The Analysis and Complementarity of Abbe Principle Application Limited in Coordinate Measurement

Fei Yetai, Shang Ping, Chen Xiaohuai, Huang Qiangxian and Yu Liandong

***Abstract*—The accuracy requirements of coordinate measuring technology are increasingly high. The effectiveness and applicability of the Abbe principle is severely limited under this case. The first-order errors are not completely eliminated even if the equipment layout meats the Abbe principle, thus they will impact the accuracy of measurement results greatly. Error correction has also lost its original effect to the micro nanometer measurement. In this paper, the analysis and complementarity of Abbe principle application limited in coordinate measurement is given; new approaches and instrument structure programs are proposed to reduce the first-order measurement error arising from inevitable instrument structure and manufacturing errors.**

***Index Terms*—Coordinate Measurement, Abbe Principle, Limitation analysis, Complementarity.**

I. INTRODUCTION

There are many factors that would lead to measurement errors, mainly from the errors of measuring instrument itself which are generated in the process of design, manufacture, assemble and adjustment. So the reasonable instrument structure is the important reasons affecting the measurement accuracy [1]. For the coordinate measurements, the layout of the measured objects, measurement standards and the working table is the key. Germany's E. Abbe lectured at the University at Jena mentioning the design principles of measuring instrument. The principle was published in 1890, and promulgated through the various editions of Czapski's "theory of optical instruments after Abbe" [2]. He thought the standard length (standard line) should be collinear with the line of the measured length (measured line) in the length measurement process. Measurement errors due to guide movement characteristics which have quadratic relationship with the angular motion are the small second-order errors. They can be ignored for low-precision measuring instruments [3]. However, because it is often difficult to meat the Abbe principle due to space constraints of the instrument structure, they have the first-order measurement errors which have linear relationship with the angular motion.

Fig.1 shows a one-dimensional measurement system which violates the Abbe principle. As can be seen from the figure, Abbe arm S exists between the standard scale 1 and measured line 2. Measuring frame 3 generates inclination angle φ for the straightness deviation of the lead rail movement. Producing a measurement error:

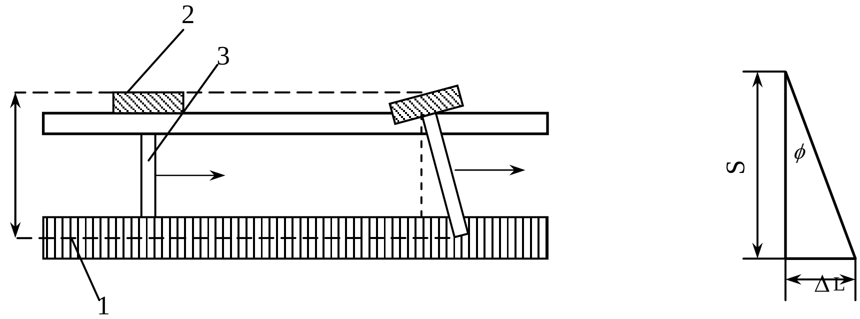

Fig.1. A one-dimensional measurement system which violates the Abbe principle

$$\Delta L = S\tan\phi \approx S\phi \tag{1}$$

As the progressing of the technology, the machining centers at home and abroad are developing toward precision and ultra-precision direction. The accuracy requirements of coordinate measuring technology are increasingly high. The effectiveness and applicability of the Abbe principle is severely limited under this case. The analyses of Abbe principle limited in high-precision coordinate measurement are given in this paper. The first-order errors are not

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completely eliminated even if the equipment layout meats the Abbe principle, and the existent second-order errors are not small, thus they will impact the accuracy of measurement results greatly. Error correction techniques are effectively used for reducing the first-order errors over the years. But about the micro nanometer measurement, error correction has also lost its original effect. Therefore, we must explore new ways to reduce the measurement error arising from inevitable instrument structure and manufacturing errors.

