Discount Pricing Model to Coordinate a Supply Chain under Stochastic Demand Environment

S. P. Sarmah*, Santanu Sinha

Abstract—**Here, we have studied a single vendor and multi buyer coordination problem through price discount mechanism. The vendor offers multiple pricing schedules to the retailers and each retailer selects a schedule that maximizes the corresponding individual profit. The model developed here takes into consideration the reaction of each buyer and ensures that the motivation of each retailer to select local optima can also lead to global optima with maximum channel profits. A numerical example is carried out and the results show that channel profit increases with the increase in the number of pricing schedules.**

Index Terms—**common order interval, discount schedule, heterogeneous buyers, supply chain coordination.**

I. Introduction

One of the important issues in supply chain management (SCM) is to coordinate between different members of a supply chain. In the literature, many contracts have been discussed to coordinate a supply chain. Among the different coordination mechanisms mentioned in the literature, quantity/price discount is a very popular and efficient mechanism often used by many organizations to coordinate the business activities [1]. Authors [2] have examined a discount-pricing model where the buyers and the supplier have a common order interval time so that supplier's finished goods are directly shipped to the buyer. Authors [3] have considered a one-vendor, multi-buyer supply chain for a single product.

From the literature, it has been found that pricing policy with single schedule is not optimal in a case where the vendor deals with many heterogeneous buyers [4]. When the buyers are heterogeneous in nature, finding the suppliers' optimal pricing schedule is difficult and this has motivated us to design a single vendor-multi-buyer coordination model with multiple price-discount schedules. In the model, each pricing schedule offered by the vendor is represented by a selling price and common order interval to influence the buyers' order policy. Both discounted price and order interval are the decision variables in the model. The vendor first decides the number of discount schedules to be offered and then optimally derive each schedule such that each buyer and the vendor receive Pareto-optimal pay-offs. Under this framework, the supplier announces its pricing scheme to all the buyers and each buyer selects the schedule that maximizes the corresponding profit/benefits. The modeling effort ensures that each buyer's selection of local optima also leads to global optima. The inherent complexity of the problem has been handled efficiently by the application of Genetic Algorithm [5].

The rest of the paper is organized as follows: Section 2 contains the notations and the modeling assumptions. Section 3 shows the mathematical model while the solution methodology is included in Section 4. A computational study is carried out in Section 5. Finally, in section 6, conclusion and future scope of work are mentioned.

II. Development of the Mathematical Model

The following notations and major assumptions are considered in the development of the model.

Notations

n	Number of buyers
μ_i	Mean annual demand for buyer *i*, where, $i = 1\ldots.n$.
σ_i^2	Variance of annual demand for buyer *i*
K_i	Fixed ordering cost for buyer *i*
h_i	Holding cost rate for buyer *i*
LT_i	Lead time for replenishment from the vendor to buyer *i*
$1-\alpha_i$	Service level of buyer *i*
z_i	Standard normal inverse value corresponding to cumulative probability of $1-\alpha_i$
A_i	Setup/ order processing cost incurred by vendor due to buyer *i*
CV	Coefficient of variation of annual demand
T_i	Order interval of buyer *i*
T_i^*	Economic Order interval of buyer *i*
P_o	Unit production/procurement cost of the seller
P	Un-discounted per-unit selling price of the seller
P_r	Selling price of the buyer/retailer
$B_i(.)$	Profit function of buyer *i* and *B* total profit function of all buyers
$S_i(.)$	Profit function of the seller due to buyer *i*

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$S=\sum_{1}^{n} S_i(.)$ Total profit function of the seller considering all the buyers

b_i Coordination benefit of buyer *I and b* is the summation of all the buyers benefits

s_i Coordination benefit of the seller due to buyer *i* and s is due to all the buyers

$\gamma=\frac{b}{s}$ An index to split the coordination benefit

In the model, the seller uses a lot for lot policy and it is assumed that $\left(P_r > P > P_o\right)$. Further, it is also assumed that all the buyers are rational and avail any discount resulting a better (or at least just the same) pay-off as compared to no-discount scenario. The modeling efforts ensure that each buyer receives a pay-off that is higher than that of under no-discount scenario. Finally, the model also aims to split the coordination benefit at an equitable fashion known as, 'social welfare solution' (i.e., $\gamma \approx 1$).

