Proceedings of the World Congress on Engineering 2010 Vol III
WCE 2010, June 30 - July 2, 2010, London, U.K.

Optimal Parameter Design by Regression Technique and Grey Relational Analysis

A. Al-Refaie, L. Al-Durgham, and N. Bata

***Abstract*—This research proposes an approach for optimizing multiple responses in the Taguchi method using regression models and grey relational analysis. In this approach, each response is transformed into signal-to-noise (S/N) ratio. The S/N ratios are then utilized to model each response with process factors and complete the responses for all factor level combinations. The grey relational analysis is then used to combine the quality response at each experiment into a single grey grade. Typically, the larger grey grade indicates better performance. Thus, the factor level with the largest level grade is selected as the optimal level for that factor. Three case studies in manufacturing applications on the Taguchi method are utilized for illustration of the proposed approach. It is concluded that the proposed approach is efficient for finding global optimal factor levels. Moreover, this approach can be used with incomplete data. Finally, the regression models can be used to determine the process factors that significantly affect quality response.**

***Index Terms*—Multiple quality responses, Taguchi method, Grey relational analysis, Regression models**

I. Introduction

Taguchi [1] method has significantly improved quality and yield in a product/process design. This method focuses on optimizing a single quality response of main interest [3-4]. In today's high-tech processes, however, products have more than one quality response of main interest. The Taguchi method primarily uses engineering judgment to decide optimal factor levels for multi-responses, which increases uncertainty during the decision-making process. Over the past few years, the optimization of multiple responses has received an increasing research attention [5-6]. Nevertheless, most of research efforts consider only the few experiments in Taguchi's orthogonal array (OA) to decide optimal factor levels. Moreover, they failed to formulate the relationship between process factors (independent factors) with the quality response (dependent factor).

An efficient mathematical technique for underplaying the relationship between the quality response and process factors is the multiple regression models [7]. Grey relational analysis has been reported efficient in transforming multiple quality responses into a single grade. Several researches [8-9] have used the grey grade for deciding the optimal factor levels. In this context, this paper proposes an approach for optimizing multiple quality responses in the Taguchi method using regression models and grey relational analysis; where the former will be used to complete the response values for all factor level combinations, whereas the latter will be used to determine optimal factor levels. This research is organized into the following sequence. Section two outlines the proposed approach steps. Section three provides illustrative case studies. Finally, the conclusions are made in section four.

Manuscript received March 18, 2010. (Write the date on which you submitted your paper for review.) This work was supported by the University of Jordan, Amman.
A. Al-Refaie is with the University of Jordan; e-mail: abbas.alrefai@ ju.edu.jo).
L. Al-Durgham is with the University of Jordan.
N. Bata is a graduate student in the Dept. of Industrial Engineering, University of Jordan.

II. Proposed Approach

Taguchi method conducts n experiments in OA to investigate f factors concurrently. Suppose q responses are of main interest. The quality response is mainly divided into three main types; the larger-the-better (LTB), the smaller-the-better (STB), and the nominal-the-best (NTB) type responses. The proposed procedure for solving the multiple response problem is described in the following steps:

Step1: for each experiment in the OA, calculate the value S/N ratio, η_{ij}, at experiment i for each response j using an appropriate equation from the following formulas:

$$\eta_{ij} = \begin{cases} -10\log_{10}\left(\frac{1}{K}\sum_{r=1}^{K}(y_{ir}^2)\right) & \text{for STB} \\ 10\log_{10}(s_i^2/\bar{y}_i^2) & \text{for NTB}, \forall i \\ -10\log_{10}\left(\frac{1}{K}\sum_{r=1}^{K}(1/y_{ir}^2)\right) & \text{for LTB} \end{cases} \quad (1)$$

where $\bar{y}_i$ and s_i are the estimated average and standard deviation of y_{ir} replicates at the ith experiment. The K denotes the number of replicates.