II. The Existence Analysis of The First-order Measurement Error in One-dimensional Measurement

One-dimensional measurement system structure is composed mainly by rail, working table, workpiece, scale and probe. Lead rail is divided into sliding rail and rolling rail, etc. [4]. Due to the movement of sliding and rolling rail type one-dimensional measurement system in vertical and horizontal direction are different. The analyses are made respectively in the following.

A. The Existence Analysis of The First-order Measurement Error in The Vertical Direction

The architecture diagram of sliding rail type (the rolling rail type) one-dimensional measurement system in the vertical direction is shown in Fig. 2. The workpiece is fixed on the working table. Measured line is collinear with the standard scale in the contact measurement, so it is in line with the Abbe principle. When the working table moves toward the dotted position in the measurement, the working table rotates θ in the vertical plane due to the errors of rail. The rotating center is O. Then, the measurement error reflected on the standard scale is:

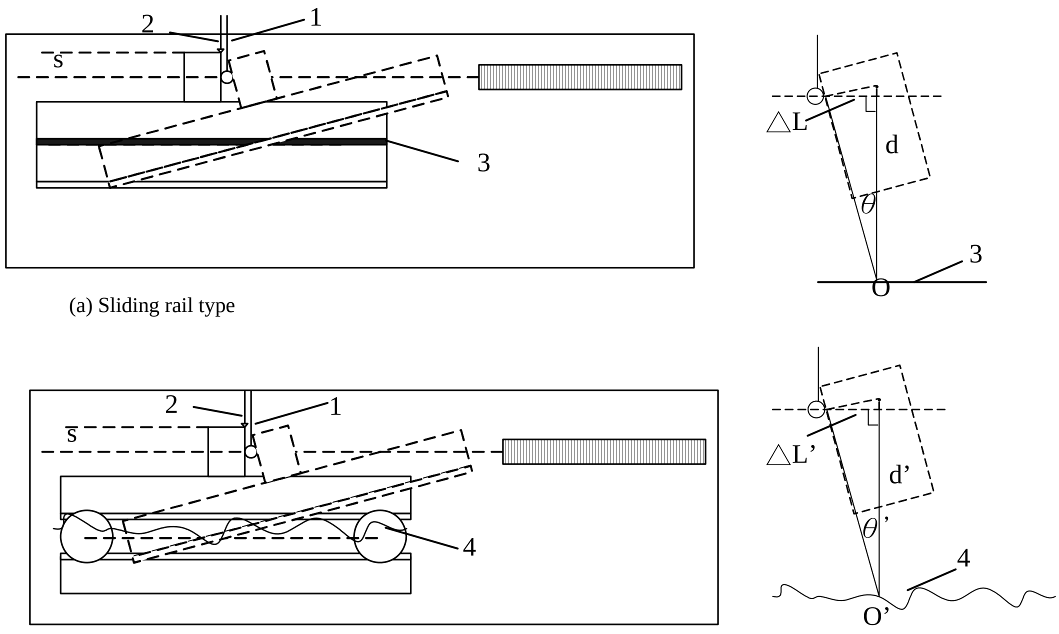
$$\Delta L = d \tan\theta \approx d\theta \tag{2}$$

The distance d between the rotating center and the standard scale line is the first-order error arm. The working table's angular motion error of the sliding rail type one-dimensional measurement system is determined by the rail straightness. The locus of the instantaneous rotating center is a straight line 3. The working table's angular motion error of the rolling rail type one-dimensional measurement system is determined by the roundness of balls, the straightness of the moving rail's upper surface and the statical rail's lower surface three factors due to the unique structure of rolling rail. The working table's rotating center O of each location is instantaneous in the vertical plane. They are connected into an irregular curve 4 shown in Fig.2 (b), so the first-order error arm d is also instantaneous.

Light beam is usually aimed at the edge of the workpiece's silhouette in the non-contact measurement. There is the Abbe arm s between the measured line and the extension line of the standard scale for different workpiece's dimension. That is to say, there not only has the first-order measurement error but also the Abbe error in the non-contact measurement system.