III. Model formulation

The mathematical model under stochastic demand environment is developed on two different cases, namely, (i) the seller does not offer any discount and (ii) the seller offers discounts to the buyers. The 'no-discount' approach is considered as the benchmark of the model to show the efficacy of the coordinated model with price discounts.

A. No-discount policy

With un-discounted sell price P of the seller, the gross profit of the buyer i is given by,
Profit = Sales revenue – order cost – inventory holding cost – safety stock cost

$$\Pi_{bi}=\left(P_r-P\right)\mu_i-\left(\frac{K_i\mu_i}{Q_i}+\frac{1}{2}h_iQ_i+h_iz_i\sigma_i\sqrt{LT_i}\right) \tag{1}$$

Differentiating (1), one gets, $T_i^*=\sqrt{\frac{2K_i}{\mu_i h_i}}$. Thus, the optimal profit of the i^{th} buyer will be,

$$\Pi_{bi}^*=\left(P_r-P\right)\mu_i-\left(\sqrt{2K_i\mu_i h_i}+h_iz_i\sigma_i\sqrt{LT_i}\right) \tag{2}$$

And, $\Pi_b^*=\sum_{i=1}^{n}\Pi_{bi}^*$. Corresponding to the ordering policy of each buyer, the seller's cumulative gross profit is given as, Profit = Revenue – Setup cost

$$\Pi_s^*=\sum_{i=1}^{n}\left[\left(P-P_o\right)\mu_i-\frac{A_i\mu_i}{Q_i^*}\right]=\sum_{i=1}^{n}\left[\left(P-P_o\right)\mu_i-A_i\sqrt{\frac{h_i\mu_i}{2K_i}}\right] \tag{3}$$

System or Channel Profit is given as,

$$\Pi_{ch}=\left(\Pi_b^*+\Pi_s^*\right)=\sum_{i=1}^{n}\left[\left(P_r-P_0\right)\mu_i-\left\{\begin{array}{l}\sqrt{2K_i\mu_i h_i}+h_iz_i\sigma_i\sqrt{LT_i}\\+A_i\sqrt{\frac{h_i\mu_i}{2K_i}}\end{array}\right\}\right]$$

B. Discount policy

In the following sub-sections, we have formulated the pricing policies with different number of price break schedules, such as, single, double and multiple price breaks - each focusing on the maximization of the channel coordination benefit with the constraint that no party is worse off as compared to the earlier un-discounted policy.

1) Discount policy with single pricing schedule

Let there be a pricing schedule of $\{p',T'\}$ where, discounted price is equal to p' and correspondingly, the common interval time is T'. The profit of the i^{th} buyer is given as,

$$\Pi'_{bi}=\left(P_r-p'\right)\mu_i-\left[\begin{array}{l}\frac{K_i}{T'}+\frac{1}{2}T'\mu_i h_i+\\h_iz_i\sigma_i\sqrt{LT_i+T'}\end{array}\right]$$

(4)

Total profit of all the buyers is obtained as, $\Pi'_b=\sum_{i=1}^{n}\Pi'_{bi}$

The gross profit for the seller is given as

$$\Pi'_s=\sum_{i=1}^{n}\left[\left(p'-P_o\right)\mu_i-\frac{A_i}{T'}\right] \tag{5}$$

Therefore, the channel profit is obtained as follows,

$$\Pi'_{ch}=\left(\Pi'_b+\Pi'_s\right)=\sum_{i=1}^{n}\left[\begin{array}{l}\left(P_r-P_0\right)\mu_i-\\\left\{\frac{K_i}{T'}+\frac{1}{2}T'\mu_i h_i+h_iz_i\sigma_i\sqrt{LT_i+T'}+\frac{A_i}{T'}\right\}\end{array}\right]$$

