Metal Cutting Optimization Using Graphical Method

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***Abstract*—Metal cutting optimization has significant practical importance. Due to high cost of CNC machines, there is a need to operate them at optimal cutting conditions. Since the cost of machining depends on cutting variables, the optimum values of these variables have to be determined before a part is put into production. The usual cutting variables are cutting speed, feed, and depth of cut. The objective criterion can be cost or time (production rate) or profit rate. The constraints can be cutting speed, feed, depth of cut, surface roughness, force, power, etc. In the present work, an attempt has been made to arrive at optimal cutting conditions in single pass plain turning by graphical method. The cost objective function is derived using extended tool life equation and constraints such as cutting speed, feed, surface roughness, force, and power are considered. A case study is undertaken and optimal point based on cost objective is identified graphically. A computer program is developed using C++ language.**

***Index Terms*— metal cutting, optimization, turning, geometric programming.**

I. Introduction

The turning is the most commonly used metal cutting operation and selection of optimal machining parameters is of great interest to field engineer. The technology developments in the past has made it possible to improve not only accuracy of job but also productivity by adopting optimization methods in the metal cutting operation. In the past, the selection of cutting parameters was made either by experienced machining expert or by use of handbooks of metal cutting; the selected parameters need not be optimal.

The cutting conditions such as cutting speed, feed rate, and depth of cut affect the production rate, product quality, and production cost of a component during turning operation. With the use of sophisticated and high cost CNC machines coupled with high labour cost, the selection of optimal cutting conditions are essential. In optimization of machining operation, the following three basic criteria are used for selection of machining parameters: (i) the minimum production cost criterion, (ii) the minimum production time or maximum production rate criterion, and (iii) the maximum profit rate criterion.

Optimality conditions for minimum cost and maximum production rate are different.

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The selection of optimal metal cutting parameters, i.e., speed, feed, and depth of cut, is usually made by constructing a mathematical model based on some optimization criterion and then to solve the problem taking into account the constraints affecting the solutions.

In general, optimization of turning problem is nonlinear problem, involving nonlinear objective function and linear/nonlinear constraints. The problem is required to be formulated with cutting speed, feed, and depth of cut as variables. Many investigators analyzed single pass with single tool plain turning operations as the problem of optimization. The single pass analysis addresses the problem of determining optimum values of cutting speed and feed rate for a given depth of cut.

II. Literature Survey

The optimization of plain turning has been investigated by many authors.

Ermer [1] illustrated geometric programming technique to determine the optimum machining conditions when solution is restricted by inequality constraints. He considered cutting velocity and feed as the variables and the cost objective only, involving constraints of the feed and the surface finish.

Eskicioglu *et al*. [2] used geometric programming for problem of optimizing single pass, plain turning operation for cost and time objectives involving velocity, feed, power, surface finish, force, deflection and tolerance constraints. They indicated the problem of optimizing a single pass turning operation is actually one degree of difficulty problem, regardless to number of constraints used, using the method of relaxed constraints.

Wu and Ermer [3] considered unconstrained single pass, plain turning problem for optimization analysis using maximum profit rate criterion involving only cutting speed as the variable.

Walvekar and Lamber [4] discussed the use of geometric programming technique to select machining variables with feed and power as constraints.

Khare and Agrawal [5] developed neural network software for optimization of cutting speed in single pass, plain turning, considering cost as well as time objective.

III. Formulation

The cost criterion for single pass is used in the present work with the assumptions as listed below.

1) Tool life is not a random variable.
2) The Taylor's modified tool life equation is considered for the formulation of problem.
3) The flank wear land curves are assumed to be linear.

4) Supports, fixtures and work holding devices are assumed rigid.

The objective function for cost criterion is expressed as a function of cutting parameters such as machine and labour rate (M), tool changing time (Tct), cost of tool (Ct), workpiece diameter (D), length of cut or pass (L), cutting speed (V), feed (f), and depth of cut (d) for the plain turning.

The formulation of objective function (cost) with suitable constraints is described below.

A. Cost per Component (COST)

The machining cost per component is made up of number of different costs, such as cost of nonproductive time (C1), cost of machining time (C2), cost of tool changing time (C3), and a tool cost per component (C4).

