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Comparative Study of Robust Controllers for Vibration Control of Flexible Structures by Combining Pre-Compressed Layer Damping and ACLD Treatment

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ABSTRACT -. It is well known fact that system parameters of the flexible structures keep on changing due to several reasons. Ordinary controllers loose their effectiveness in changed situations and do not guarantee the stability of the closed loop system. However, controllers designed based on robust control theory, not only maintain the closed loop stability of the perturbed system with a large variation in system parameters but also maintain the best performance. H_∞ Loop shaping controller is designed and implemented on a smart flexible structure treated with pre-compressed layer damping and ACLD treatment. It outperforms Linear Quadratic Gaussian and standard H_∞ controller both in terms of robust stability and robust performance. Relative merits and demerits of the μ – synthesis based controller are also discussed.

Index Terms- Robust control, ACLD, H_∞ control, Loop Shaping, mu-synthesis

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I. INTRODUCTION AND PROBLEM FORMULATION

It is a well known fact that all flexible structure are subjected to change in system parameters with the passage of time due to change in operating conditions and environmental factors. Ordinary fixed controllers designed based on nominal parameters loose their effectiveness due to variation in system parameters. The modern robust controllers based on sophisticated $\mathbf{H_2}$ and $\mathbf{H_\infty}$ optimization theory are free from such defects as discussed in reference [1]. In robust control methodology one designs a central controller which gives robust stability and robust performance for a family of system with perturbed parameters. In this direction, Jee et al [2] used $\mathbf{H_\infty}$ control design scheme based matrix fraction stability condition. A cantilever beam was used in the study. Kang et al [3] presented a robust vibration control for flexible SCARA type robot manipulators based on mu-synthesis theory.

Baz [4] developed the robust controller for ACLD treated beam by minimizing the $\mathbf{H_2}$-norm of the transfer functions. Afterwards, Crassidis et al [5] discussed the performance of $\mathbf{H_\infty}$ controllers at different operating frequencies and temperatures. Chang et al [6] presented a model reduction method and uncertainty modeling for the design of a low order $\mathbf{H_\infty}$ robust controller for suppression of smart panel vibrations.

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For ACLD based systems the difference between theoretical and experimental results is higher. So, there is a need to develop controllers which are robust to these parametric errors. So the natural choice is to look for a robust controller. Working in the same direction, Li et al [7] used mu-synthesis technique for the vibration control of plate – like structures. Liu et al [8] designed and implemented $\mathbf{H}_\infty$ robust controller to accommodate uncertainties of the ACLD parameters.

Xie et al [9] investigated the robust vibration control of a thin plate covered with a controllable constrained layer damping. Caracciolo et al [10] developed robust controller based on mixed $\mathbf{H}_2/\mathbf{H}_\infty$ optimization. Hu et al [11] designed and applied robust controllers to suppress the vibrations of a circular plate. Stavroulakis et al [12] reduced the rotor system vibrations using magnetic bearings. Very recently, Lim et al [13] proposed linear matrix inequality based robust controller for an active mass damper. For robust vibration control of smart structures, $\mathbf{H}_\infty$ and mu-synthesis based controllers (MU controller) have been generally tried. $\mathbf{H}_\infty$ Loop Shaping Design Procedure ($\mathbf{H}_\infty$ -LSDP) is an important robust control methodology. In the present work $\mathbf{H}_\infty$ -LSDP based robust controller has been applied and the results are compared with standard $\mathbf{H}_\infty$, MU controller and Linear Quadratic Gaussian (LQG) controller.

II. SYSTEM DESCRIPTION

a.) Finite Element Modeling

A FEM based model of the structural system is obtained by using energy principles along with Hamilton method. The beam is allowed to move axially. When the bolts are not tightened, the beam will be under simply supported beam conditions. If the bolts are tightened to a high value such that slope formed at the free ends is zero and the beam behaves as fixed- fixed beam with axial movement allowed. Under the various loads, applied by tightening the loads, different boundary conditions can be developed artificially (fig 1).