ISBN: 978-988-18210-8-9
ISSN: 2078-0958 (Print); ISSN: 2078-0966 (Online)

Step 2: Formulate the multi linear regression for response j using the calculated η_{ij} in step 1. Then, obtain the η_{ij} values for all factor level combinations. That is,

$$\eta_j = f(x_1, x_2, \ldots, x_f) \tag{2}$$

Step 3: Let z_{ij} $(0 \leq z_{ij} \leq 1)$ represents the normalized η_{ij} for jth QCH at experiment i. Calculate the z_{ij} values for each response using Eq. (3).

$$z_{ij} = \frac{\eta_{ij} - \min\{\eta_{ij}\}}{\max\{\eta_{ij}\} - \min\{\eta_{ij}\}}, \; \forall i, \; \forall j \tag{3}$$

The $\max\{\eta_{ij}\}$ is the maximum value of η_{ij} .The $\min\{\eta_{ij}\}$ is the minimum value of η_{ij} . Then calculate ξ_{ij} as follows:

$$\xi_{ij} = \frac{\Delta_{\min} - \zeta\Delta_{\max}}{\Delta_{ij} - \zeta\Delta_{\max}} \; \forall i, \forall j \tag{4}$$

where ζ is the distinguishing coefficient ranges between zero and one; usually ζ equals 0.5. Also, the Δ'_{ij} is the difference of the absolute value between the ideal setting, η_{oj} , and η_{ij} . The $\Delta_{\min}$ and $\Delta_{\max}$ are the smallest and largest values of all the Δ_{ij} from all responses.

Step 4: Let γ_i denoted the grey relational grade for the ith experiment from all q responses. Calculate γ_i using Eq. (5).

$$\gamma_i = \frac{1}{q}\sum_{j=1}^{q} \xi_i \qquad \forall i \tag{5}$$

Step 5: Let $\overline{\gamma}_{lf}$ denotes the average of γ_i values at level l of factor f. Calculate the $\overline{\gamma}_{lf}$ values for all factor levels. For each factor f decide the optimal level as the level that maximizes the $\overline{\gamma}_{lf}$ for this factor.

Step 6: Compare the obtained results using the proposed approach with those obtained by other approaches in previous literature.

III. Illustration

Three case studied, previously studied in literature, are adopted for illustrations.

A. *Optimization of the sputtering process parameter*

Chen *et. al* [10] optimized the sputtering process parameters of GZO films using the grey–Taguchi method. The factors studied were (A) R.F. power, (B) sputtering pressure, (C) deposition time, (D) substrate temperature, and (E) post-annealing temperature. Three responses were of main interest including deposition rate (y_1, LTB), electrical resistively (y_2, STB), and structural, morphological and optical transmittance (y_3, LTB). The L_{18} array shown in Table 1 was utilized in experimental work.

Step1: The η_{ij} values were calculated for each response and the results are also displayed in Table1.

Step 2: The multiple linear regression equations are obtained for the three responses. They are formulated as:

η_1 = 10.378 + 0.079916A + 0.3899B + 0.00073C + 0.00127D + 0.000756E, *R-Sq (adj)* = 92.6%

η_2 = - 27.641 + 0.105861A + 0.1834B + 0.00952C + 0.01925D + 0.010952E, *R-Sq (adj)* = 93.0%

η_3 = 39.2060 - 0.0023569A - 0.05405B - 0.0046972C + 0.0010519D + 0.0005642E, *R-Sq (adj)* = 84.6%

Step 3-5: The regression equations in step 2 are utilized to estimate the η_{ij} for the 3^5 full factorial design, which as factor level combinations. The z_{ij} and γ_i values are calculated for all combinations. Finally, the $\overline{\gamma}_{lf}$ values are calculated then displayed in Table 2.

Step 6: From Table 2, the combination of optimal factor levels is identified as $A_3B_1C_1D_3E_3$. The response values at this combination are calculated then tabulated with those obtained using grey analysis [10] are tabulated in Table 3. It is found that the proposed approach provided almost similar results as the grey analysis.

ISBN: 978-988-18210-8-9
ISSN: 2078-0958 (Print); ISSN: 2078-0966 (Online)

Table 1. The experimental results for case 1.

