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# On Sensitivity of EWMA Control Chart for Monitoring Process Dispersion

Saddam Akber Abbasi *

*Abstract*—**Control chart is the most important Statistical Process Control (SPC) tool used to monitor reliability and performance of manufacturing processes. Variability EWMA charts are widely used for the detection of small shifts in process dispersion. For ease in computation all the variability EWMA charts proposed so far are based on asymptotic nature of control limits. It has been shown in this study that quick detection of initial out-of-control conditions can be achieved by using exact or time varying control limits. Moreover the effect of fast initial response (FIR) feature, to further increase the sensitivity of variability EWMA charts for detecting process shifts, has not been studied so far in SPC literature. It has been observed that FIR based variability EWMA chart is more sensitive to detect process shifts than the variability charts based on time varying or asymptotic control limits.**

***Keywords: EWMA; Process Variability, Fast Initial Response, Monte Carlo Simulations; Run Length Distribution***

## 1 Introduction

Control chart introduced by Walter A. Shewhart in 1920's, is the most important Statistical Process Control (SPC) tool used to monitor reliability and performance of manufacturing processes. The basic purpose of implementing control chart procedures is to detect abnormal variations in the process (location & scale) parameters. Although firstly proposed for manufacturing industry, control charts have now been applied in a wide variety of disciplines, such as in nuclear engineering [8], health care [26], education [25] , analytical laboratories [15] etc. Shewhart type control charts are mostly used, process location is usually monitored by $\overline{X}$ chart while for monitoring process dispersion $R$ or $S$ charts are widely used. Research has shown that, due to memoryless nature of Shewhart control chars, they do not perform well for the detection of small and moderate shifts in process parameters. When quick detection of small shifts is desirable, cumulative sum (CUSUM) and exponentially weighted moving average (EWMA) charts act as superior alternatives to Shewhart charts.

Since the introduction of EWMA chart by Roberts [20], many researchers have examined these charts from different perspectives. To mention but a few of these see for example [11, 17, 23, 3, 14, 2, 22] and references therein. In contrast to Shewhart type charts which are only based on information of the current observations, EWMA charts make use of information about current as well as historical observations by adopting a varying weight scheme, assigning highest weight to the most recent observations and the weights decreasing exponentially for less recent observations. This helps in earlier detection of small shifts in process (location and scale) parameters (for details see [16]). Monitoring process variability using EWMA chart has also attracted the attention of many researchers in past, some important contributions are [27, 7, 6, 12, 24] and [4]. Recently Shu and Jiang [22] proposed a new EWMA chart for monitoring process dispersion namely NEWMA chart and showed that NEWMA chart outperformed Crowder and Hamilton [6] proposed variability EWMA chart in terms of average run length.

All the variability EWMA schemes proposed so far are based on asymptotic nature of control limits. Ease of computation has been reported as the main reason for using asymptotic limits but the use of these limits make the EWMA chart insensitive to start up quality problems. It should be noted that exact control limits of EWMA charts should be time varying approaching asymptotic limits as time increases (see [16]). When the process is initially out of control, it is extremely important to detect the sources of these out-of-control conditions as early as possible for the implementation of corrective actions at an early stage. This can be achieved by using exact limits (that are dependent on time) instead of asymptotic control limits. Further increase in the sensitivity of time varying variability EWMA scheme can be achieved by using FIR feature, which has not been investigated so far in SPC literature. This study aims at investigating variability EWMA chart performance using asymptotic, time varying and FIR based control limits. The comparison has been made on the basis of run length characteristics such as average run length (ARL), median run length

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(MDRL) and standard deviation of run length distribution (SDRL).

For the purpose of investigating the affect of time varying control limits and FIR feature on variability EWMA chart performance, I select the most efficient NEWMA chart recently proposed by Shu and Jiang [22] in Journal of Quality Technology. The design of NEWMA chart is established using time varying and FIR based control limits and their performance is compared with asymptotic NEWMA chart. The rest of the study is organized as follows. The following section briefly introduces structure of NEWMA chart and further presents the design of NEWMA chart using time varying control limits (TNEWMA chart). The next section compares run length characteristics of NEWMA and TNEWMA charts. The effect of FIR feature is then investigated and compared to asymptotic and time varying EWMA schemes. To get a better insight on the run length distribution of these charts, run length curves are also presented. The paper finally ends with concluding remarks.

## 2 TNEWMA Chart

In this section we briefly describe the structure of NEWMA chart as was proposed by Shu and Jiang [22]. Further the time varying control limit structure of NEWMA chart is then established.