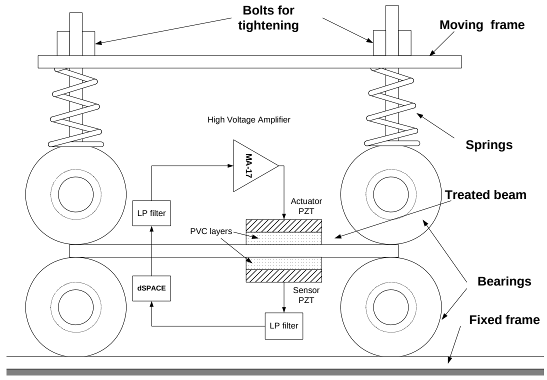

Figure 1:-Schematic view of the clamped beam with variable boundary conditions

In FEM approach, the actual system (i.e. flexible structure) is modeled as n-degree-of-freedom mechanical system whose generalized co-ordinates are represented by a $n \times 1$ vector $\mathbf{q}$ and $n \times n$ mass matrix $\mathbf{M}$ and stiffness matrix $\mathbf{K}$; both matrices are positive definite and symmetric. The equation of motion is given as

$$\mathbf{M}\ddot{\mathbf{q}} + \mathbf{K}\,\mathbf{q} = \mathbf{f} \tag{1}$$

where $\mathbf{f}$ is the $n \times 1$ vector of generalized co-ordinates. Since the matrix $\mathbf{K}$ is a complex matrix, eq. (1) cannot be used directly for controller design and simulation purpose using standard simulation tools. Although certain methods are available (like GHM, Biot method etc) in which $\mathbf{K}$ is a real matrix, certain other problems are encountered. In these approaches damping matrix $\mathbf{D}$ comes into picture to accurately represent the dynamics of the system.

b.) Grey Box Subspace System Identification

Even though matrix $\mathbf{K}$ is complex, it is easy to construct the frequency response function (FRF) from the input -to- output data using $\mathbf{M}$ and $\mathbf{K}$ matrices with the help of standard MATLAB toolboxes.

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Reduced order system transfer function can be identified from FRF data using standard system identification methods. The system can be written in the form written as below

$$\dot{\mathbf{x}}(t)=\mathbf{A}\mathbf{x}(t)+\mathbf{B}\mathbf{u}(t)=\begin{pmatrix} 0 & I \\ -C & -\Lambda \end{pmatrix}\mathbf{x}(t)+\begin{pmatrix} \mathbf{0} \\ \mathbf{B}_2 \end{pmatrix}\mathbf{u}(t) \tag{2a}$$

$$\mathbf{y}(t)=\mathbf{C}\mathbf{x}(t)=\begin{pmatrix} \mathbf{B}_2^{\mathbf{T}} & \mathbf{0} \end{pmatrix}\mathbf{x}(t) \tag{2b}$$

where C and $\Lambda \in \mathbb{R}^{n \times n}$, and $\mathbf{B_2} \in \mathbb{R}^{n \times m}$ has to be estimated if inputs and outputs are collocated, which is there in our present work. Here m represents the number of inputs of the system. For a single input single output (SISO) system, with single mode consideration, the above model can be written as

$$\begin{aligned} \dot{\mathrm{x}}(t) &= \begin{pmatrix} 0 & 1 \\ -\omega^2 & -2\xi\omega \end{pmatrix} x(t)+\begin{pmatrix} 0 \\ \phi_{12} \end{pmatrix}\mathrm{u}(t) \\ \mathrm{y}(t) &= \begin{pmatrix} 1 & 0 \end{pmatrix}\mathrm{x}(t) \end{aligned} \tag{3}$$

where ω, ξ and ϕ are the natural frequency, damping ratio and mode shape (of the considered mode) of the structure at the location where actuator is attached respectively. ϕ_{12} is obtained by multiplying the mass normalized mode shapes at the sensor and actuator location. By considering the various modes of interest, the model above can be extended to multi mode case easily. Let $\mathbf{G}(j\omega)$ represents the FRF of the continuous time system obtain from FEM data. For accurately identifying the system using feed through term by correcting the model shown in relation (3) by the following equation

$$\begin{aligned} \mathbf{y}(t) &= \mathbf{C}\,\mathbf{x}(t) + \mathbf{D}\,\mathrm{u(t)} \\ &= \begin{pmatrix} \mathbf{B}_2^{\mathbf{T}} & \mathbf{0} \end{pmatrix}\mathbf{x}(t)+\mathbf{D} \end{aligned} \tag{4}$$

The value of **D** is varied from zero to certain positive constant until the FRF of the identified model matches with the obtained from FEM analysis.

c.) Linear Fractional Transformation Model of uncertain system

Every single mode of a structural system is represented in transfer function form by equation

$$\frac{\mathrm{y}}{\mathrm{u}}=\frac{\phi_{12}}{\mathrm{s}^2+2\xi\omega\mathrm{s}+\omega^2} \tag{5}$$

where φ_{12} , ξ and ω represent the product of mode shapes at actuator and sensor locations, damping ratio and natural frequency of the structural system. Figure (2) shows the graphical presentation of LFT based model of the structure for a single mode.