Exp i	Factor levels					Response replicate (y_{irj})						S/N ratio (η_{ij})		
	A	B	C	D	E	y_{i11}	y_{i12}	y_{i21}	y_{i22}	y_{i31}	y_{i32}	η_{i1}	η_{i2}	η_{i3}
1	50	0.13	30	25	none	4.5	4.7	14.9	15.3	88.4	88.4	13.2	-23.6	38.9
2	50	0.67	60	50	100	5.6	5.6	9.8	9.7	87.7	87.7	15.0	-19.8	38.9
3	50	1.33	90	100	200	5.0	4.9	7.9	7.8	88.1	88.1	13.8	-17.9	38.9
4	100	0.13	30	50	100	9.6	9.3	5.4	5.6	89.2	89.3	19.5	-14.8	39.0
5	100	0.67	60	100	200	11.1	11.3	4.6	4.3	87.1	87.0	21.0	-13.0	38.8
6	100	1.33	90	25	none	10.0	10.0	6.5	6.6	84.7	84.7	20.0	-16.4	38.6
7	200	0.13	60	25	200	19.9	20.2	1.6	1.7	86.6	86.6	26.0	-4.1	38.8
8	200	0.67	90	50	none	21.6	21.6	1.9	2.0	82.3	82.4	26.7	-5.7	38.3
9	200	1.33	30	100	100	20.9	20.9	1.8	1.6	85.6	85.3	26.4	-4.6	38.6
10	50	0.13	90	100	100	4.8	4.6	7.3	7.0	87.6	87.6	13.4	-17.1	38.9
11	50	0.67	30	25	200	5.0	4.9	6.9	7.1	89.1	89.1	13.9	-16.9	39.0
12	50	1.33	60	50	none	4.9	4.8	7.8	7.7	87.4	87.4	13.8	-17.8	38.8
13	100	0.13	60	100	none	9.7	9.7	6.1	5.9	87.0	87.0	19.7	-15.6	38.8
14	100	0.67	90	25	100	11.1	11.6	6.0	5.8	83.7	83.7	21.1	-15.4	38.5
15	100	1.33	30	50	200	10.7	10.8	5.5	5.7	88.4	88.3	20.6	-14.9	38.9
16	200	0.13	90	50	200	19.5	19.4	1.0	1.1	83.1	83.1	25.8	-0.6	38.4
17	200	0.67	30	100	none	22.1	22.0	1.2	1.3	85.7	85.7	26.9	-1.7	38.7
18	200	1.33	60	25	100	20.5	20.5	1.4	1.3	83.9	83.7	26.2	-2.8	38.5

Table 2. Optimal factor levels for case study 1.

Factor	Level		
	1	2	3
A	0.434033	0.473801	0.742785
B	0.588138	0.545844	0.516637
C	0.566654	0.548717	0.535247
D	0.537476	0.546641	0.566502
E	0.529256	0.549362	0.572001

Table 3. Optimal response values for case study 1.

Response	Initial condition $A_2B_2C_1D_3E_1$	Grey analysis [10] $A_3B_2C_1D_3E_3$	Proposed approach $A_3B_1C_1D_3E_3$
y_1 (LTB)	21.8055	22.1884	21.6571
y_2 (STB)	1.6098	1.2510	1.2653
y_3 (LTB)	85.7302	86.8512	87.1436

ISBN: 978-988-18210-8-9
ISSN: 2078-0958 (Print); ISSN: 2078-0966 (Online)

B. Optimization of turning operations

Lin [11] employed the Taguchi Method and grey relational analysis to optimize turning operation with multiple performance characteristics, including: tool life (y_1, LTB), cutting force (y_2, STB), and surface roughness (y_3, STB). Three turning controllable factors: (A) cutting speed, (B) feed rate, and (C) depth of cut. The L_9 orthogonal array is shown in Table 4. The η_{ij} values are calculated for the three responses at all the nine experiments. The results are also displayed in Table 4. Then, the regression equations are estimated are found