Assume the quality variable of interest say $X$ follows normal distribution with mean $\mu_t$ and variance $\sigma_t^2$ (i.e $X \sim N(\mu_t, \sigma_t^2)$). Let $S_t^2$ represents the sample variance and $\delta_t$ represents the ratio of process standard deviation $\sigma_t$ and its true value $\sigma_0$ at time period $t$ (i.e $\delta_t = \sigma_t/\sigma_0$). Suppose $Y_t = ln(S_t^2/\sigma_0^2)$, for an in control process i.e. $\sigma_t = \sigma_0$, $Y_t$ is approximately normally distributed with mean $\mu_Y$ and variance $\sigma_Y^2$ where

$$\mu_Y = ln(\delta_t^2) - \frac{1}{n-1} - \frac{1}{3(n-1)^2} + \frac{2}{15(n-1)^4} \qquad (1)$$

and

$$\sigma_Y^2 = \frac{2}{n-1} + \frac{2}{(n-1)^2} + \frac{4}{3(n-1)^3} - \frac{16}{15(n-1)^5} \qquad (2)$$

Note that when the process is in control the statistic

$$Z_t = \frac{Y_t - \mu_Y|\sigma_t = \sigma_0}{\sigma_Y} \qquad (3)$$

is exactly a standard normal variate. When the process is out of control, $Z_t \sim N(\gamma_t, 1)$, where $\gamma_t = \left(ln(\sigma_t^2/\sigma_0^2)\right)/\sigma_Y$ ([22]). The EWMA statistic for monitoring process variability used by [22] is based on resetting $Z_t$ to zero whenever its value becomes negative i.e $Z_t^+ = max(0, Z_t)$. NEWMA chart is based on plotting the EWMA statistic

$$W_t = \lambda\left(Z_t^+ - \frac{1}{2\pi}\right) + (1-\lambda)W_{t-1} \qquad (4)$$

and gives an out of control signal whenever $W_t > UCL_a$ where

$$UCL_a = L_a\sqrt{\frac{\lambda}{2-\lambda}}\sigma_{Z_t^+} \qquad (5)$$

Barr and Sherrill [1] showed that

$$\sigma^2_{Z_t^+} = \left(\frac{1}{2} - \frac{1}{2\pi}\right) \qquad (6)$$

We will see that exact variance of $W_t$ is time varying and hence the exact control limit should be dependent on time approaching $UCL_a$ as $t \to \infty$. Define $Z_t' = Z_t^+ - \frac{1}{2\pi}$, we can write $W_t$ as

$$W_t = \lambda Z_t' + (1-\lambda)W_{t-1} \qquad (7)$$

By continuous substitution of $W_{t-i}, i = 1, 2, \ldots, t$; the EWMA statistic $W_t$ can be written as (see [20] and [16]):

$$W_t = \lambda\sum_{i=0}^{t-1}(1-\lambda)^i Z'_{t-i} + (1-\lambda)^t W_0 \qquad (8)$$

Taking variance on both sides of Equation 4 we obtain

$$Var(W_t) = \lambda^2\sum_{i=0}^{t-1}(1-\lambda)^{2i}Var(Z'_{t-i}) + (1-\lambda)^{2t}Var(W_0) \qquad (9)$$

For independent random observations $Z_t'$, $var(Z_t') = var(Z'_{t-i}) = \sigma^2_{Z_t^+}$. After a bit of simplification, we have

$$Var(W_t) = \sigma^2_{Z_t^+}\left(\lambda^2\left[\frac{1-(1-\lambda)^{2t}}{1-(1-\lambda)^2}\right]\right) \qquad (10)$$

which further simplifies to

$$Var(W_t) = \sigma^2_{Z_t^+}\left(\left(\frac{\lambda}{2-\lambda}\right)\left[1-(1-\lambda)^{2t}\right]\right) \qquad (11)$$

For the rest of study we will refer to the variability EWMA chart based on exact variance of $W_t$ given in Equation (11) as TNEWMA chart. TNEWMA chart gives an out of control signal whenever $W_t > UCL_t$, where

$$UCL_t = L_t\sqrt{\frac{\lambda[1-(1-\lambda)^{2t}]}{2-\lambda}}\sigma_{Z_t^+} \qquad (12)$$

$UCL_t$ converges to $UCL_a$ as $t \to \infty$, the convergence being slow for smaller values of $\lambda$.

## 3 Comparison of Run Length Characteristics of NEWMA and TNEWMA Charts

To evaluate the performance of control charts, average run length (ARL), which is the mean of run length distribution, is the most important and widely used measure.

