Optimization for Efficient Supply Chain Using Swarm Intelligence: an Outsourcing Perspective

N C Hiremath, Sadananda Sahu, and M K Tiwari

Abstract - **The thrust of global economy drives the organizations to outsource process, parts, and labor, virtually anywhere in the world and get the desired combination of low cost and high quality. However, ineffective utilization of shipment practices in supply chain prevents to achieve the anticipated outsourcing benefits. In this research, a three stage inventory model is developed to address the outsourcing issues with different shipment policies. Owing to inherent computational complexities of the problem with higher dimensions, various deterministic approaches practically fail. Proposed work, therefore, utilizes a nature inspired evolutionary algorithm, namely particle swarm optimization (PSO), to solve the problem. This paper applies enhanced particle swarm optimization (EPSO), a variant of particle swarm optimization for solution purpose. The results obtained delineate efficacy to handle the fluctuations in the possible shipment options and simultaneously deciding the optimal shipment policies.**

Keywords: Inventory issue, Multilevel Inventory Model, Outsourcing, Particle Swarm Optimization.

I. INTRODUCTION

Outsourcing represents a major trend not only in manufacturing segments but also in other sectors of corporate world. The key factors that engender the outsourcing of manufacturing jobs are identified to be the low wages, the improved quality with low production costs, the relaxation of various international trade barriers, and the reduction in export costs. Owing to reduced gap in terms of cost of production and process planning with competitors; companies are now focusing on effective utilization of shipment practices to increase their margin of profit. Earlier researchers proposed several shipment policies for different stages of integrated inventory systems. However, integrated inventory system that considers all the factors affecting the inventory management of the outsourcing firms were not addressed. For a single-vendor single-buyer integrated system, [1] presented a shipment policy for determining the vendor's production batch and successive shipments sizes.

The inventory models developed by [2]-[5] implicitly assumed that the transportation cost is a part of the ordering cost. Reference [6] suggested that this assumption is invalid as transportation cost can be affected by the routing decisions and the selected shipment size.

In this paper, a multi-level inventory model is developed for procurement of outsourced components. It encompasses the need to avoid shipment decisions biased to the transportation discount options, therefore, re-emphasizing the significance of integrated inventory systems. The model also assimilates the formulation of inventory level at a stage where two different shipment policies are adopted resulting in two different frequencies of the inflow and outflow of the stock. The optimal values of these shipping frequencies are also achieved through this model. The model is flexible in deriving the optimal shipment sizes without any assumption on the holding costs.

Another salient feature of this model exists in its solution methodology. The problem of the type defined above requires inspection of all feasible solutions for determination of its optimal solution. However, with slight increase in the values of problem parameters, the search domain of the problem instance increases exponentially. This leads to enormous computational complexity which cannot be handled by deterministic approaches. Therefore, for such problems, various artificial intelligence based stochastic search techniques have been proposed in the literature. These techniques are: Genetic Algorithm (GA) [7], Ant Colony Optimization (ACO) [8]-[9], and Particle Swarm Optimization (PSO) [10].

Owing to faster convergence, better exploration and exploitation abilities, and consistent performance in producing near-optimal results, this paper utilizes particle swarm optimization (PSO) as its basic search mechanism. PSO, first introduced by [11], is inspired by the natural behaviour of flocking birds. The movement of each particle of the swarm depends upon the particle's cognitive and social components. The cognitive component motivates the particle to attain the best position found by it so far, whereas the social component moves the particle toward the global optimum.

However, to enhance the search capability of PSO, various improvements have been suggested in its basic structure. Reference [12] proposed EPSO, an enhanced version of PSO which provides the particles with additional information through a primitive component apart from the social and cognitive components used in the basic PSO. The model is solved by adopting EPSO strategy and deriving the optimal decisions to achieve the minimum total channel cost. It leads to better search for the near-optimal solution in reduced computational time.

Based on these premises, this research discusses the following objectives:

1. To obtain the optimal shipment frequencies for three–stage outsourcing.
2. Use of evolutionary algorithm for deriving optimal shipment quantities.
3. Incorporating exporting discount in the export size decision making.

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4. Inventory management of a system involving different frequencies of inflow and outflow of stock.

The remainder of the paper is explained in the following sections.