(6)

Hence, coordination benefit of the i^{th} buyer is found as,

$$b_i=\left(\Pi'_{bi}-\Pi_{bi}^*\right)$$
$$=\left[\begin{array}{l}\left(P-p'\right)\mu_i-\left\{\frac{K_i}{T'}+\frac{1}{2}T'\mu_i h_i+h_iz_i\sigma_i\sqrt{LT_i+T'}\right\}\\-\left\{\sqrt{2K_i\mu_i h_i}+h_iz_i\sigma_i\sqrt{LT_i}\right\}\end{array}\right]$$

(7)

Coordination benefit of all the buyers, $b=\sum_{i=1}^{n}b_i$. Similarly, coordination benefit of the seller is,

$$s=\left(\Pi'_s-\Pi_s^*\right)=\sum_{i=1}^{n}\left[\left(p'-P_o\right)\mu_i-\frac{A_i}{T'}\right]-\Pi_s^*$$
$$=\sum_{i=1}^{n}\left[\left(p'-P\right)\mu_i-\frac{A_i}{T'}-A_i\sqrt{\frac{h_i\mu_i}{2K_i}}\right]$$

(8)

Thus, system-wide coordination benefit is given as, $C=(b+s)$. The objective of the model is,

$$\textit{Maximize}, \quad C(p',T'), \tag{9}$$

such that, (i) $b_i \geq 0$, (ii) $s \geq 0$, and (iii) $\gamma \approx 1$, where, $\gamma = b/s$.

2) Discount policy with two pricing schedules

Here, a discount pricing model with two schedules $\{p_1, T_1\}$ and $\{p_2, T_2\}$, i.e. $\{p_i, T_i\}$, i = 1, 2, are considered respectively. In this case, the profit of the i^{th} buyer can be written as,

$$\Pi''_{bi} = \left(\lambda_{i1}\Pi''_{bi(1)} + \lambda_{i2}\Pi''_{bi(2)}\right), \tag{10}$$

where, λ_{i1} and λ_{i2} are binary numbers (0/1) such that, $(\lambda_{i1}+\lambda_{i2})=1$, and,

$$\Pi''_{bi(j)} = (P_r - p_j)\mu_i - \left[\frac{K_i}{T_j} + \frac{1}{2}T_j\mu_i h_i + h_i z_i \sigma_i \sqrt{LT_i + T_j}\right]$$

$$\text{for } (j=1,2), \tag{11}$$

Thus, coordination benefit of the i^{th} buyer,

$$b_i = \left(\lambda_{i1}\Pi''_{bi(1)} + \lambda_{i2}\Pi''_{bi(2)}\right) - \Pi_{bi}^{*} \tag{12}$$

Similarly, profit of the seller,

$$\Pi''_s = \sum_{i=1}^{n}\left[\lambda_{i1}\left\{(p_1 - P_o)\mu_i - \frac{A_i}{T_1}\right\} + \lambda_{i2}\left\{(p_2 - P_o)\mu_i - \frac{A_i}{T_2}\right\}\right] \tag{13}$$

And, coordination benefit of the seller, $s = \left(\Pi''_s - \Pi_s^{*}\right)$ (14)

Thus, system-wide coordination benefit is given as, $C=(b+s)$. The objective of the model is,

$$\text{Max.} \quad C(p_1, T_1, p_2, T_2), \tag{15}$$

such that, (i) $b_i \geq 0$, (ii) $s \geq 0$, (iii) $\gamma \approx 1$, where, $\gamma = b/s$, (iv) $(\lambda_{i1}+\lambda_{i2})=1$,

(v) $$\lambda_{i1} = \begin{cases} 1 & \text{if } B_i(p_1,T_1) > B_i(p_2,T_2), \\ 0 & \text{otherwise} \end{cases}$$

Modeling with more number of discount pricing schedules is a straightforward extension of the above mentioned procedure.