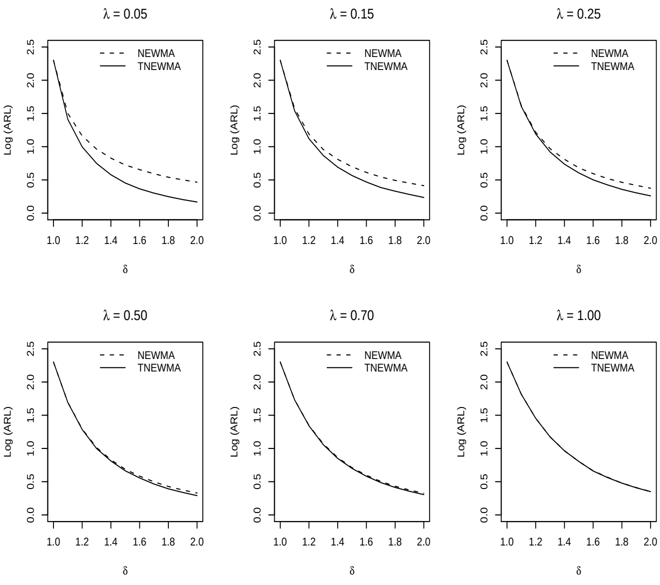


Figure 1: ARL comparison of NEWMA and TNEWMA charts for different values of $\lambda$ when $\text{ARL}_0 = 200$

Table 1: Run length characteristics of NEWMA chart when $ARL_0 = 200$

| | | $\lambda$ | | | | |
|---|---|---|---|---|---|---|
| | | 0.05 | 0.15 | 0.3 | 0.7 | 1 |
| Delta | $L$ | *1.569* | *2.148* | *2.432* | *2.650* | *2.693* |
| 1.0 | **ARL** | **199.69** | **200.24** | **201.39** | **199.82** | **199.52** |
| | MDRL | 136.00 | 139.00 | 142.00 | 139.00 | 137.00 |
| | SDRL | 197.62 | 200.98 | 203.09 | 196.87 | 202.39 |
| 1.1 | **ARL** | **31.68** | **37.69** | **43.26** | **54.11** | **65.19** |
| | MDRL | 24.00 | 28.00 | 31.00 | 38.00 | 46.00 |
| | SDRL | 26.26 | 34.30 | 41.50 | 55.14 | 64.62 |
| 1.2 | **ARL** | **14.52** | **15.48** | **17.30** | **22.22** | **28.44** |
| | MDRL | 12.00 | 12.00 | 13.00 | 16.00 | 20.00 |
| | SDRL | 10.05 | 12.12 | 15.41 | 21.10 | 27.88 |
| 1.3 | **ARL** | **9.21** | **9.07** | **9.57** | **11.73** | **14.97** |
| | MDRL | 8.00 | 7.00 | 7.00 | 8.00 | 10.00 |
| | SDRL | 5.49 | 6.43 | 7.55 | 11.04 | 14.43 |
| 1.4 | **ARL** | **6.72** | **6.45** | **6.47** | **7.31** | **9.23** |
| | MDRL | 6.00 | 5.00 | 5.00 | 5.00 | 7.00 |
| | SDRL | 3.67 | 4.09 | 4.71 | 6.43 | 8.85 |
| 1.5 | **ARL** | **5.30** | **4.98** | **4.78** | **5.15** | **6.38** |
| | MDRL | 5.00 | 4.00 | 4.00 | 4.00 | 5.00 |
| | SDRL | 2.69 | 2.89 | 3.26 | 4.32 | 5.93 |
| 1.6 | **ARL** | **4.52** | **4.10** | **3.87** | **3.95** | **4.61** |
| | MDRL | 4.00 | 4.00 | 3.00 | 3.00 | 3.00 |
| | SDRL | 2.15 | 2.25 | 2.42 | 3.10 | 4.03 |
| 1.7 | **ARL** | **3.91** | **3.50** | **3.27** | **3.19** | **3.69** |
| | MDRL | 4.00 | 3.00 | 3.00 | 2.00 | 3.00 |
| | SDRL | 1.74 | 1.81 | 1.95 | 2.40 | 3.16 |
| 1.8 | **ARL** | **3.47** | **3.11** | **2.85** | **2.70** | **3.01** |
| | MDRL | 3.00 | 3.00 | 2.00 | 2.00 | 2.00 |
| | SDRL | 1.46 | 1.52 | 1.63 | 1.90 | 2.48 |
| 1.9 | **ARL** | **3.17** | **2.83** | **2.51** | **2.37** | **2.58** |
| | MDRL | 3.00 | 3.00 | 2.00 | 2.00 | 2.00 |
| | SDRL | 1.30 | 1.34 | 1.38 | 1.62 | 2.00 |
| 2.0 | **ARL** | **2.92** | **2.59** | **2.29** | **2.12** | **2.24** |
| | MDRL | 3.00 | 2.00 | 2.00 | 2.00 | 2.00 |
| | SDRL | 1.13 | 1.18 | 1.21 | 1.41 | 1.69 |