B. The Existence Analysis of The First-order Measurement Error in The Horizontal Direction

There is also the first-order measurement error of the one-dimensional measurement system in horizontal direction. The architecture diagram of sliding rail type one-dimensional measurement system in the horizontal direction is shown in Fig. 3.The straightness of the two rails and the roundness of balls decide that the working table has angular motion error in the horizontal plane. The locus of instantaneous rotating center is an irregular curve DD shown in Fig.3. When the working table moves from location A to B in the measurement, the working table rotates θ, the rotating center is P. When the straightness of the two rails



(a) Sliding rail type

(b) Rolling rail type

Fig.2. The architecture diagram of one-dimensional measurement system in the vertical direction 1. Contact probe 2. Not-contact acquiring light beam 3~4. The locus of instantaneous rotating center

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and the roundness of balls are exactly the same, the rotating center P is on the standard scale line, the first-order error arm d is zero. That is to say, the first-order measurement error is zero. But this is almost impossible. When the instantaneous rotating center P deviates from the standard scale line, the measurement error is: $\Delta L = d\tan\theta \approx d\theta$. The first-order measurement error of sliding rail type one-dimensional measurement system in the horizontal direction is the same with the rolling rail type. Not to repeat here.

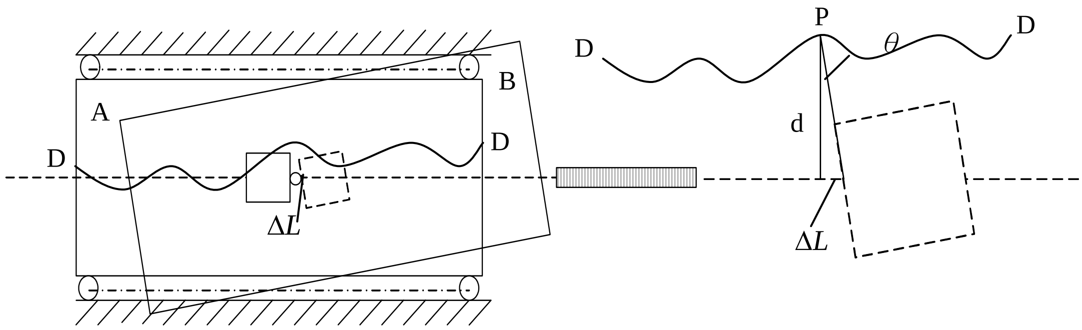

Fig.3.The architecture diagram of rolling rail type one-dimensional measurement system in the horizontal direction

From the above analysis we can see that one-dimensional measurement system has first-order measurement error ΔL in the vertical and horizontal direction. The first-order measurement error $\Delta L = 25nm$, when the rotation angle $\theta = 1''$, the first-order error arm $d = 5mm$. This shows that even the measurement systems are in line with the Abbe principle, they can not completely eliminate the impact of the first-order measurement error. Table Ⅰ shows a few examples of the first-order measurement errors produced by different parameters. Under normal circumstances, the first-order error arm will be larger than $1mm$, and then the first-order measurement error will be more than $5nm$ at least. This is not allowed for nano-precision measurement.

III. The Existence Analysis of The First-order Measurement Error in Two-dimensional Measurement

Most of the traditional two-dimensional positioning platforms are stacked structure as shown in Fig.4. Stacked structure is composed of two-layer rails and measurement platform. The upper structure moves on the lower structure by the driver formatting X axis movement; while the lower structure moves on the base of rail by the driver formatting Y axis movement. Two one-dimensional systems are stacked into the two-dimensional positioning platform in this structure actually. Measured line is collinear with the line of the standard scale, so it is in line with the Abbe principle. Individually, X, Y dimension in the horizontal and vertical plane both have the first-order measurement errors resulting from the first-order error arm as the one-dimensional measurement system.