II. PROBLEM DESCRIPTION

It has been a very common practice in manufacturing sectors to retain their core competencies and then outsource the production of other components to various established manufacturers in the industry.

Generally, procurement of the outsourced components involves three stages

- Production of the components at the manufacturing units (MUs).
- The collection of the manufactured components at the exporting point (EP) or the collection centre. This centre refers to best location in the country to export the components to the assembly plant.
- Assembling the imported components at the assembly plant (AP)

Two types of shipment strategies called inland shipment policy (MUs to EP) and export shipment policy (EP to AP) are adopted in these stages for successful completion of outsourcing. The inland shipment policy governs the shipment from the manufacturing units to the exporting point, and, the export shipment policy governs the shipments from the exporting point to the assembly point.

Following assumptions have been considered for realization of the proposed model:

a) The production rate of each of the components is greater than demand rate of the components.
b) The consumption rate of the components at the assembly plant is continuous and constant. Only the required numbers of components are produced by the manufacturing units that is transported to and consumed by the assembly plant during the time period T_c.
c) No shortage of components occurs at the assembly plant.
d) The exporting frequency is less than or equal to the inland shipping frequency ($m \le n$)
e) All the shipment sizes are integer numbers greater than or equal to 1. ($n,m \ge 1$)
f) The inland shipment policy assumes instantaneous replenishment, and the transportation time is constant.
g) No damages or defects occur in the components at any stage of the procurement process.

III. MATHEMATICAL FORMULATION

Some important notations are given in the appendix.

The proposed Inventory based Multi-level Outsourcing Model (IMOM), assimilates different costs involved in the procurement of the outsourced components. With the flow of demand and order, inventory at each stage changes over the period and affects the total channel cost. The total channel cost is expressed as the sum of the following costs:

1. The total average holding costs of the manufacturers, collector/exporter and the assembly plant during the time period T_c.

Average holding cost for the time period T_c

$$= \begin{array}{l}\text{Average time weighted inventory } \times T_c \\ \times \text{ holding cost per unit per } T_c\end{array}$$

2. Setup costs by the manufacturers, and ordering cost of the exporting point and the assembly plant.
3. Exporting cost of the components from the collection centre to the assembly unit.

A. Components of Total Channel Cost

The average time weighted inventory of manufacturing unit of component j during the:

1st time interval T of $T_c = P_j T^2/2$

2nd time interval T of $T_c = [P_j T^2/2+(P_j T-q_{fj})T]$

3rd time interval T of $T_c = [P_j T^2/2+(2P_j T-q_{fj}-q_j)T]$

Total time weighted inventory during the time T_c of a manufacturing unit of component j

$$= A.P_j T^2 + B.q_{fj}T + C.q_j T + D.q_{lj}T \qquad (1)$$

Where $A = (k_j^{\,2} - (1-f_j)^2)/2,$

$B = 1\text{-}k_j,$

$C = \{(n\text{-}k_j)(n\text{-}k_j\text{-}1) - (k_j - 1)(k_j - 2)\}/2,$

$D = (n\text{-}k_j)$ And $T = T_c\ /n$.

Thus, total time weighted inventory during the time T_c of all the manufacturing units comes out to be

$$= \sum_{j=1}^{w} A.P_j T^2 + B.q_{fj}T + C.q_j T + D.q_{lj}T \qquad (2)$$

Where w is the total number of manufacturing units of various outsourced components.

Hence, the total holding cost during the time T_c of all the manufacturing units comes out to be

$$= \sum_{j=1}^{w} \left[(A.P_j T^2 + B.q_{fj}T + C.q_j T + D.q_{lj}T\ ).T_c.H_{pj} \right] \qquad (3)$$

Total average inventory holding cost at the Exporting Point (EP)

Average time weighted inventory at the collection center
= time weighted ([average inflow of inventory]
- [average outflow of inventory])

$$\begin{aligned}&=[q_{ls}T_c+(n-1)q_{fs}T_c+(1+2+3+...+n-2)q_sT]\\&-[r_1(m-1)t+r_2(m-2)t+...+r_{m-1}(1)t+Q_su]\\&=[(q_{ls}+(n-1)q_{fs})T_c+(n-1)(n-2)q_sT/2]\\&-[r_1(m-1)t+r_2(m-2)t+...+r_{m-1}(1)t+Q_su]\\&=[(q_{ls}+(n-1)q_{fs}/n)T_c+(n-1)(n-2)q_sT_c/2n]\\&-[(r_1(m-1)+r_2(m-2)+....+r_{m-1})T_c/m+Q_s.u]\end{aligned} \tag{4}$$