ARL indicates the average number of observations after which an out of control signal is detected by a control chart. In this study Monte Carlo simulation approach with 10,000 iterations is used to approximate run length distributions of the NEWMA and TNEWMA charts following the methods of [11, 13, 18, 28]. Note that [9] and [21] indicates that even 5000 replications are enough for finding ARL's in many control chart settings with in an acceptable error rate. To get a better insight of the performance of proposed charts, median and standard deviation of run length distribution are also provided. The summary of the run length characteristics of NEWMA and TNEWMA charts is reported in Tables 1 and 2 for different values of smoothing parameter $\lambda$. In the following tables ARL denotes average run length, SDRL denotes standard deviation of the run length distribution and MDRL denotes median of the run length distribution. Control chart multiples $L_a$ and $L_t$ are so chosen as to give the same in control average run length of 200 (i.e $ARL_0 = 200$) for both the charts. The results in Tables 1 and 2 indicate that for smaller values of $\lambda$ (which is most popular choice for EWMA charts), out-of-control ARL ($ARL_1$) of the TNEWMA chart is significantly lower than $ARL_1$ of NEWMA chart, see for example $ARL_1 = 9.93$ for TNEWMA chart using $\lambda = 0.05$ and $\delta = 1.2$ while for NEWMA chart $ARL_1 = 14.52$ for same values of $\lambda$ and $\delta$. It indicates that TNEWMA chart requires on average nearly five less observations as compared to NEWMA chart to detect a shift of $1.2\sigma$ in process variability when $\lambda = 0.05$. MDRL of the TNEWMA chart is also lower than MDRL of the NEWMA chart while there is a slight increase in SDRL of the TNEWMA chart as compared to NEWMA chart for lower values of $\lambda$ and $\delta$. Figure 1 presents ARL comparison of NEWMA and TNEWMA charts for some choices of $\lambda$. In each plot shift $\delta$ is plotted on horizontal axis while ARL is plotted on vertical axis in logarithmic scale for better visual comparison. The effect of using time varying control limits can be clearly seen from Figure 1, particularly for smaller values of $\lambda$. As expected, ARL of TNEWMA chart starts to converge towards ARL of NEWMA chart with an increase in $\lambda$. At $\lambda = 1$, ARL performance of both the charts is nearly same. Moreover Figure 2 shows percentage decrease in $ARL_1$ of TNEWMA chart as compared to NEWMA chart for certain choices of $\lambda$ and $\delta$. We can see that the difference in $ARL_1$ of both the charts is bigger for smaller values of $\lambda$ and higher values of $\delta$. The difference tends to reduce as $\lambda$ increases and $\delta$ decreases. Hence the use of exact control limits also improves variability EWMA chart performance for detecting shifts of higher magnitude.

## 4 Effect of Fast Initial Response on Variability EWMA Chart

We have seen in the previous section that the use of time varying control limits as compared to asymptotic limits significantly improves out of control run length behavior of EWMA charts used for the purpose of monitoring process variability. Further increase in the sensitivity of EWMA chart to detect shifts in variability can be achieved by using FIR feature. The FIR feature was introduced firstly by [10] for the CUSUM charts to detect out of control signals more quickly at process startup by assigning some non zero constant to the starting value. [11] proposed the idea of applying the FIR feature to