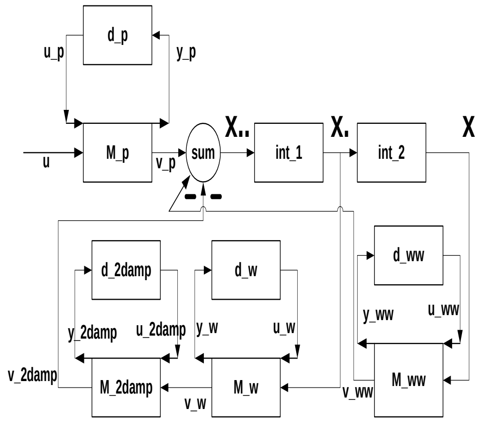

Figure 2:- LFT representation for a single mode

By adding different modes the model can be generated for multiple modes.$\mathbf{W_1}$ and $\mathbf{W_2}$ are the weighing transfer functions used for controller design.

III. OPTIMAL AND ROBUST CONTROL DESIGNS

a.) $\mathbf{H}_\infty$ Controller Design

The $\mathbf{H}_\infty$ solution formulae use solutions of two algebraic Riccati equations (ARE). An algebraic Riccati equation

$$E^T X + XE - XWX + Q = 0 \tag{6}$$

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where $\mathbf{W}=\mathbf{W}^T$ and $\mathbf{Q}=\mathbf{Q}^T$, uniquely corresponds to a Hamiltonian Matrix $\begin{bmatrix} \mathbf{E} & \mathbf{-W} \\ \mathbf{-Q} & \mathbf{-E}^T \end{bmatrix}$. The stabilizing solution **X**, if it exists, is a symmetric matrix which solves the ARE and is such that **E-W X** is a stable matrix. The stabilizing solution is denoted as

$$\mathbf{X} = \mathbf{Ric}\begin{bmatrix} \mathbf{E} & \mathbf{-W} \\ \mathbf{-Q} & \mathbf{-E}^T \end{bmatrix} \quad (7)$$

b.) H_∞ Loop Shaping Controller Design

The H_∞ robust stabilization against such perturbations and the consequently developed design method, the H_∞ LSDP, could relax the restrictions on the number of right-half plane poles and produce no pole-zero cancellations between the nominal model and controller designed. This method does not require an iterative procedure to obtain an optimal solution and thus raises the computational efficiency.

c.) μ Controller design by D-K iteration method

For robust stability and robust performance, it is required to find a stabilizing controller **K** such that

$$\sup_{\omega\in\mathbb{R}} \mu\left[\mathbf{M(P,K)}(j\omega)\right] < 1 \quad (8)$$

An iterative method was proposed to solve which is called the D-K iteration μ-synthesis method and is based on solving the following optimization method, for a stabilizing controller **K** and a diagonal constant scaling matrix **D.**

V. RESULTS AND COMPARISION

First of all the nominal system corresponding to intermediate system with zero pre-stress conditions are chosen. Systems corresponding to nearly simply supported and nearly fixed - fixed boundary conditions can be taken as the perturbed systems around the nominal system. The intermediate boundary conditions at which the first four natural frequencies are 21.9, 67.4, 129.7 and 247.1 Hz is chosen as nominal system. Perturbed taken with boundary conditions as nearly simply supported with first four natural frequencies as 12.8, 52.2, 112.5 and 226.3 Hz. The perturbed systems are modeled in LFT form. LQG, standard H_∞, H_∞ LSDP and MU -synthesis based controllers are designed by using weighing functions. In the nominal CL system, the reduction in amplitude is 3.5dB, 5.3dB, 7.2dB for the first, second and fourth mode respectively (figure 3). The magnitude at the third mode is negligible and hence is not apparent in this figure. Figure (4) shows the FRF of the perturbed closed loop system.

Nominal performance is established at all the frequencies, by all the controllers, as μ value is less than unity. However, performance of the perturbed CL system degrades as μ value reaches above unity for some frequencies. Nominal and robust performance with LSDP controller is best at all the modes w.r.t. other controllers.