Average holding cost of the collection center during time T_c

$$\begin{aligned}&=\{[(q_{ls}+(n-1)q_{fs})T_c+(n-1)(n-2)q_sT_c/2n]\\&-[(r_1(m-1)+r_2(m-2)+...+r_{m-1})T_c/m+Q_su]\}.T_c.H_c\end{aligned} \tag{5}$$

Total average inventory holding cost at the assembly plant (AP)

For component j, during different time interval total average inventory holding cost at the assembly plant is found to be as follows:

1st time interval t of $T_c = Dt^2/2+r_0t$

2nd time interval t of $T_c =Dt^2/2+(r_0t+r_1t-Dt^2)$

3rd time interval t of $T_c\ Dt^2/2+(r_0t+r_1t+r_2t-2Dt^2)$

(m-1)th time interval t of

$T_c =Dt^2/2+(r_0t+r_1t+...+r_{m-2}t-(m-2)Dt^2)$

mth time interval t of

$T_c =Dt^2/2+(r_0t+r_1t+...+r_{m-1}t-(m-1)Dt^2)$

Total time weighted inventory during the time T_c of the assembly unit/importer (the sum of all the above equations derived for m intervals)

$$\begin{aligned}&=[mr_0+(m-1)r_1+(m-2)r_2+...+2r_{m-2}+r_{m-1}]\\&\times[t-(m^2-2m)Dt^2/2]\end{aligned} \tag{6}$$

Average holding cost at the assembly unit during time T_c

$$\begin{aligned}&= \{[mr_0+(m-1)r_1+(m-2)r_2+...+2r_{m-2}+r_{m-1}]\\&\quad t-(m^2-2m)Dt^2/2\}.T_c.H_a\}\end{aligned} \tag{7}$$

The total average holding cost is the sum of the average holding costs of the manufacturing unit, collection center and the assembly plant (i.e. sum of equations 3, 6 & 7).

Hence, the total average holding cost comes out to be:

$$\begin{aligned}&=\sum_{j=1}^{w}\left[(A.PT^2+B.q_{fj}T+C.q_jT+D.q_{fj}T).T_c.H_{pj}\right]\\&\quad+\{[(q_{ls}+(n-1)q_{fs})T_c+(n-1)(n-2)q_sT_c/2n]\}\\&-[(r_1(m-1)+r_2(m-2)+...+r_{m-1})T_c/m+Q_su]\}.T_c.H_c\\&+\{[mr_0+(m-1)r_1+(m-2)r_2+...+2r_{m-2}+r_{m-1}]t\\&-(m^2-2m)Dt^2/2\}.T_c.H_a\end{aligned} \tag{8}$$

Setup costs and the ordering costs

Total setup costs of all the manufacturing units $S_0 =\sum_{j=1}^{w}S_j$ (9)

Total ordering costs of the collection center during T_c = n x C_{oc} (10)

Total ordering costs of the assembly unit during T_c = m x C_{oa} (11)

Exporting cost

Exporting cost is also considered to be a major cost in this model. The exporting cost for each shipment is calculated on the basis of discounting system.

-Total Exporting cost for the m shipments during the time

$$T_c =\sum_{x=1}^{m}(r_x \text{ x } C_{r_x}) \tag{12}$$

Where r_x is the size of the x^{th} export to the assembly plant

x=1,2,3,...,m

C_{r_x} is the exporting cost of the r_x shipment

B. Objective Function

Minimization of Total Channel Cost

= Minimization (Total average holding cost units + Total ordering costs of the collection center + Total ordering costs of the assembly unit + Total exporting cost) + Total setup costs of the manufacturing
= Minimization of

$$\begin{aligned}&\{\sum_{j=1}^{w}\left[(A.PT^2+B.q_{fj}T+C.q_jT+D.q_{fj}T).T_c.H_{pj}\right]\\&\quad+\{[(q_{ls}+(n-1)q_{fs})T_c+(n-1)(n-2)q_sT_c/2n]\\&-[(mr_m+r_1(m-1)+r_2(m-2)+...+r_{m-1})T_c/m+Q_su]\}\\&\times T_c.H_c+\{[mr_0+(m-1)r_1+(m-2)r_2+...+2r_{m-2}+r_{m-1}]\\&\times t-(m^2-2m)Dt^2/2\}\times\ T_c.H_a+n \text{ x } C_{oc}\ +m \text{ x } C_{oa}\\&+\sum_{i=1}^{m}(r_i \text{ x } C_{r_i})\ \}+\sum_{j=1}^{w}S_j\}\end{aligned} \tag{13}$$