Table 2: Run length characteristics of TNEWMA chart when $ARL_0 = 200$

| | | λ | | | | |
|---|---|---|---|---|---|---|
| | | 0.05 | 0.15 | 0.3 | 0.7 | 1 |
| Delta | $L$ | 1.649 | 2.164 | 2.440 | 2.652 | 2.693 |
| 1.0 | **ARL** | **199.85** | **202.30** | **201.37** | **201.63** | **199.73** |
| | MDRL | 124.00 | 136.50 | 137.00 | 141.00 | 140.00 |
| | SDRL | 233.87 | 212.73 | 202.42 | 199.74 | 194.83 |
| 1.1 | **ARL** | **25.80** | **34.71** | **42.42** | **54.01** | **65.18** |
| | MDRL | 16.00 | 25.00 | 30.00 | 38.00 | 45.00 |
| | SDRL | 28.83 | 34.31 | 41.58 | 53.70 | 63.66 |
| 1.2 | **ARL** | **9.93** | **13.18** | **16.65** | **21.96** | **28.43** |
| | MDRL | 6.00 | 10.00 | 12.00 | 15.00 | 20.00 |
| | SDRL | 10.36 | 12.53 | 15.26 | 21.30 | 28.12 |
| 1.3 | **ARL** | **5.61** | **7.37** | **8.66** | **11.39** | **14.94** |
| | MDRL | 4.00 | 6.00 | 6.00 | 8.00 | 11.00 |
| | SDRL | 5.64 | 6.54 | 7.73 | 10.90 | 14.40 |
| 1.4 | **ARL** | **3.78** | **4.92** | **5.67** | **7.09** | **9.23** |
| | MDRL | 3.00 | 4.00 | 4.00 | 5.00 | 7.00 |
| | SDRL | 3.49 | 4.21 | 4.69 | 6.26 | 8.67 |
| 1.5 | **ARL** | **2.84** | **3.68** | **4.14** | **5.01** | **6.38** |
| | MDRL | 2.00 | 3.00 | 3.00 | 4.00 | 5.00 |
| | SDRL | 2.49 | 3.01 | 3.28 | 4.40 | 5.82 |
| 1.6 | **ARL** | **2.32** | **2.94** | **3.27** | **3.82** | **4.58** |
| | MDRL | 2.00 | 2.00 | 3.00 | 3.00 | 3.00 |
| | SDRL | 1.90 | 2.25 | 2.48 | 3.16 | 4.17 |
| 1.7 | **ARL** | **2.00** | **2.43** | **2.68** | **3.07** | **3.65** |
| | MDRL | 1.00 | 2.00 | 2.00 | 2.00 | 3.00 |
| | SDRL | 1.50 | 1.81 | 1.95 | 2.41 | 3.08 |
| 1.8 | **ARL** | **1.77** | **2.14** | **2.26** | **2.59** | **3.01** |
| | MDRL | 1.00 | 2.00 | 2.00 | 2.00 | 2.00 |
| | SDRL | 1.23 | 1.53 | 1.64 | 1.92 | 2.47 |
| 1.9 | **ARL** | **1.60** | **1.91** | **2.03** | **2.27** | **2.56** |
| | MDRL | 1.00 | 1.00 | 2.00 | 2.00 | 2.00 |
| | SDRL | 1.04 | 1.28 | 1.35 | 1.60 | 2.06 |
| 2.0 | **ARL** | **1.47** | **1.72** | **1.84** | **2.02** | **2.24** |
| | MDRL | 1.00 | 1.00 | 1.00 | 2.00 | 2.00 |
| | SDRL | 0.91 | 1.09 | 1.17 | 1.33 | 1.64 |

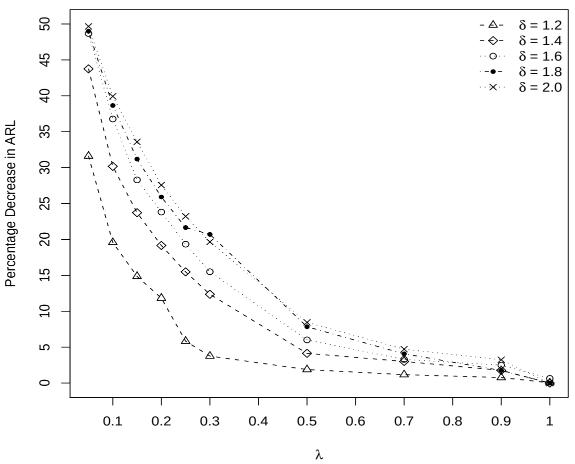


Figure 2: Percentage decrease in out-of-control ARL of TNEWMA chart as compared to NEWMA chart when $ARL_0 = 200$