IV. Solution Procedure

Genetic Algorithm (GA) is applied here to efficiently solve this complex problem. The following steps are taken to solve the problem.

Step 1: Initialize the population for the variable set $\{p_i, T_i\}$

Step 2: Evaluation of fitness function

Step 2.1: For each individual buyer *i*: compute coordination benefit (b_i) corresponding to each group: (p_1,T_1), (p_2,T_2),...., (p_k,T_k).

Step 2.2: Assign the i^{th} buyer in the group such that b_i is maximum

Step 2.3: Calculate s_i and s

Step 2.4: Evaluate fitness value from objective function and constraint functions for each of these solutions

Step 3: Perform crossover, mutation operation on population.

Step 4: Repeat step 2 to step 3 till termination criteria is reached.

We have formatted the problem in MS Excel and have applied a GA tool, 'EVOLVER' (source: *http://www.palisade.com.au/evolver*) embedded in MS Excel as an add-in feature and optimized the variables towards deriving maximum channel coordination benefit with the constraint that neither the buyers and nor the seller receive lesser profit as compared to no-discount 'benchmark' scenario.

V. Numerical Example

The following data (Table 1) are considered for the numerical study. The assumed data set is very similar to that of [2](Chakravarty and Martin, 1988). All the problems are solved on a Pentium IV computer with 512MB RAM and 1.83GHz CPU.

Table 1: Data set for the buyers

Buyer(i)	K(i) \$ per order	μ(i)	*h(i)* \$ per unit Per year
1	58	414	2.98
2	66	400	3.00
3	99	211	2.95
4	67	351	2.90
5	52	1341	3.00
6	77	550	2.84
7	58	1340	2.75
8	68	959	2.95
9	51	217	3.02
10	100	1485	2.90

Table 2: Data set for the seller

A_i	P_o	P	P_r	LT_i	1-α_i
\$500 per set up	\$15 per unit	\$25 per unit	\$40 per unit	30 days	95%

A. No-discount scenario (CV=0.0)

Table 3 shows the total profit received by all the buyers, the seller and the system as a whole, when CV=0.0. This is used as the benchmark solution to calculate the coordination benefit at CV=0.0.

Table 3: Profit when CV=0.0

Profit			γ
Buyer	Vendor	System	
103924.9	53887.3	157812.3	1.9

B. Price discount policy under complete demand information (CV=0.0)

Table 4 shows, the coordination benefits received by all the buyers, the seller and the system as a whole, when CV=0.0. It also shows how the coordination benefit increases with the increase in number of groups. However, we have not found any optimal solution with more than four number of discount schedules and maximum payoffs are received with four number of discount schedules. Table 4 also shows how the buyers are included in different groups.

Table 4: Coordination benefit when CV=0.0

No. of Group(s)	Optimal Pricing Schedule	Buyer (i)	Coordination benefit			γ
			Buyer	Vendor	System	
1	{23.93, 0.73}	all	4184.37	4184.37	8368.74	1.00
2	{24.12, 0.55} {23.55, 1.00}	5, 7, 8, 10 1, 2, 3, 4, 6, 9	4559.93	4538.93	9098.86	1.005
3	{23.64, 0.93} {23.36, 1.15} {24.11, 0.55}	1, 2, 6 3, 4, 9 5, 7, 8, 10	4587.49	4574.98	9162.47	1.003
4	{23.64, 0.92} {23.04, 1.39} {24.11, 0.55} {23.34, 1.15}	1, 2, 6 3 5, 7, 8, 10 4, 9	4596.71	4581.38	9178.10	1.003

C. Price discount policy under incomplete demand information

Similarly, we have examined how the coordination benefit is influenced with the change in CV values and in the following table we have shown the result only with respect to CV = 0.05. Other values of CV for which numerical results are obtained are 0.01, 0.25, 0.50, and 1.0

