram.

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The non-zero elements p_{ij} are given below:

$p_{01} = p_1, p_{02} = p_2, p_{03} = p_3, p_{04} = p_4, p_{05} = p_5,$

$p_{06} = p_6, p_{07} = p_7, p_{08} = p_8, p_{09} = p_9,$

$p_{10} = 1, p_{20} = 1, p_{30} = 1, p_{40} = 1, p_{50} = 1, p_{60} = 1,$

$p_{70} = 1, p_{80} = 1, p_{90} = 1$ (19-37)

By these transition probabilities it is also verified that:

$p_{01} + p_{02} + p_{03} + \ldots + p_{09} = 1$ (38)

$p_{i0} = 1$ for $i = 1, 2, \ldots, 9$ (39)

The mean sojourn time (μ_i) in the regenerative state '*i*' is defined as the time of stay in that state before transition to any other state. If *T* denotes the sojourn time in the regenerative state '*i*', then:

$\mu_i = E(T) = \Pr[T > t]$

Thus $\mu_0 = \int_0^\infty e^{-\lambda t} dt = \frac{1}{\lambda}$, $\mu_1 = \int_0^\infty \overline{I_r(t)} dt = \frac{1}{\alpha_1}$,

$\mu_2 = \frac{1}{\alpha_2}; \mu_3 = \frac{1}{\alpha_3}; \mu_4 = \frac{1}{\beta_1}; \mu_5 = \frac{1}{\beta_2}; \mu_6 = \frac{1}{\beta_3}; \mu_7 = \frac{1}{\gamma_1};$

$\mu_8 = \frac{1}{\gamma_2}; \mu_9 = \frac{1}{\gamma_3}$ (40-49)

The unconditional mean time taken by the system to transit for any regenerative state '*j*' when it (time) is counted from the epoch of entrance into state '*i*' is mathematically stated as:

$m_{ij} = \int_0^\infty t dQ_{ij}(t) = -q_{ij}^{*\prime}(0)$ (50)

Thus, $m_{01} + m_{02} + m_{03} + \ldots + m_{09} = \mu_0$ (51)

$m_{10} = \mu_1, m_{20} = \mu_2, m_{30} = \mu_3, m_{40} = \mu_4, m_{50} = \mu_5, m_{60} = \mu_6,$

$m_{70} = \mu_7, m_{80} = \mu_8, m_{90} = \mu_9$ (52-61)

V. The Mathematical Analysis

A. Mean Time to Unit Failure

Regarding the failed states as absorbing states and employing the arguments used for regenerative processes, the following recursive relation for $\phi_i(t)$ is obtained:

$\phi_0(t) = Q_{01}(t) + Q_{02}(t) + Q_{03}(t) + Q_{04}(t) + Q_{05}(t) + Q_{06}(t) + Q_{07}(t) + Q_{08}(t) + Q_{09}(t)$ (62)

Solving the above equation for $\phi_o^{**}(s)$ by taking Laplace Stieltje's Transforms and using the determinant method, the following is obtained:

$\phi_0^{**}(s) = \frac{N(s)}{D(s)}$ (63)

where $N(s) = \phi_o^{**}(s)$ and $D(s) = 1$

Now the mean time to system failure (MTSF) when the unit started at the beginning of state 0, is:

$\text{MTSF} = \lim_{s \to 0} \frac{1 - \varphi_0^{**}(s)}{s} = \frac{N}{D}$ (64)

where $N = \mu_0$ and $D = 1$ (65)

B. Availability Analysis of Unit of the Plant

Using the probabilistic arguments and defining $A_i(t)$ as the probability of unit entering into upstate at instant t, given that the unit entered in regenerative state i at t=0, the following recursive relations are obtained:

$A_0(t) = M_0(t) + q_{01}(t) © A_1(t) + q_{02}(t) © A_2(t) + q_{03}(t) © A_3(t) + q_{04}(t) © A_4(t) + q_{05}(t) © A_5(t) + q_{06}(t) © A_6(t) + q_{07}(t) © A_7(t) + q_{08}(t) © A_8(t) + q_{09}(t) © A_9(t)$