Table 3: Run Length Characteristics of FNEWMA chart when $ARL_0 = 200$

| | | λ | | | | |
|---|---|---|---|---|---|---|
| | | 0.05 | 0.15 | 0.3 | 0.7 | 1 |
| Delta | $L$ | 1.740 | 2.241 | 2.530 | 2.736 | 2.784 |
| 1.0 | **ARL** | **199.24** | **199.63** | **201.74** | **200.29** | **199.61** |
| | MDRL | 94.00 | 117.00 | 122.00 | 114.00 | 109.00 |
| | SDRL | 263.25 | 244.59 | 242.92 | 249.86 | 251.52 |
| 1.1 | **ARL** | **21.17** | **28.27** | **34.36** | **41.54** | **50.85** |
| | MDRL | 7.00 | 14.00 | 15.00 | 14.50 | 16.00 |
| | SDRL | 29.33 | 36.66 | 46.10 | 58.65 | 74.19 |
| 1.2 | **ARL** | **8.02** | **9.16** | **10.72** | **13.07** | **15.19** |
| | MDRL | 4.00 | 4.00 | 5.00 | 4.00 | 4.00 |
| | SDRL | 10.14 | 12.57 | 14.46 | 20.49 | 27.47 |
| 1.3 | **ARL** | **4.08** | **4.77** | **5.29** | **5.88** | **7.07** |
| | MDRL | 2.00 | 2.00 | 3.00 | 3.00 | 3.00 |
| | SDRL | 5.15 | 5.83 | 6.68 | 8.35 | 11.57 |
| 1.4 | **ARL** | **2.76** | **3.28** | **3.39** | **3.56** | **3.96** |
| | MDRL | 1.00 | 2.00 | 2.00 | 2.00 | 2.00 |
| | SDRL | 3.18 | 3.60 | 3.84 | 4.31 | 5.63 |
| 1.5 | **ARL** | **2.11** | **2.42** | **2.54** | **2.51** | **2.80** |
| | MDRL | 1.00 | 1.00 | 2.00 | 2.00 | 2.00 |
| | SDRL | 2.07 | 2.38 | 2.51 | 2.56 | 3.34 |
| 1.6 | **ARL** | **1.80** | **1.99** | **2.06** | **2.12** | **2.19** |
| | MDRL | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 |
| | SDRL | 1.57 | 1.74 | 1.83 | 1.90 | 2.16 |
| 1.7 | **ARL** | **1.58** | **1.70** | **1.80** | **1.79** | **1.87** |
| | MDRL | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 |
| | SDRL | 1.20 | 1.32 | 1.44 | 1.38 | 1.57 |
| 1.8 | **ARL** | **1.44** | **1.55** | **1.58** | **1.61** | **1.61** |
| | MDRL | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 |
| | SDRL | 0.96 | 1.10 | 1.11 | 1.12 | 1.16 |
| 1.9 | **ARL** | **1.34** | **1.43** | **1.46** | **1.48** | **1.50** |
| | MDRL | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 |
| | SDRL | 0.79 | 0.91 | 0.93 | 0.93 | 0.98 |
| 2.0 | **ARL** | **1.26** | **1.34** | **1.38** | **1.39** | **1.41** |
| | MDRL | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 |
| | SDRL | 0.66 | 0.79 | 0.81 | 0.80 | 0.84 |

EWMA control structures using two one-sided EWMA charts. [19] used the FIR approach proposed by [11] for time varying control limits and showed superior performance of their proposed scheme as compared to [11] FIR Scheme. Both these schemes were criticized as they require the use of two EWMA charts instead of one for monitoring changes in process parameters. [23] presented another FIR scheme for EWMA charts, his proposal is based on further narrowing the time varying control limits by using an exponentially decreasing FIR adjustment which is defined as

$$FIR_{adj} = 1 - (1 - f)^{1+a(t-1)} \tag{13}$$

where $a$ is known as the adjustment parameter and is chosen such that the FIR adjustment has very little affect after a specified time period say at $t = 20$, we have $FIR_{adj} = 0.99$. The effect of this FIR adjustment decreases with time and makes the control limit a proportion $f$ of the distance from the starting value [23]. By comparing run length characteristics, [23] showed that his proposed FIR scheme outperformed the previous FIR schemes by [11] and [19]. The FIR adjustment used by [23] is very attractive and has also been recently used by [5] for applying FIR feature to generally weighted moving average control charts. In this section I examine the affect of FIR on the performance of variability EWMA chart. The time varying variability EWMA chart using FIR will be referred as FNEWMA chart for the rest of study. FNEWMA chart signals an out-of-control condition whenever $W_t$ exceeds $UCL_f$, where $UCL_f$ is given as

$$UCL_f = L_f \left(1 - (1 - f)^{1+a(t-1)}\right) \sqrt{\frac{\lambda[1 - (1 - \lambda)^{2t}]}{2 - \lambda}} \sigma_{Z_t^+} \tag{14}$$

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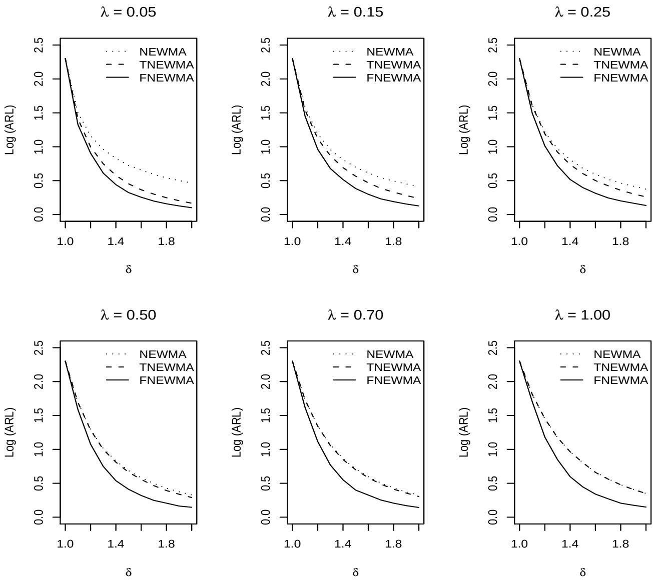