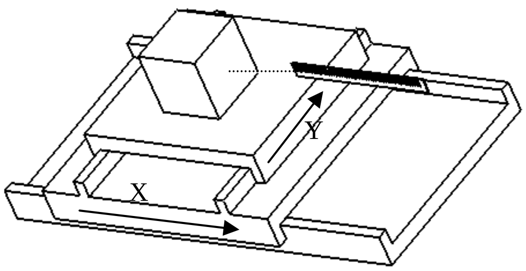

Fig.4. The model of stacked structure two-dimensional measurement system

In addition, we can see the distance H between the lower rail 2 and the measured line 1 is greater because of this stacked structure from the architecture diagram in Fig.5. That means the first-order error arm may be longer, the first-order measurement error caused by the straightness of the rail and the roundness of balls will be significant from the (2) and Ⅰ. For example, H is usually $> 10mm$, if $\theta = 0.5'' - 1''$, then the resulting first-order measurement error will reaches $25nm$ - $50nm$. The impact of the first-order measurement error can not be allowed for precision two-dimensional measurement system. Error correction techniques can be used to reduce the impact of them for the two-dimensional measurement systems which have stable structure.

TABLE Ⅰ. Example of the first-order measurement error produced by rail motion error

d (mm) / △L(nm) / θ (″)	0.1	1	5	10	15
1	0.5	5	25	50	75
2	1	10	50	100	150
3	1.5	15	75	150	225
4	2	20	100	200	300

IV. The First-order Measurement Error Correction Effect Analysis

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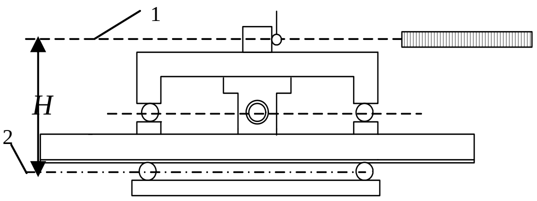

Fig.5. The architecture diagram of rolling rail type two-dimensional measurement system

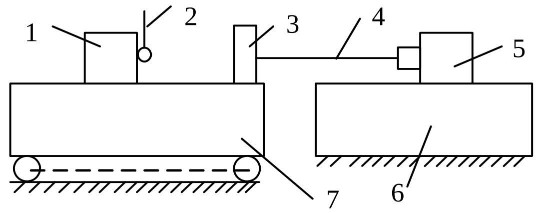

Fig.6. Schematic diagram of the common measurement system 1. Workpiece 2. Probe 3. Reflecting prism 4. Measuring light beam 5. Laser 6. Pedestal 7. Rail

The emergence of error compensation technology makes the accuracy of coordinate measurement systems guaranteed. The first-order measurement error caused by the rail straightness can be amended by software after measuring the rotation angle and the first-order error arm based on the above analysis. But the effectiveness of this technology is limited. It is not applicable in the high-precision measurements, particularly in nano and sub-nano precision measurement. The limit error of ΔL in (2) is as follows:

$$\delta_{\Delta L} = \pm\sqrt{\left(\frac{\partial f}{\partial d}\right)^2 (\delta_d)^2 + \left(\frac{\partial f}{\partial \theta}\right)^2 (\delta_\theta)^2} = \pm\sqrt{\theta^2 (\delta_d)^2 + d^2 (\delta_\theta)^2}. \quad (3)$$

If the accuracy requirements of the measurement are $\delta_{\Delta L} = 30nm, \theta = 1'', d = 10mm,$ according to the error of detecting instrument is 1 / 10 of the error of detected instrument and the principle of equal error distribution, the allowable errors calculated respectively are:

$$|\delta_d| < 0.5mm \quad |\delta_\theta| < 0.05''$$

If $\delta_{\Delta L} = 10nm,$ then $|\delta_d| < 0.15mm, |\delta_\theta| < 0.015''$

The current instruments are very difficult to achieve such a high-precision measurement of the rotation angle and the first-order error arm; In addition, the complex law of the first-order error arm's instantaneous change due to the instantaneous center is difficult to grasp. So it is difficult to meet the requirements through on-line real-time detection to error correction for the high-precision coordinate measuring systems.

