Integration of Quality Cost to Economic Service Life

Jirarat Teeravaraprug

Abstract— **Using of the machine for quite a while, the machine seems to be deteriorated and it may not function exactly well. It may result in customer dissatisfaction and quality cost. This paper deals with the determination of economic service life when considering quality cost. The cost of quality is measured by using a well-known quality loss function, quadratic loss function. The result shows that integrating quality cost to the model of economic service life would make the economic service life shorter.**

Index Terms—**Quality cost, economic service life, quality loss function.**

I. INTRODUCTION

You have probably seen a 50 year-old automobile still in service. Provided it received proper repair and maintenance, almost anything can be kept operating for an extended period of time. If we can keep any cars operating for an almost indefinite period, why don't we spot more old cars on the street? There may be several reasons. Some people get tired of driving the same old car. Others may want to keep a car as long as it will last, but they realize that the repair and maintenance costs will become excessive. This is an example of studying economic service life given by [1].

Economic service life is defined as the period of useful life that minimizes the equivalent annual cost of an asset. Normally, to determine the equivalent annual cost of an asset, two types of costs are considered. One type is the investment of an asset and the other type is an asset's operating and maintenance (O&M) costs. In the case of using an asset for a long period of time, the equivalent annual cost of the investment seems to be low whereas that of the operating and maintenance costs tend to be high. On the other hand, in the case of using an asset for a short period of time, the equivalent annual cost of the investment seems to be high whereas that of the operating and maintenance costs tend to be low. Obtaining economic service life then needs to trade-off between those two types of costs.

This paper is not considering an economic service life for any asset. The asset taking into account of this paper is a machine or equipment that can produce goods or services. New machine or equipment normally provides goods or services about the customers' desired target. This paper assumes that only one quality characteristic is taken into an account that is the customer satisfaction. Using of the machine for quite a while, the machine seems to be

deteriorated and it may not function exactly well. It may result in customer dissatisfaction and quality cost. Loss function is then utilized to measure the customer dissatisfaction and quality cost. A popular quadratic loss function is assumed to be the good measure of customer dissatisfaction in this paper. An optimization model of determining an economic service life for a machine with the use of quadratic loss function is provided. A numerical example is given to illustrate the use of the model.

II. ECONOMIC SERVICE LIFE

Economic service life can be called as minimum cost life, retirement life and replacement life. Normally, the topic "economic service life" is seen in Engineering Economy text book. Almost no researcher gives an attention in the topic. It is about how to determine an asset life that minimizes the overall costs. The overall costs comprise initial investment cost, the negative value of trade-in value, and operating and maintenance cost. Each cost generally is calculated by using annual worth (AW) value. Annual worth is a popular analysis technique and applicable to a variety of engineering economy studies. It is the equivalent uniform annual worth of all estimated receipts and disbursements during the life-cycle of the project or alternative. Therefore, the total annual worth (TAW) of the overall costs can be calculated as

$$
\begin{aligned}
\text{TAW} = \ & \text{AW of Initial Investment} \\
& - \text{AW of Trade-in Value} \\
& + \text{AW of Operating and Maintenance Cost} \quad (1)
\end{aligned}
$$

Generally, with each passing year of an asset use, the equivalent annual worth value of operating and maintenance cost increases whereas the equivalent annual worth value of the asset's initial investment decreases. Lastly, the actual trade-in amount or salvage value decreases relative to the first cost. Fig. 1 shows the difference between annual worth value of initial investment and that of trade-in value; and also the annual worth of operating and maintenance cost. It can be seen that the minimum point is the economic service life.
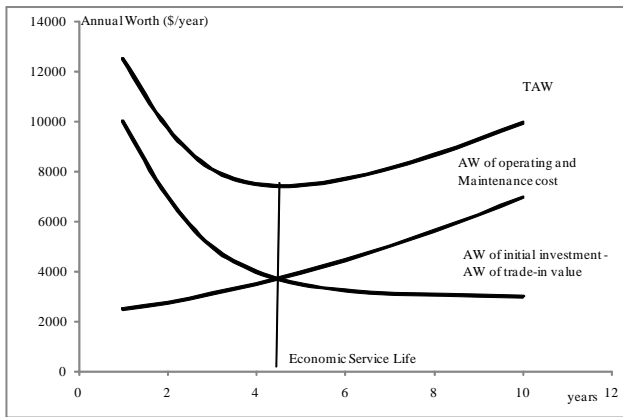
Fig. 1 Annual worth of costs and economic service life

Fig. 2 Quadratic loss function

III. LOSS FUNCTION

Due to product performance variation, a quality evaluation is needed. One of the quality evaluation systems is based on the concept of quality cost. Quality cost is the loss to the customer incurred when the product performance deviates from the customer-desired point. The loss may be estimated by the quality loss function. The quality loss function is a way to quantify the quality cost of a product on a monetary scale when a product or its production process deviates from the customer-desired value for one or more key characteristics. The quality cost includes long-term losses related to poor reliability and the cost of warranty, excess inventory, customer dissatisfaction, and eventually loss of market share.