η_1 = 80.221 - 0.08113A - 28.672B + 2.696C,
R-Sq(adj) = 86.7%

η_2 = - 43.650 + 0.009842A - 26.622B - 7.3871C,
R-Sq(adj) = 95.5%

η_3 = 6.397 - 0.00953A - 65.64B - 1.896C,
R-Sq(adj) = 73.8%

The η_{ij} values are estimated for all 3^3 (=27) factor level combinations. The z_{ij} and γ_i values are then obtained for all combinations. Finally, the $\overline{\gamma}_{lf}$ values are estimated then tabulated in Table 5, where the combination of optimal factor levels is $A_1B_1C_1$. Table 6 compares the response values at $A_1B_1C_1$ with those obtained at initial conditions and using grey analysis [11]. From this table, it is noted that the proposed approach and the grey relational analysis provide the same results.

C. Optimization of Inconel on machining of CNC WEDM process

Ramakrishnan *et al.* [12] modeled and multiple response optimization of Inconel 718 on machining of CNC WEDM process. Four process factors were investigated, including: (A) pulse in time, (B) delay time, (C) wire feed speed, and (D) ignition current, on two responses: (y_1) material removal rate and (y_2) surface roughness of wire electro-discharge machining (WEDM) process.

Table 5. Optimal factor levels for case study 2.

Factor	Level		
	1	2	3
A	0.611023	0.540273	0.504265
B	0.727724	0.517716	0.410121
C	0.603815	0.542011	0.509735

Table 6. Optimal response values for case study 2.

Response	Initial condition $A_2B_2C_2$	Grey analysis [11] $A_1B_1C_1$	Proposed method $A_1B_1C_1$
y_1 (LTB)	1048.926	2689.147	2689.147
y_2 (STB)	564.507	278.079	278.079
y_3 (STB)	3.475	1.159	1.159

Table 7 displays the experimental results. Utilizing the η_{i1} and η_{i2} values in Table 7, the regression models for the two responses are formulated as:

η_1 = 30.7153 + 6.1200A - 0.18435B - 0.07380C + 0.06063D, *R-Sq(adj)* = 96.1%

η_2 = - 8.57-2.15A + 0.254B - 0.0739C - 0.0878D, *R-Sq(adj)* = 99.2%

The η_{ij}, z_{ij} and γ_i values are estimated for all 3^4 factor level combinations. Finally, the $\overline{\gamma}_{lf}$ values are estimated then displayed in Table 8, where the combination of optimal factor levels is $A_3B_3C_1D_1$. Table 9 compares the response values at $A_3B_3C_1D_1$ with those obtained at initial conditions and using multi-response S/N ratio approach [12]. From this table, it is noted that the proposed approach provides almost similar results with the multiple response S/N ratio approach.

Table 4. The experimental results for case study 2.

Exp. *i*	Factor			Response (y_{ij})			S/N ratio		
	A	B	C	y_{i1}	y_{i2}	y_{i3}	η_{i1}	η_{i2}	η_{i3}
1	135	1	1	2645	263	1.239	68.4485	-48.3991	-1.8614
2	135	2	2	2060	704	1.921	66.2773	-56.9515	-5.6705
3	135	3	3	1733	1198	9.443	64.7760	-61.5691	-19.5022
4	210	1	3	1310	593	2.641	62.3454	-55.4611	-8.4354
5	210	2	1	1198	389	4.513	61.5691	-51.799	-13.0893
6	210	3	2	734	854	7.490	57.3139	-58.6292	-17.4896
7	285	1	2	854	335	0.908	58.6292	-50.5009	0.8383
8	285	2	3	765	857	4.184	57.6732	-58.6596	-12.4318
9	285	3	1	216	464	9.695	46.6891	-53.3304	-19.731

ISBN: 978-988-18210-8-9
ISSN: 2078-0958 (Print); ISSN: 2078-0966 (Online)

Table 7. Orthogonal array and the experimental results

Exp. i	A	B	C	D	y_1	y_2	η_{i1}	η_{i2}
1	0.6	4	8	8	46	3.2	46	3.2
2	0.6	6	12	12	48	3.3	48	3.3
3	0.6	8	15	16	42	3.3	42	3.3
4	0.8	4	12	16	56	3.8	56	3.8
5	0.8	6	15	8	50	3.4	50	3.4
6	0.8	8	8	12	52	3.2	52	3.2
7	1.2	4	15	12	70	4.2	70	4.2
8	1.2	6	8	16	74	3.8	74	3.8
9	1.2	8	12	8	64	3.4	64	3.4

Table 8. Optimal factor levels for case study 3.