V. The Complementarity of Abbe Princinple

Abbe principle has great limitations or does not have universal applicability on high-precision measuring systems according to the above analysis. The parallel structures which are multi-Abbe principle have been adopted in the modern wide range measurement system. That is inconsistent with the Abbe principle. However, the measurement system structure shown in Fig.6 is often mistakenly considered to meet the Abbe principle due to the standard line using the laser as a standard is collinear with the measured line. But in fact the angular motion error of the rail will lead to significant first-order measurement errors, therefore it do not meet the Abbe principle. So the Abbe principle needs to be complemented new concept.

The new concept of the Abbe principle should be: the standard length (standard line) should be placed in extension line of the measured length (measured line), while the standard quantity and measured quantity or standard quantity and the targeting reading system of the measured quantity must be settled on the same mobile station or device. This new content will make Abbe principle more scientific, rigorous and complete. The measurement system structure based on this concept will have the characteristics of " the same line" and "the same station". That is to say, the significant first-order measurement errors and small second-order errors due to angular motion error do not exist for having this "double same (DS) "feature. The measurement systems and structures which are compliance with the new concept of the Abbe principle strictly will achieve the ability of high-precision measurement. In the traditional measurement systems, the requirement of "the same line" and "the same station" can be easily achieved for graduated scales used as measurement standards. But in most of the modern measurement systems, wavelength is used as measurement standards, the light sources are fixed and can not be moved. Therefore the requirement of "the same station" can not be achieved, that is to say, they do not meet the Abbe principle.

VI. Approach to Reducing The First-order Measurement Error

A. *Using Two-dimensional "Coplane" Structure and Layout*

The rotation angle of working table and the first-order error arm are the main parameters determining one-dimensional measurement error. Thus we can reduce the error through these two aspects. The rotation angle error caused by the rail straightness and the balls roundness only can be reduced through improving the quality of processing and assembly; the first-order error arm can be reduced through designing the new structure and layout of the measurement system. It is the main way to reduce the first-order measurement errors effectively based on the structural principle. In view of the above, "coplane" structure two-dimensional measurement system shown in Fig.7 is proposed. The oriented surface of the X, Y two-dimensional movement rail is coincident with the measurement surface of working table. The first-order measurement errors due to not coincide can be eliminated based on this "coplane" orientation [5]. The measured parts are fixed on the surface of working table in the measurement. The Abbe error of the small workpiece caused by the height difference can be ignored for small thickness. But the Abbe errors can not be

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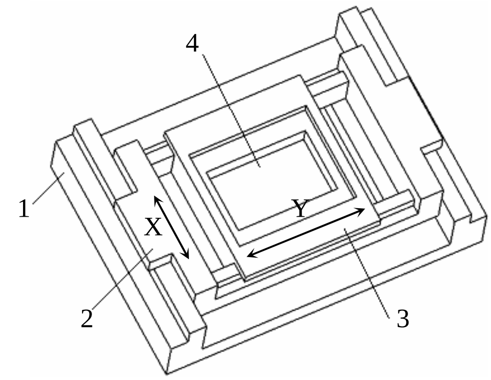

Fig.7. The model of "coplane" structure two-dimensional measurement system 1. X axis slide 2. Y axis slide 3. Working slide platform 4. Working table

ignored for a certain high workpiece. Two solutions are proposed in this case. One is the measurement surface of working table can be designed below the "coplane" position, then select the gaskets of appropriate thickness according to the measured height to ensure that the measured surface and the rail-oriented surface coincide; Another approach is the measurement surface of working table can be designed into Z-displacement component, but its displacement control accuracy is undemanding, the coincidence degree of the measured surface and the rail-oriented surface just be controlled less than $0.5mm$. For example, there is the first-order error arm $d = 0.5mm$ due to not coincide of the two rail-oriented surfaces and movement rotating angle of the working table $\theta = 1''$, the impact of resulting error $\Delta L = d\tan\theta \approx d\theta = 2.50nm$ is very small and could be ignored for two-dimensional measurement platform, and the rail with this precision is easy to processed. The first-order measurement error caused by straightness error in the rail's vertical plane close to zero as the coincidence degree of two surfaces can be controlled smaller in actual processing and assembly. Then the higher precision measurements can be achieved.