Even though researchers attempt to construct many types of quality loss functions, there is a general consensus that quadratic loss function may be a better approximation for the measurement of customer dissatisfaction of product quality. Assuming that t is the customer-desired point, the quadratic loss function (L) is defined as Eq. (2) and Fig. 2.

$$L = k(x-t)^2, \qquad (2)$$

where k is a positive loss coefficient based on estimated losses at a given specification limit and x is a quality performance of the considered quality characteristic. Mathematical details of the loss function can be found in [2]. Hence, the well-known expected quality cost based on quadratic loss function is

$$E[L] = k\left[(\mu-t)^2 + \sigma^2\right], \qquad (3)$$

where μ and σ are the mean and standard deviation of quality performance of the considered quality characteristic.

IV. THE MODEL

This paper aims to find an economic service life when considering quality cost. As stated before, the overall costs comprise initial investment cost, operating and maintenance cost, and also the negative value of trade-in value and each cost generally is calculated by using annual worth (AW) value. Therefore, the total annual worth of overall costs (TAW) can be written as:
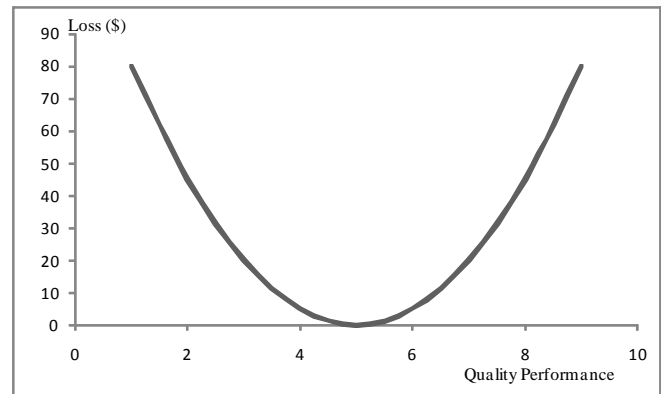
$$TAW = Inv(A/P,i,n) - SV(A/F,i,n)$$
$$+ \sum_j OC_j(P/F,i,j)(A/P,i,n), \qquad (4)$$

where Inv is an initial investment of the asset, $(A/P,i,n)$ is the capital-recovery factor (CRF) using for transforming present value to be annual value of n period with the discounted rate i, SV is the trade-in value or salvage value, $(A/F,i,n)$ is the sinking-fund factor (SFF) using for transforming future value of period n to be annual worth of n period with the discounted rate i, OC_j is operating and maintenance cost of period j with the discounted rate i, $(P/F,i,j)$ is the single-payment factor (SPPWF) using for transforming future value of period j to be present value with the discounted rate i.

Incorporating the quality loss function to the model in (4), the overall costs turn to be:

$$TAW = Inv(A/P,i,n) - SV(A/F,i,n)$$
$$+ \sum_j OC_j(P/F,i,j)(A/P,i,n)$$
$$+ \sum_j \left[k_j(\mu_j - t_j)^2 + \sigma_j^2\right](P/F,i,j)(A/P,i,n). \qquad (5)$$

The last term of Eq.(5) is the expected value of quality cost when measuring by quadratic loss function. k_j represents loss coefficient of period j, μ_j is the mean of quality performance of period j, t_j is the customer-desired target value of period j, and σ_j^2 is the variance of quality performance in period j. It is assumed that in every period, the mean of quality performance would be equal to its customer-desired target value due to maintenance. TAW then turns to be:

$$TAW = Inv(A/P,i,n) - SV(A/F,i,n)$$
$$+ \sum_j OC_j(P/F,i,j)(A/P,i,n)$$
$$+ \sum_j \sigma_j^2(P/F,i,j)(A/P,i,n). \qquad (6)$$

The objective is to find an economic service life (*n*) that minimize the total annual worth of overall costs (TAW) as shown in Eq. (6). The numerical example presented in the next section would illustrate the use of the model.