Factor	Level		
	1	2	3
A	0.503286	0.497314	0.598393
B	0.517335	0.526903	0.557841
C	0.573775	0.527449	0.497768
D	0.549350	0.529367	0.520275

Table 9. Optimal response values for case study 3.

Response	Initial condition $A_2B_2C_2D_2$	Multi-response S/N ratio [12] $A_3B_3C_1D_2$	Proposed method $A_3B_3C_1D_1$
y_1 (LTB)	52.163	68.562	66.674
y_2 (STB)	3.430	3.453	3.316

IV. Conclusions

This research proposed an approach for optimizing multiple quality responses using the regression models and grey relational analysis. Three case studies were provided for illustration, in all of which the proposed approach was found efficient. As a result, this approach can be effectively utilized for optimizing multiple quality responses in a wide range of applications on the Taguchi method. This approach also is found efficient for determining the global optimal combination of factor levels. Future research will apply this approach for determining optimal factor levels with fuzzy outputs.

References

[1] G. Taguchi, *Taguchi Methods. Research and Development*. Dearborn, MI, American Suppliers Institute Press, 1991, Vol. 1.

[2] A.R. Khoei, I. Masters, D.T. Gethin, "Design optimization of Aluminium recycling processes using Taguchi technique," *Journal of Materials Processing Technology*, 2002, 127, 96-106.

[3] M-H C. Li, A. Al-Refaie, and C.Y. Yang, "DMAIC Approach to Improve the Capability of SMT Solder Printing Process," *IEEE Transactions on Electronics Packaging Manufacturing*, 2008, 31(2), 126-133.

[4] A. Al-Refaie, "Optimizing SMT performance using comparisons of Efficiency between different systems technique in DEA," *IEEE Transactions on Electronics Packaging Manufacturing*, 2009, 32(4), 256 – 264.

[5] H.C. Liao, Y.K. Chen, "Solving multi-response problem in the Taguchi method by DEA based ranking method. International," *Journal of Quality & Reliability Management*, 2002, 19 (7), 825-837.

[6] A. Al-Refaie, T. H. Wu, and M.H.C. Li, "DEA approaches for solving the multi-response problem in Taguchi method," *Artificial Intelligence for Engineering Design, Analysis and Manufacturing*, 2009, 23, 159-173.

[7] D.C. Montgomery, *Design and Analysis of Experiments.* John Wiley, 2009.

[8] A. Al-Refaie, "Grey-DEA approach for solving the multi-response problem in Taguchi Method," *Proceedings of the Institution of Mechanical Engineering-Part B. Journal of Engineering Manufacture*, 2010, 224(B1), 147-158.

[9] A. Al-Refaie, M.H.C. Li, and K.C. Tai, "Optimizing SUS 304 wire drawing process by grey analysis utilizing Taguchi method," *Journal of University of Science and Technology Beijing*, 2008, 15(6), 714-722.

[10] C.C. Chen, C.C. Tsao, Y.C. Lin, and C.Y. Hsu, "Optimization of the sputtering process parameters of GZO films using the Grey-Taguchi method," *Ceramics International*, 2009, article in press.

[11] C.L. Lin, "Use of the Taguchi method and grey relational analysis to optimize turning operations with multiple performance characteristics," *Materials and Manufacturing Processes*, 2004, 19(2), 209-220.

[12] R. Ramakrishnan, and L. Karunamoorthy, "Modeling and multiple-response optimization of Inconel 718 on machining of CNC WEDM process," *Journal of Material s Processing Technology*, 207, 343-349.