$A_1(t) = q_{10}(t) © A_0(t)$

$A_2(t) = q_{20}(t) © A_0(t)$

$A_3(t) = q_{30}(t) © A_0(t)$

$A_4(t) = q_{40}(t) © A_0(t)$

$A_5(t) = q_{50}(t) © A_0(t)$

$A_6(t) = q_{60}(t) © A_0(t)$

$A_7(t) = q_{70}(t) © A_0(t)$

$A_8(t) = q_{80}(t) © A_0(t)$

$A_9(t) = q_{90}(t) © A_0(t)$ (66-76)

Where $M_0(t) = e^{-\lambda t}$

Taking Laplace Transforms of the above equations and solving them for $A_0^*(s)$, the following is obtained:

$A_0^*(s) = \frac{N_1(s)}{D_1(s)}$ (77)

Therefore, the steady-state availability of the unit is given by

$A_0 = \lim_{s \to 0} sA_0^*(s) = \frac{N_1}{D_1}$ (78)

Where $N_1 = \mu_0$, and $D_1 = \mu_o + \sum_{i=1}^{9} p_i \mu_i$

VI. Particular Case

For the particular case, it is assumed that the failure and repair rates are exponentially distributed and therefore the following have been assumed:

$g_{1i}(t) = \alpha_1 e^{-\alpha_1 t}$, $g_{2i}(t) = \alpha_2 e^{-\alpha_2 t}$, $g_{3i}(t) = \alpha_1 e^{-\alpha_1 t}$

$g_{1e}(t) = \beta_1 e^{-\beta_1 t}$, $g_{2e}(t) = \beta_2 e^{-\beta_2 t}$, $g_{3e}(t) = \beta_3 e^{-\beta_3 t}$

$g_{1m}(t) = \gamma_1 e^{-\gamma_1 t}$, $g_{2m}(t) = \gamma_2 e^{-\gamma_2 t}$, $g_{3m}(t) = \gamma_3 e^{-\gamma_3 t}$,

$p_{01} = p_1$, $p_{02} = p_2$, $p_{03} = p_3$, $p_{04} = p_4$, $p_{05} = p_5$, $p_{06} = p_6$,

$p_{07} = p_7$, $p_{08} = p_8$, $p_{09} = p_9$

and $p_{i0} = 1$ for i = 1, 2, 3, …, 9.

$\mu_0 = ?\frac{1}{\lambda}$ $\mu_1 = ?\frac{1}{\alpha_1}$ $\mu_2 = ?\frac{1}{\alpha_2}$ $\mu_3 = ?\frac{1}{\alpha_3}$

$\mu_4 = ?\frac{1}{\beta_1}$ $\mu_5 = ?\frac{1}{\beta_2}$ $\mu_6 = ?\frac{1}{\beta_3}$

$\mu_7 = ?\frac{1}{\gamma_1}$ $\mu_8 = ?\frac{1}{\gamma_2}$ $\mu_9 = ?\frac{1}{\gamma_3}$

Using the data as summarized in section 1 for the above particular case and expressions obtained in (65) & (78), the following values of unit effectiveness are estimated:

Mean time to unit/evaporator number 7 failure = 36845 hours.

Availability of the unit/evaporator number 7 (A_0) = 0.999585.

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VII. CONCLUSIONS

The model incorporates the breakdown situations of the unit/evaporator number 7 of the desalination plant and offers a scientific basis for probabilistic maintenance analysis. It has been achieved that the expected time for which the unit/evaporator number 7 is in operation before it completely fails is about 36845 hours. Also, the probability that the unit/evaporator number 7 will be able to operate within the tolerances for a specified period of time is 0.999585 which certainly would meet the annual maintenance norms fixed for the plant.

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