V. NUMERICAL EXAMPLE

A machine has an initial investment as $25,000, and its operating and maintenance cost (OC) and trade-in value are shown in Table 1. The discounted rate is assumed 10 percent per year. In the case of not considering quality cost, Table 2 shows the annual worth of initial investment, trade-in value, operating and maintenance cost, and total overall costs. It can be seen that the total overall costs of year 5 is minimum. That means the economic service life is 5 years.

In the case of considering quality cost, it is assumed that the variance of quality performance is enlarged in each passing year of the machine use. A linear function is utilized to capture the increment of variance. The interception and coefficient of the linear function are 20 and 3, respectively. The quadratic loss is used to measure the quality cost due to deviation of quality performance. The loss coefficient is 100. Table 3 shows the costs when considering quality cost. It can be seen that the economic service life turns to be 4 years. Fig. 3 shows the costs of the example both with and without considering quality cost.

Table 4 shows the sensitivity analysis of TAW when

changing the coefficient of the variance linear function. It is found that the economic service life is changed from 4 years to 3 years when the coefficient is changed from 5 to 7. Then the economic service life is constant at 3 years. Table 5 shows the sensitivity analysis of loss coefficient when changing the loss coefficient from 100 to 300. It is shown that the economic service life is changed from 4 years to 3 years when the loss coefficient is changed from 180 to 200.

VI. DISCUSSION

Based on the numerical example, it can be seen that by integrating quality cost to the economic service life, it does not show highly affect in the analysis. The reason of that is TAW around economic service life seems to be insensitive. Fig. 1 shows that TAW is almost flat around the economic service life. However, by integrating quality cost, the economic service life is changed from 5 years to 4 years. Further, if the variance coefficient and loss coefficient are increased, the economic service life seems to be shorter.

REFERENCES

[1] C. S. Park, *Contemporary Engineering Economics*, Addison-Wesley, CA., 1997.
[2] B. R. Cho and M. S. Leonard, "Identification and extensions of quasiconvex quality loss functions," *Int J Reliability Qual Saf Eng*, vol. 4, pp. 191-204.

Table 1 Numerical example data

year	1	2	3	4	5	6	7	8	9	10
Trade-in value	15000	13000	11000	9000	8000	5000	5000	1000	800	600
OC	6100	6800	7800	9300	11400	14000	14000	25000	30000	35000

Table 2 Annual worth of initial investment, trade-in value, OC, and TAW when not considering quality cost

Year	1	2	3	4	5	6	7	8	9	10
Investment	27,500	14,405	10,053	7,887	6,595	5,740	5,135	4,686	4,341	4,069
Trade-in value	-15000	-6190	-3323	-1939	-1310	-648	-527	-87	-59	-38
OC	6,100	6,433	6,846	7,375	8,034	8,807	9,355	9,376	10,895	12,407
TAW wo Quality Cost	18,600	14,648	13,576	13,322	13,319	13,900	13,963	13,975	15,177	16,438

Table 3 Annual worth of initial investment, trade-in value, OC, and TAW when considering quality cost