C. Constraints

The objective function formulated in equation (13) is subjected to the following constraints:

- The exporting frequency (m) is less than or equal to the inland shipping frequency (n).

$$m \le n \tag{14}$$

- The total size of all the inland shipments and the total size of all the exports should be equal to the demand during the time period T_c .i.e.

$$q_1+q_2+q_3+...+q_n=q_{fs}+(n-2)q_s+q_{ls}=Q_s \tag{15}$$

$$r_1+r_2+r_3+...+r_{m-1}+r_m=Q_s \tag{16}$$

This is due to the fact that during the time period T_c the exporter receives Q_s set of components and exports the same quantity.

- Any export size cannot exceed the stock available with the exporter/collection centre.

$$r_x \le q_{ls}+q_{fs}+(Z_x-1)q_s-(r_1+r_2+r_3+...+r_{x-1}) \tag{17}$$

$r_x \le$ (Initial inventory + (stock received - stock exported) before the r_x shipment)

Initial inventory at the starting of the time period $T_c = q_{ls}$

Stock received before the r_x shipment = $q_{fs}+(Z_x-1)q_s$

Stock exported before the r_x shipment = $r_1+r_2+r_3+...+r_{x-1}$

Where $x=1,2,3,...,m$ and

Z_x is the integer value which gives the number of inland shipments made before the x^{th} export from the collection centre to the assembly plant.

i.e. Z_x = Integer part of ($\frac{n}{m}\text{萸}$)

- The minimum size of an export is the size which could avoid the shortage at the assembly plant until the next export arrives.
  To avoid shortage at the assembly plant the following conditions should be satisfied

$$r_1 \ge Dt-r_0$$

$$r_2 \ge 2Dt-(r_0+r_1)$$

$$r_3 \ge 3Dt-(r_0+r_1+r_2)$$

$$\cdots\cdots\cdots\cdots\cdots$$

$$r_{m-1} \ge (m-1)Dt-(r_0+r_1+r_2..+r_{m-2})$$

But from equation (16), $r_m=mDt-(r_1+r_2+,...,+r_{m-2}+r_{m-1})$ hence, the general equation for the export shipment size comes out to be:

$$r_x \ge (D\text{荻})(x)-(r_0+r_1+r_2+r_3+...+r_{x-1}) \tag{18}$$

Where $x=1,2,3,...,m-1$

Owing to the formulation and associated constraints, it is imperative to solve with a solution methodology that can search the solution space with due consideration of problem sensitivity.

IV. SOLUTION METHODOLOGY

The proposed methodology solves the multivariate and computationally complex problem, as the search space is large and sensitive to the changes in the variable values. For example, let the range of inland shipments (n) lies between 1 and 10. For each 'n' there are 10 possibilities of m as $m \le n$. Moreover, for each value of 'm' there are different possible combinations of export shipments $r_1, r_2, r_3, ..., r_m$. Since cost associated with each of these is dissimilar, it is expensive in terms of computational cost to check every possible combination of the decision variables. The total number of decision variables varies with the number of exports (i.e. m+2).

The deterministic methods like dynamic programming, minimal cut, and branch and bound algorithm are not suitable for solving this model. Hence, nature inspired evolutionary algorithm is used to obtain the near-optimal solution. Due to the global and local exploration abilities, faster convergence, and consistency in the performance of PSO, the proposed model utilizes its variant EPSO. The canonical PSO can be represented as follows:

$$\vec{V}_i(t+1) = w\vec{V}_i(t) + c_1 r_c(\vec{P}_{ib} - \vec{X}_i(t)) + c_2 r_s(\vec{P}_g - \vec{X}_i(t)) \tag{19}$$

$$\vec{X}_i(t+1) = \vec{X}_i(t) + \vec{V}_i(t+1) \tag{20}$$

Where $\vec{X}_i(t)$ and $\vec{V}_i(t)$ represent the position and velocity of a particle i at time t, w is the inertia weight of the initial velocity, c_1 and c_2 are cognitive and social acceleration constants, $r_c=[0,1]$ and $r_s=[0,1]$ are random parameters with uniform distributions, and $\vec{P}_{ib}$ refers to the particle's best position and $\vec{P}_g$ refers to the global best position of the swarm.