Figure 3: ARL comparison of NEWMA, TNEWMA and FNEWMA charts for different values of $\lambda$ when $\mathrm{ARL}_0 = 200$

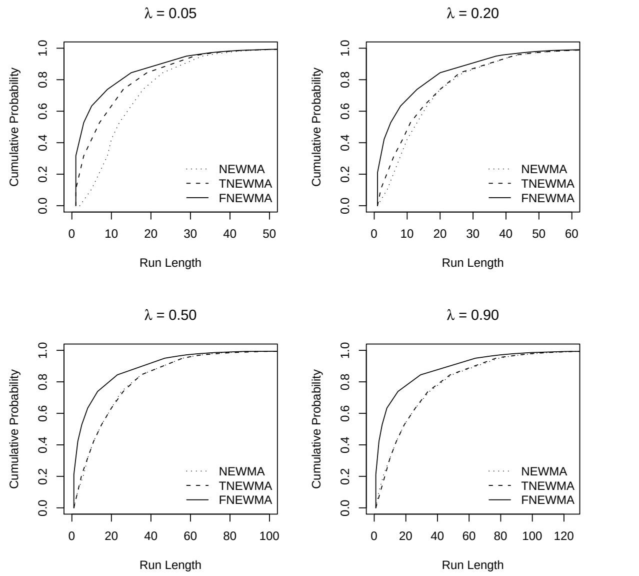


Figure 4: Run length curves of NEWMA, TNEWMA and FNEWMA charts for different values of $\lambda$ when $\delta = 1.2$ and $ARL_0 = 200$

To obtain a substantial benefit from FIR feature, $f$ should be fairly small. In this study I used $f = 0.5$ and limited the effect of FIR adjustment till $t = 20$ following [23] and [5]. The run length characteristics of FNEWMA chart are reported in Table 3. $ARL_0$ for FNEWMA chart is also fixed at 200 by using appropriate $L_f$ values for different choices of $\lambda$. By comparing results in Tables 1, 2 and 3, we can observe the superior run length performance of FNEWMA chart as compared to NEWMA and TNEWMA charts. For example, for FNEWMA chart $ARL_1 = 10.72$ using $\lambda = 0.3$ and $\delta = 1.2$, while the corresponding $ARL_1$ for TNEWMA and NEWMA charts are 16.65 and 17.30 respectively. It indicates that FNEWMA chart requires on average nearly six less observations as compared to NEWMA and TNEWMA charts to detect a shift of $1.2\sigma$ in process variability when $\lambda = 0.3$. Figure 3 provides ARL comparison of NEWMA, TNEWMA and FNEWMA charts for some choices of $\lambda$. We can easily observe that $ARL_1$ of FNEWMA chart is consistently lower then $ARL_1$ of both NEWMA and TNEWMA charts for every choice of $\lambda$. The difference seems greater for higher values of $\lambda$ which is consistent with the findings of [23]. The lower $ARL_1$ values of FNEWMA chart helps in quicker detection of shifts in process variability. To get a more insight on the run length distribution of NEWMA, TNEWMA and FNEWMA charts, Figure 4 presents run length curves (RLC's) of these charts for certain values of $\lambda$ using $\delta = 1.2$. We can observe that for smaller values of $\lambda$, RLC's of TNEWMA chart are higher than RLC's of NEWMA chart indicating that TNEWMA chart has greater probability for shorter run lengths for these $\lambda$ values. The superiority of FNEWMA chart over NEWMA and TNEWMA charts is also clear for all values of $\lambda$. Note that this high probability at shorter run lengths helps significantly in quick detection of shifts in process variability.

## 5 Conclusions

This study examines the performance of variability EWMA chart using asymptotic, time varying and FIR based control limits. It has been shown that shift detection ability of variability EWMA chart can be improved by using exact or time varying limits instead of asymptotic control limits, particularly for smaller values of smoothing parameter $\lambda$. FIR feature has also shown to contribute significantly in further increasing the sensitivity of the EWMA chart to detect shifts in process variability. Although computations have been performed using NEWMA chart but results can be generalized for other variability EWMA charts discussed in Section 1. This study will help quality practitioners to choose a more sensitive variability EWMA chart.

## References

[1] D.R. Barr and E.T. Sherrill, *Mean and variance of truncated normal distributions*, The American Statistician 53 (1999), pp. 357–361.

[2] P.K. Carson and A.B. Yeh, *Exponentially Weighted Moving Average (EWMA) control charts for mon-*

*itoring an analytical process*, Industrial and Engineering Chemistry Research 47 (2008), pp. 405–411.