B. The locus of Working Table's instantaneous rotating center modeling and correction principles

About the two-dimensional measurement systems in line with "coplane" structure, the instantaneous rotating center of the working table exists due to the rail straightness, which resulting in the first-order measurement error exists. Instant center locus equation of working table is derived by the method of separating instant center in any position. The first-order measurement error of the working table's any measurement points caused by the rail straightness can be calculated by the equation, then it can be amended by the software. There are calculation errors of the first-order measurement arm because that the geometrical center of the working table is acted as the rotating center in traditional methods of separation and amendments. Now the problem is solved through the method above [6].

Calculating schematic of the working table's instant center is shown in Fig.8. Assume that one point B is taken in working table, it moves to the point B' when the working table rotating θ related the instantaneous rotating center O due to the impact of rail straightness error. The position

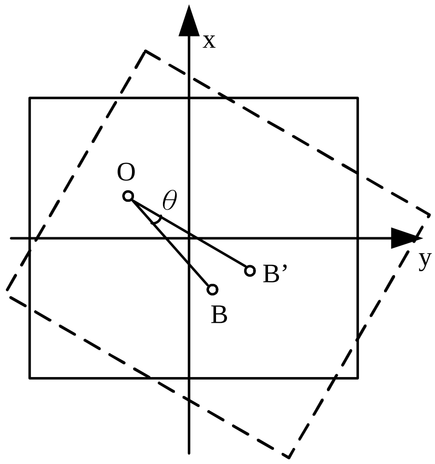

Fig.8. Calculating schematic of the working table's instant center

errors of B are the coordinate differentials of point B and point B' respectively in X , Y direction. Assume that the coordinates of the measured point B are (X_B, Y_B) , and then the coordinates of B' point can be obtained by the coordinate transform matrix following.

$$\begin{bmatrix} X_{B'} - X_O \\ Y_{B'} - Y_O \end{bmatrix} = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} X_B - X_O \\ Y_B - Y_O \end{bmatrix} \tag{4}$$

The coordinate errors are the differentials of $(X_{B'}, Y_{B'})$ and (X_B, Y_B) .

In addition to high-precision micro-nano three-dimensional measurement system, finding new methods and designing new structures are needed to achieve the measurement requirements.

The author has proposed a novel architecture design principle for three-dimensional measurement system, which can reduce the first-order measurement errors effectively. It will be further discussed in the author's published.

VII. CONCLUSION

From the above analysis of one-dimensional and two-dimensional measurement system, the first-order measurement error always exists for the traditional layout of the measurement system even meet the Abbe principle, it has a great impact on measuring accuracy of the precision measurement system, and it is difficult to compensated. Therefore, this paper presents a new layout, mechanical structure and solutions to reduce the errors. Two-dimensional "coplane" layout and the software compensation method are taken in the two-dimensional measurement system, so the impacts on the measurement accuracy caused by the first-order measurement errors are reduced. The analysis is significant for reducing the measurement error of coordinate measuring system.

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REFERENCES

[1] Y. T. Fei, *Error theory and data processing.* Machine Press, China, 2004, pp. 3–4.

[2] C. Evans, *Precision engineering: an evolutionary view.* Cranfield Press, 1989.

[3] W. K. Shi, X. F. Yu, *Detection technology.* China Machine Press, 2000, pp. 44.

[4] Z. B. Pu, B. G. Wang, *Measurement and control instrument design.* China Machine Press, 2001, pp. 116–134.

[5] W. L. Wang, Y. T. Fei, and K. C. Fan, "Investigation of nanometer XY positioning stage," Proceedings of the 1st IEEE International Conference on Nano / Micro Engineered and Molecular Systems, Jan. 2006, Zhuhai, China, pp. 320–324.

[6] H. T. Yang, Y. T. Fei, and X. H. Chen, "The high precision error separating principle and technique of the guide strip of nanometer measuring machine' s working table," Journal of Shanghai Jiaotong University, vol. 40, No. 12, Dec. 2006, pp. 2066－2069.

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