In addition to the cognitive and social components used in the equation (20), a primitive component was introduced by [12] in EPSO. Its mathematical representation of velocity update can be given as follows

$$\begin{aligned}\vec{V}_i(t+1) &= w\vec{V}_i(t) + (c_1 r_c(\vec{P}_{ib} - \vec{X}_i(t)) \\ &+ (c_2 r_s(\vec{P}_g - \vec{X}_i(t)) + c_3 r_n(\vec{P}_g - \vec{P}_{ib})\end{aligned} \tag{21}$$

Where $$w = (w_i - w_f) \times \frac{(I - E)}{I} + w_f \tag{22}$$

$$c_1 = (c_{1f} - c_{1i}) \times \frac{E}{I} + c_{1i}, \tag{23}$$

$$c_2 = (c_{2f} - c_{2i}) \times \frac{E}{I} + c_{2i} \tag{24}$$

$$c_3 = 0.5 \ .$$

w_i and w_f are the initial and final inertia weights, I is the total number of iterations and E is the current iteration. Reference [13] suggested that the optimal solution can be improved when w_i and w_f are taken as 0.9 and 0.4 respectively. The value for c_1 changes from (c_{1i}) 2.5 to (c_{1f}) 0.5 whereas c_2 changes from (c_{2i}) 0.5 to (c_{2f}) 2.5.

Fig. 1 depicts the use of EPSO to determine the best export shipment policy for a given number of inland shipments (n) and exports (m). An export shipment policy consists of m shipments of sizes $r_1, r_2, r_3, ..., r_m$. The export shipment policies are used as the position vectors of the particles of the swarm. Each particle contains 'm' bits to represent the shipments sizes of all the exports during the cycle time $\mathrm{T_c}$. For example, if three exports are made during the cycle time $\mathrm{T_c}$, the position of the particle would be a three dimensional vector, each representing the size of the exports.

To obtain the best export shipment policy for a given number of inland shipments and exports, initial population containing particles of feasible export policies are generated. The total channel cost generated by the particle is called as the fitness of the particle. The particle with least total channel cost is said to have the maximum fitness value. The initial positions of the particles are set as their individual best position called "the particle best position". The position of the particle with best fitness value is set as "the global best position", while its fitness value is termed as the "global best cost". The particles move to new positions with the velocities governed by equation (21). Before calculating the fitness at the new particle positions, it is ensured that the shipment values lie between upper and lower limits provided by the equations (17) and (18) respectively. In case, a shipment is found to cross the limits, it is replaced by the nearest feasible shipment size. When the particles satisfy the feasibility condition, the fitness for each of the particle is re-calculated and accordingly the particles' best and global best positions are updated. The process of attaining new velocities, updating particle positions and their fitness is continued until all the particles converge to a single policy. When the termination condition is reached, the global best position is obtained as the best export policy for the given number of inland shipments (n) and exports (m). The overall solution methodology for the outsourcing problem is depicted in Fig. 2.

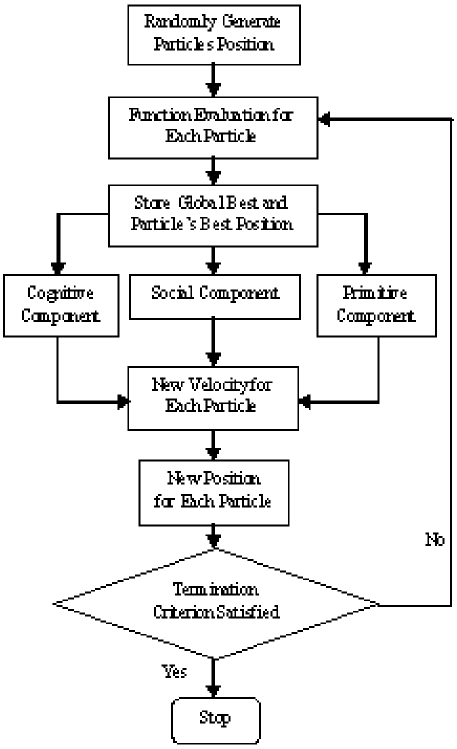

Fig. 1 Flow chart for EPSO (Pandey et al. 2007)