The cost of nonproductive time (C1) includes loading and unloading time of the job and tool as well as the idle time. If T1 is the total nonproductive time and M is a machine and labour rate, then

$$C1 = M \cdot T1 \quad (1)$$

The cost of machining time (C2) is obtained by multiplying the machine and labour rate (M) with machining time (Tm) per component as given below.

$$C2 = M \cdot Tm \quad (2)$$

The cost for tool changing time per component (C3) is calculated as:

$$C3 = M \cdot Tct \cdot (Tm / T) \quad (3)$$

The tool cost per component (C4) is determined as

$$C4 = Ct \cdot (Tm / T) \quad (4)$$

The total cost per component, excluding the material cost can be expressed as

$$COST = C1 + C2 + C3 + C4$$

$$COST = M \cdot T1 + M \cdot Tm + (M \cdot Tct + Ct) \cdot (Tm / T) \quad (5)$$

The Taylor's modified tool life equation considered for formulation of optimization problem is

$$V \cdot f^{a1} \cdot d^{a2} \cdot T^{a3} = K \quad (6)$$

The machining time of a component having diameter D and length L for a pass can be written as:

$$Tm = L / (f \cdot N) = L \cdot f^{-1} \cdot N^{-1}$$

$$\text{or } Tm = (\pi \cdot D \cdot L / 1000) \cdot V^{-1} \cdot f^{-1} \quad (7)$$

Using equations (5) to (7), the objective function for cost criterion is expressed as

$$COST = K_{01} \cdot V^{-1} \cdot f^{-1} + K_{02} \cdot V^{(1/a3)-1} \cdot f^{(a1/a3)-1} + M \cdot T_1 \quad (8)$$

Where, $K_{01} = M \cdot (\pi \cdot D \cdot L / 1000)$ and

$$K_{02} = (M \cdot T_{ct} + C_t) \cdot (\pi \cdot D \cdot L / 1000) \cdot (1/K)^{1/a3} \cdot d^{a2/a3}$$

B. Constraints

The following physical constraints and limitations due to various considerations are used in the formulation of optimization model.

Machine tool speed restriction

Any conventional machine tool has finite number of discrete rotational speeds starting from some minimum to a maximum value. For CNC machines also, the speed can vary continuously in a specified range. The optimal cutting speed (V) or rotational speed (N) should be within the available extreme limits.

Machine tool feed restriction

Feed has more significant impact on tool life than depth of cut. In general, the maximum feed in rough turning is limited by the maximum force that the cutting tool, machine tool, work piece and fixture are able to withstand. The maximum feed in finish turning operation is limited by the surface finish requirement. The selected feed should be within the maximum and minimum feed available on the machine tool.

Depth of cut restriction

The depth of cut has an important effect on the tool life. But, in general, the tool life is less affected by changes in depth of cut than by changes in speed or feed. The selection of maximum depth of cut is dependent on tool material and geometry, the cutting force, the available machine power, and the surface finish requirement. The minimum depth of cut depends on the selected machine which allows particular depth of cut setting. Hence, the depth of cut should not exceed the limiting values.

Surface roughness restriction

The value of surface roughness obtained on a component after machining depends on tool geometry, cutting speed, feed, depth of cut, etc. The effect of tool geometry on surface roughness is not considered in the present work.

Power restriction

The power required to cut depends on the cutting parameters selected. Because of the power capacity of the selected machine, the cutting parameters should be selected keeping in view power restrictions of selected machine.

Temperature restriction

The temperature developed during cutting depends on the cutting parameters; hence, the selection of cutting parameters has to be made such that the temperature evolved may not cross the maximum allowed temperature.

Cutting force restriction

The cutting force required to cut metal depends on the cutting parameters selected. The cutting force is restricted because it affects the cutting operation. The accuracy of job depends on the force because the force may deflect either job or tool or both.

The constraints considered [6] in the present work are shown in Table I.

Table I. List of constraints considered

(1) Speed : $V_{min} \le V \le V_{max}$
(2) Feed: $f_{min} \le f \le f_{max}$
(3) Depth of cut: $d_{min} \le d \le d_{max}$
(4) Surface roughness: $14785\ V^{-1.52} \cdot f^{1.004} \cdot d^{0.25} \le SR_{max}$
(5) Power: $0.0372 \cdot V^{0.91} \cdot f^{0.78} \cdot d^{0.75} \le PO_{max}$
(6) Temperature: $74.96 \cdot V^{0.4} \cdot f^{0.2} \cdot d^{0.105} \le TEMP_{max}$
(7) Force: $844 \cdot V^{-0.1013} \cdot f^{0.725} \cdot d^{0.75} \le FO_{max}$

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IV. Methodology

The authors [1], [2], [7] - [10] have applied geometric programming technique for metal cutting optimization in turning operation. Abuelnaga and EL- Dardiry [11] stated that geometric programming is one of the best methods developed in optimization theory. Gopalkrishnan [8] accepted that the metal cutting optimization problem involving more than two constraints is cumbersome to solve using geometric programming, however Eskicioglu [2] indicated that such problem involving any number of constraints are possible to be solved using method of relaxed constraints. The geometric programming is popularly accepted as an optimization method for such problems.

A graphical presentation of turning optimization problem, as an illustration can be helpful for understanding methodology of searching optimal solution. For given objective, “iso-contours” are drawn together with constraint functions. User can visually inspect the relative position of contours with respect to constraint, optimal point, effect of constraints, and input parameters on optimal solution.