[3] L.K. Chan and J. Zhang, *Some issues in the design of EWMA charts*, Communications in Statistics Part B: Simulation and Computation 29 (2000), pp. 207–217.

[4] G.M. Chen, S.W. Cheng, and H.S. Xie, *Monitoring process mean and variability with one EWMA chart*, Journal of Quality Technology 33 (2001), pp. 223–233.

[5] W.C. Chiu, *Generally weighted moving average control charts with fast initial response features*, Journal of Applied Statistics 36 (2009), pp. 255–275.

[6] S.V. Crowder and M. Hamilton, *Average run lengths of EWMA controls for monitoring a process standard deviation*, Journal of Quality Technology 24 (1992), pp. 44–50.

[7] R. Domangue and S.C. Patch, *Some omnibus exponentially weighted moving average statistical process monitoring schemes*, Technometrics 33 (1991), pp. 299–313.

[8] S.L. Hwang, J.T. Lin, G.F. Liang, Y.J. Yau, T.C. Yenn, and C.C. Hsu, *Application control chart concepts of designing a pre-alarm system in the nuclear power plant control room*, Nuclear Engineering and Design 238 (2008), pp. 3522–3527.

[9] M.J. Kim, *Number of replications required in control chart monte carlo simulation studies*, Ph.d. diss, University of Northern Colorado, 2005.

[10] J.M. Lucas and R.B. Crosier, *Fast initial response for CUSUM quality control schemes: Give your CUSUM a head start*, Technometrics 24 (1982), pp. 199–205.

[11] J.M. Lucas and M.S. Saccucci, *Exponentially Weighted Moving Average control schemes. properties and enhancements*, Technometrics 32 (1990), pp. 1–12.

[12] J.F. MacGregor and T.J. Harris, *The exponentially weighted moving variance*, Journal of Quality Technology 25 (1993), pp. 106–118.

[13] P. Maravelakis, J. Panaretos, and S. Psarakis, *An examination of the robustness to non normality of the EWMA control charts for the dispersion*, Communications in Statistics: Simulation and Computation 34 (2005), pp. 1069–1079.

[14] P.E. Maravelakis, J. Panaretos, and S. Psarakis, *EWMA chart and measurement error*, Journal of Applied Statistics 31 (2004), pp. 445–455.

[15] P. Masson, *Quality control techniques for routine analysis with liquid chromatography in laboratories*, Journal of Chromatography A 1158 (2007), pp. 168–173.

[16] D.C. Montgomery, 4th ed., Wiley, New York 2001.

[17] D.C. Montgomery, J.C.C. Torng, J.K. Cochran, and F.P. Lawrence, *Statistically constrained economic design of the EWMA control chart*, Journal of Quality Technology 27 (1995), pp. 250–256.

[18] A.S. Neubauer, *The EWMA control chart: Properties and comparison with other quality- control procedures by computer simulation*, Clinical Chemistry 43 (1997), pp. 594–601.

[19] T. Rhoads, D. Montgomery, and C. Mastrangelo, *A fast initial response scheme for the Exponentially Weighted Moving Average control chart*, Quality Engineering 9 (1996), pp. 317–327.

[20] S.W. Roberts, *Control chart tests based on geometric moving averages*, Technometrics 1 (1959), pp. 239–250.

[21] J.R. Schaffer and M.J. Kim, *Number of replications required in control chart Monte Carlo simulation studies*, Communications in Statistics: Simulation and Computation 36 (2007), pp. 1075–1087.

[22] L. Shu and W. Jiang, *A new EWMA chart for monitoring process dispersion*, Journal of Quality Technology 40 (2008), pp. 319–331.

[23] S.H. Steiner, *EWMA control charts with time-varying control limits and fast initial response*, Journal of Quality Technology 31 (1999), pp. 75–86.

[24] Z.G. Stoumbos and M.R.J. Reynolds, *Robustness to non normality and autocorrelation of individual control charts*, Journal of Statistical Computation and Simulation 66 (2000), pp. 145–187.

[25] Z. Wang and R. Liang, *Discuss on applying SPC to quality management in university education*, in *Proceedings of the 9th International Conference for Young Computer Scientists, ICYCS 2008*, 2008, pp. 2372–2375.

[26] W.H. Woodall, *The use of control charts in healthcare and public-health surveillance*, Journal of Quality Technology 38 (2006), pp. 89–104.

[27] A.W. Wortham and L.J. Ringer, *Control via exponential smoothing*, Transportation and Logistic Rev 7 (1971), pp. 33–39.

[28] L. Zhang and G. Chen, *EWMA charts for monitoring the mean of censored Weibull lifetimes*, Journal of Quality Technology 36 (2004), pp. 321–328.