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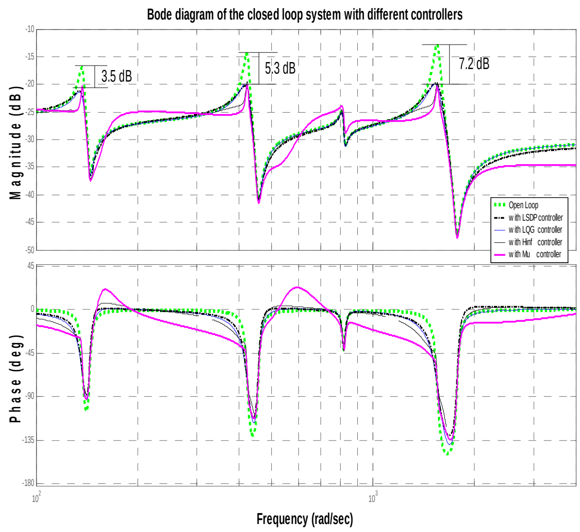

Figure 3:- Frequency response function of nominal open and closed loop system with different controllers

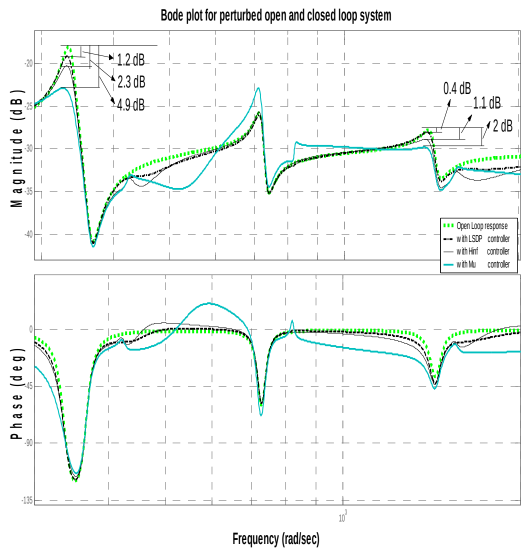

Figure 4:- Frequency response function of perturbed open and closed loop system with different controllers

By considering the pole zero maps, for standard H_∞ , H_∞ LSDP and MU controllers it is obvious that the three controllers are stable (because all the poles are on the left hand plane) as well as stabilizing (i.e. CL system is stable).

Figure (5) shows the Plot for comparison of nominal and robust performance with H_∞ controller. MU controller is best in terms of modal amplitude reduction for second and fourth modes (which are actually dominant) as seen from this figure (observe amplitude reduction in dB for all the controllers).

VI. CONCLUSIONS

In the present work, relative merits and demerits of LQG, standard H_∞, H_∞ LSDP and MU-synthesis controllers have been discussed. System parameters are varied through artificial means. Followings are some of the important conclusions

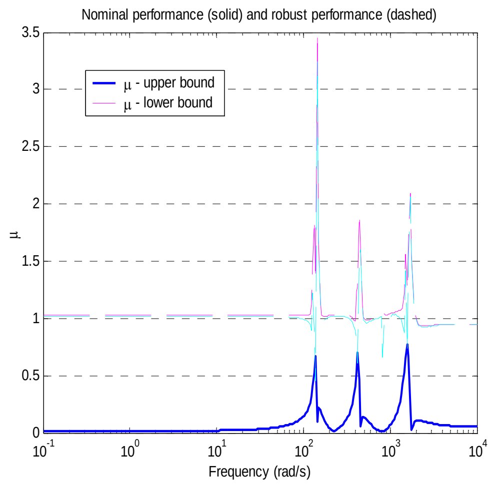

Figure 5:- Plot for comparison of nominal and robust performance with H_∞ controller

1. It was observed that LQG controllers should never be used for systems with expected large variation in system parameters.
2. Analog standard H_∞ can be tried to give good stability and performance for systems with minor perturbations. However, for systems

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with expected large variations from nominal model, MU controller is the best choice. Best CL performance is achievable with this controller; however at the expense of large control energy i.e. control energy utilization is worst with this control strategy.

3. H_∞ Loop shaping based controller not only outperforms in terms of stability but also maintains best performance even for a moderate variation in system parameters with minimum amount of control energy utilization.

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