The authors have also used geometric programming method with relaxed constraints approach. The geometric programming is based on “the arithmetic - geometric mean inequality relationship”. Constraints are considered one by one, assuming other constraints as relaxed [2]. The other constraints are then involved and checked for feasibility and optimality. If any constraint is not satisfied, the constraint in question is considered as tight and the solution is modified. The process is continued until all the constraints are satisfied simultaneously. The original primal problem, with any objective and any one constraint, such as maximum feed, surface roughness or force is converted into dual geometric programming problem with zero degree difficulty. The dual problem will have maximization objective function with linear constraints in three dual variables, each corresponding to each term of objective function and constraint. The values of dual variables are obtained solving three simultaneous linear equations in dual variables. The values in turn give optimal value of objective function of dual, and hence primal. From the values of dual variables corresponding to terms of objective function and the optimal value of objective function, the values of variables of primal objective function, i.e. speed and feed are obtained.

In general, the objective function depends on several variables and it is difficult to visualize graphically the results for such multidimensional problem, even for a case of single pass turning. A simplified, two variable problem has been considered, for understanding graphically, such optimization problem. The objective function is assumed to depend on speed and feed for a given depth of cut.

V. Graphical Method

A computer program in C++ is developed to plot objective function curves as well as constraint lines for a given case.

Based on constraint lines, feasible region can be identified. Using “iso-cost” curves, the optimal point, giving optimal solution, can be located.

The graphical method is presented for a case, considering the following data:

Machine and labour rate (M) = 2 Rs/min,

Length of the job (L) = 500 mm,

Diameter of the job (D) = 250 mm,

Tool Changing time (T_{ct}) = 5 min,

Cost of a tool (C_t) = 100 Rs./Edge,

Depth of cut (d) = 1 mm,

Handling time (T_1) = 1.5 min,

Minimum revolution of spindle (n_{min}) = 50 rpm,

Minimum revolution of spindle (n_{max})= 500 rpm,

Maximum depth of cut (d_{max}) = 5 mm,

Minimum feed (f_{min}) = 0.05 mm/rev,

Maximum feed (f_{max}) = 1.5 mm/rev,

Maximum available power (PO_{max}) = 5 kW,

Maximum force (FO_{max}) = 300 N,

Average surface roughness (SR_{max}) = 10 µm, and

Maximum temperature ($TEMP_{max}$) = 800 °C.

Based on given data, objective functions for chosen costs and constraints can be plotted on log-log scale and feasible region is identified as shown in Fig. 1.

If feed is the only constraint, the tocuhing point of a cost curve with fmax line (point “1”, Fig. 1) is optimal point. It is not feasible solution as it is away from the feasible region. If surface roughness is the only constraint, point “2” is the optimal point, which is also not feasible. Similarly, if the force is the only constraint, the point “3” is the optimal point. As point “3” is on the boundary of the feasible region, it gives optimal solution for the case considered.

The developed program can be used for getting optimal solution of a single pass, plain turning operation with any given data. The effect of change in particular data on optimal solution can also be visualized graphically. The redundant constraint can be identified.

The developed program is so flexible that it can be used to get optimal solution with any changes in data.

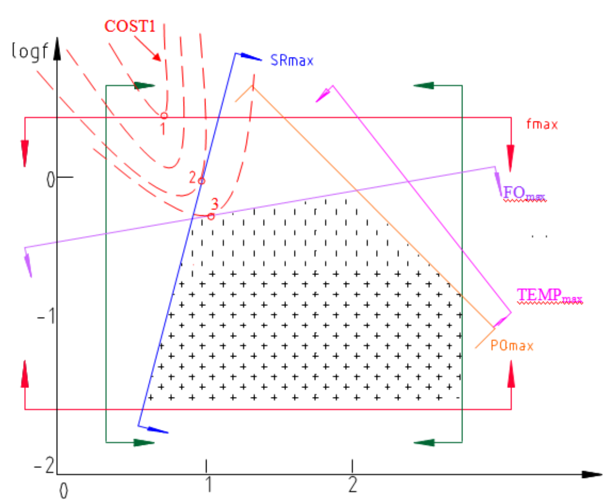

Fig. 1. Graphical presentation.

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VI. Conclusions

For a given case, the objective equation and constraints are plotted on log- log scale as well as the constant cost curves are drawn for given cost.

As can be seen from this graph, working at higher feed will always be economical. Under constraints like feed, power, and surface finish, feasible region is identified and optimal point can be located. The point '3' shown on Fig. 1 is optimal point.

The graphical method provides easier way to decide whether a particular constraint is loose or tight at optimality. The number of constraints is not a limitation for this method.

A graphical presentation as discussed here has a definite advantage of simplicity. It can be applied to any case with any number of constraints. The other criteria of optimization can also be tackled in the similar way.

The developed computer program can be used for getting optimal solution of single pass, plain turning operation with any given data. The effect of change in particular data on optimal solution can also be visualized graphically.

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