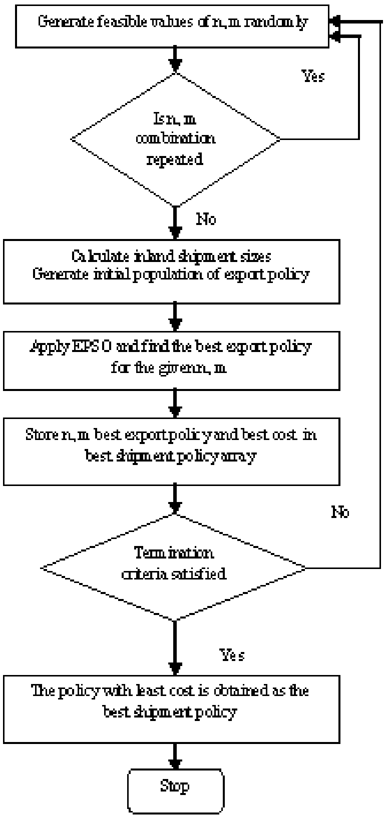

Fig. 2 Flow Diagram for the solution methodology

V. EXAMPLE

The proposed model is delineated by simulating a case of car manufacturer. A program is written to solve the problem utilizing technical computing language tool, Matlab (Version 7.1.0.246 R (14) service pack 3). The optimum number of *n* and *m* are determined along with the shipment sizes.

VI. RESULTS and DISCUSSION

In the undertaken problem, the maximum number of inland shipments (n_{max}) is found to be 30 as at most one inland shipment can take place in a day. For each combination of ‘n’ and ‘m’, 20 particles representing 20 feasible shipment policies are generated. These policies converge to a best export policy on application of EPSO. The best twenty five particles exhibit the 25 best export strategies.

EPSO convergence curve of export polices is presented for n=29 and m=18, (as shown in Fig. 3) to illustrate the role of export shipment policy in cutting down the total channel cost.

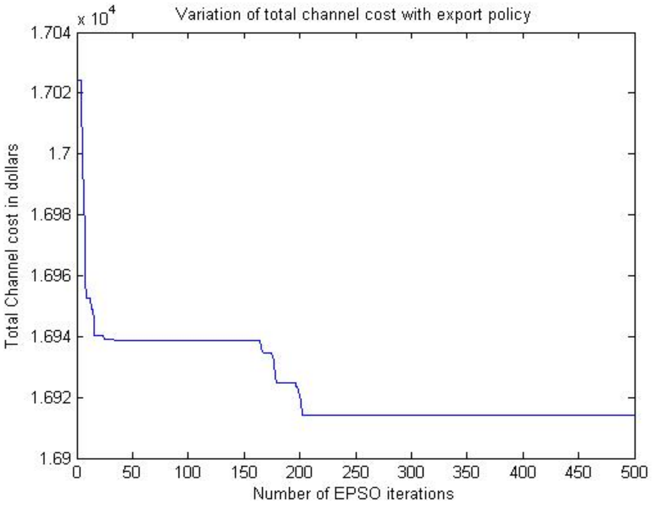

Fig.3 Convergence of export policies using EPSO

APPENDIX

C_{oc} , C_{oa} - ordering costs for the EP and the AP for each shipment respectively

d_j - Number of the component j required for the assembly of one product

D - Demand rate of the product at the assembly plant. (D =Q/T_c)

f_j - Fraction in the k_j time interval T at which the production of the component j stops during the time T_c

H_a - holding costs at the assembly plant expressed as cost per unit per cycle time

H_c - holding costs at the exporting point expressed as cost per unit per cycle time

H_{pj} - holding cost for the manufacturing unit of the component j for a unit inventory/unit time.

k_j - Index of the time interval T in which the production of the component j stops during the time T_c

m - Number of exports made from the exporting point to the assembly plant

n - Number of inland shipments from the manufacturing units to the exporting point during the time interval T_c

P_j - Production rate of the component j where j=1, 2, 3, w and w is number of manufacturing units

T_c - Cycle time

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