year	1	2	3	4	5	6	7	8	9	10
Inv-Trade-in	12,500	8,214	6,730	5,948	5,285	5,092	4,608	4,599	4,282	4,031
OC	6,100	6,433	6,846	7,375	8,034	8,807	9,355	9,376	10,895	12,407
Loss	6,900	7,329	7,743	8,143	8,529	8,901	9,259	9,604	9,935	10,253
TAW w Quality Cost	25,500	21,976	21,319	21,466	21,848	22,801	23,222	23,579	25,112	26,691
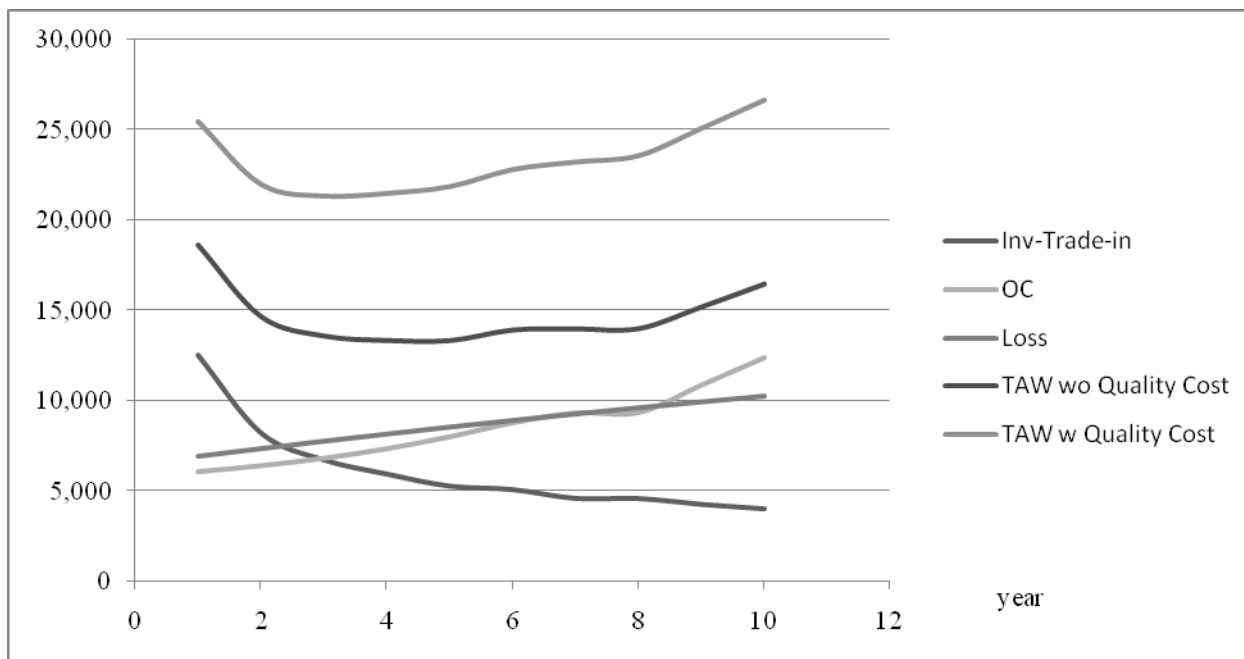
Fig. 3 Annual worth of overall costs

Table 4 Sensitivity analysis of variance coefficient

year	1	2	3	4	5	6	7	8	9	10
3	20900.00	17090.48	16156.80	16036.82	16161.83	16866.66	17049.36	16781.84	18123.43	19513.66
5	21100.00	17385.71	16544.11	16513.06	16723.86	17511.37	17773.68	17448.07	18873.16	20341.83
7	21300.00	17680.95	16931.42	16989.29	17285.88	18156.08	18498.01	18114.30	19622.88	21170.00
9	21500.00	17976.19	17318.73	17465.52	17847.91	18800.79	19222.33	18780.53	20372.60	21998.17
11	21700.00	18271.43	17706.04	17941.76	18409.93	19445.50	19946.65	19446.77	21122.33	22826.35
13	21900.00	18566.67	18093.35	18417.99	18971.96	20090.21	20670.98	20113.00	21872.05	23654.52
15	22100.00	18861.90	18480.66	18894.23	19533.98	20734.93	21395.30	20779.23	22621.77	24482.69
17	22300.00	19157.14	18867.98	19370.46	20096.01	21379.64	22119.62	21445.46	23371.50	25310.86
19	22500.00	19452.38	19255.29	19846.69	20658.03	22024.35	22843.95	22111.69	24121.22	26139.03

Table 5 Sensitivity analysis of loss coefficient

year	1	2	3	4	5	6	7	8	9	10
100	20900.00	17090.48	16156.80	16036.82	16161.83	16866.66	17049.36	16781.84	18123.43	19513.66
120	21360.00	17579.05	16672.99	16579.69	16730.44	17460.07	17666.66	17343.23	18712.71	20128.70
140	21820.00	18067.62	17189.18	17122.56	17299.05	18053.48	18283.95	17904.63	19301.98	20743.75
160	22280.00	18556.19	17705.38	17665.43	17867.65	18646.90	18901.25	18466.02	19891.26	21358.79
180	22740.00	19044.76	18221.57	18208.30	18436.26	19240.31	19518.55	19027.42	20480.53	21973.84
200	23200.00	19533.33	18737.76	18751.17	19004.87	19833.72	20135.85	19588.81	21069.81	22588.89
220	23660.00	20021.90	19253.96	19294.04	19573.48	20427.14	20753.14	20150.20	21659.08	23203.93
240	24120.00	20510.48	19770.15	19836.91	20142.08	21020.55	21370.44	20711.60	22248.36	23818.98
260	24580.00	20999.05	20286.34	20379.78	20710.69	21613.96	21987.74	21272.99	22837.63	24434.02
280	25040.00	21487.62	20802.54	20922.65	21279.30	22207.38	22605.03	21834.39	23426.91	25049.07
300	25500.00	21976.19	21318.73	21465.52	21847.91	22800.79	23222.33	22395.78	24016.18	25664.12