Table 5: Coordination benefit when CV=0.05

No. of Group(s)	Optimal Pricing Schedule	Buyer (i)	Coordination benefit			γ
			Buyer	Vendor	System	
1	{23.94, 0.67}	all	3650.21	3650.21	7300.42	1.00
2	{24.10, 0.53} {23.44, 1.00}	5, 7, 8, 10 1, 2, 3, 4, 6, 9	4065.36	4032.82	8098.18	1.008
3	{23.60, 0.89} {24.09, 0.53} {23.29, 1.11}	1, 2 5, 7, 8, 10 3, 4, 6, 9	4084.94	4017.73	8102.66	1.017
4	{23.59, 0.89} {22.95, 1.34} {24.09, 0.53} {23.27, 1.12}	1, 2, 6 3 5, 7, 8, 10 4, 9	4128.75	4046.92	8175.68	1.02

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D. Results and discussions

The results show how the performance of the supply chain in terms of system profit increases with increase in the number of price schedules (see Fig. 1). However, with more number of price schedules, the solution space becomes narrower and beyond a particular value, the solution becomes highly complex and computationally intractable. Another important observation is that coordination benefit decreases with the increase in demand variability. Fig. 2 shows the impact of demand variability on coordination benefit.

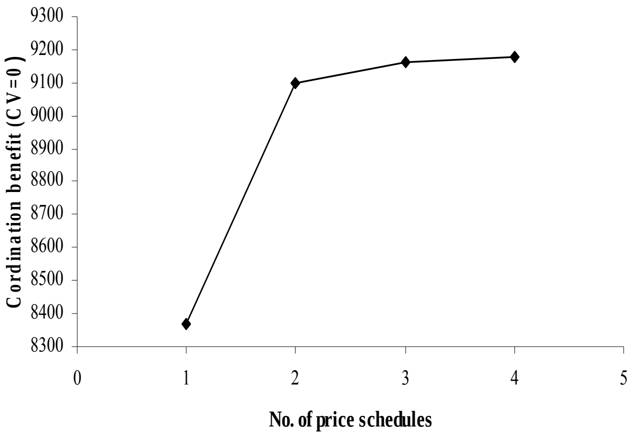

Figure 1: Coordination Benefit Vs No. of price schedules

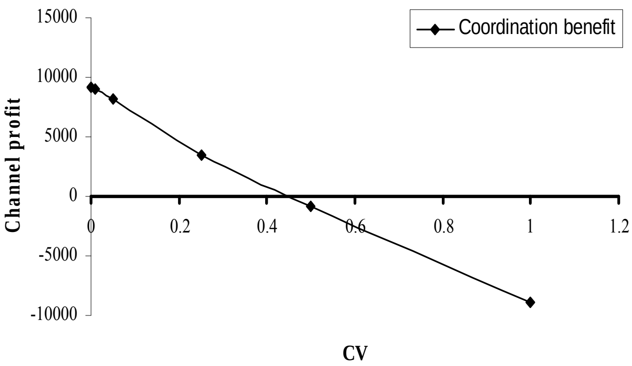

Figure 2: Coordination Benefit Vs CV

VI. Conclusions

In this paper, we have formulated pricing policies with different number of price break schedules each focusing on the maximization of the channel coordination benefit with the constraint that no party is worse off as compared to the earlier un-discounted policy. The components of each price schedule are, a discounted price and correspondingly a common (group) order interval time. The results show that with the increase in number of pricing schedule, the channel profit (coordination benefit) also increases up to a particular value for a specific problem; after which further improvement becomes computationally complex and hence intractable. The results also show that under the stochastic demand environment, the system profit is maximum when coefficient of variation (CV) = 0 (i.e. deterministic case) and with increase in the value of CV, the performance starts to deteriorate. Therefore, coordination through discount policy may not be an efficient mechanism to enhance channel profitability when CV is high. The present model can have relevant industrial application for its simple and efficient approach.

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