Short-run Process Control Based on Non-conformity Degree

Majid Aminnayeri, Elnaz Asghari Torkamani, Mehdi Davodi, Faraz Ramtin

***Abstract*— Statistical Process Control (SPC) is an approach that uses statistical techniques to monitor the process. The techniques of quality control are widely used in controlling any kinds of processes. One of these processes is the short processes. In short run processes often do not have enough data in each run to produce good estimates of the process parameters. This will cause the reduction of the performance and efficiency of control charts. A common solution to this problem is considering a single machine or process to produce many different parts, or different products. In this paper a new method based on non-conformity degree and fuzzy membership functions has been developed for controlling these processes. This method is simple and useable.**

***Index Terms*—Short-run processes, specification limits, non-conformity degree, membership function.**

I. INTRODUCTION

Control charts are widely used for monitoring and examining in a production process by investigating causes of variability. The power of these charts lies in their ability to detect process shifts and abnormal conditions. By reducing the number of losses the cost of the production will reduce and the quality of the product will be improved. In 1924, Walter Shewhart designed the first control chart for controlling a single quality characteristic. The chart contains three parts: a center line (CL) that represents the average value of a quality characteristic corresponding to in-control state, and two other parallel lines, called upper control limit (UCL) and lower control limit (LCL), which are chosen to assure that if the process is in-control, nearly all of the sample points will fall between them [1]. Let w be the sample statistic which measures some quality characteristic, μ_w and σ_w are its mean and standard deviation, respectively. Then the center line (CL), the upper control limit (UCL) and the lower control limit (LCL) in the relevant are defined as follows:

$$\begin{cases} UCL = \mu_w + k\,\sigma_w , \\ CL = \mu_w , \\ LCL = \mu_w - k\,\sigma_w . \end{cases} \tag{1}$$

where k is the distance of control limits from the center line, expressed in standard deviation units.

From 1924, many approaches have been suggested to improve the performance of the control charts proposed by Shewhart such as: CUSUM charts, EWMA charts, adoptive control charts and fuzzy control charts.

The base of control charts is the data which are gathered from a production process. These data represent the various levels of the quality characteristic associated with the product. For many problems, the data that gathered from the processes could not be so precise. This uncertainty may come from the measurement system including operators, gauges and environmental conditions [2]. Also, Standard control charting techniques rely upon a sufficiently large amount of data to reliably estimate process parameters, such as the process means (μ) and process standard deviations (σ).

In short run processes often do not have enough data in each run to produce good estimates of the process parameters. This will cause the reduction of the performance and efficiency of control charts. A common solution to this problem is considering a single machine or process to produce many different parts, or different products.

As deciding on that the process is in-control or out-of-control, is based on these data, by approving this uncertainty and shortage of data cannot judge about the condition of the process precisely. Fuzzy set theory is a useful tool to handle this uncertainty and vagueness [2]. Zadeh introduced the fuzzy set theory in 1965 [3]. There have been many efforts to apply the ideas of fuzzy sets to statistical problems [4, 5]. Many studies were done to combine statistical methods and fuzzy set theory.

Recently, some new results on the applications of fuzzy sets in statistical quality control have been presented. Engin et al. [6] have combined fuzzy sets with genetic algorithms to determine sample size in attribute control charts. Faraz and Bameni Moghadam [7] and Fazel Zarandi *et al*. [8] have constructed fuzzy control charts for the process average of a variable quality characteristic.

The main approach in all process control tools is detecting the fractions or the number of nonconforming units. The base of short run control charts is the same. The deviation from nominal constructs the data of these charts. Amirzadeh *et al.* [9] constructed p-charts using fuzzy approach and degree of nonconformity. They proposed a method which differs from the control chart introduced by Fazel Zarandi *et al.* in the

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precision of both input and output of the model. Whereas Fazel Zarandi *et al*. used fuzzy-valued data to construct their control chart; they used real-valued data and treat quality as a fuzzy set. The control limits obtained by fuzzy-valued data are fuzzy, but by real-valued data they are precise.

In this paper the approach of nonconformity degree is developed for controlling short run processes and all the processes can be plotted in the same short run control chart, with rescaled plot points. This approach can be applied in the case of variable standard deviation too. The fuzzy number that is used in this research has trapezoidal membership function. The trapezoidal fuzzy number $T(a,b,c,d)$ where $a \le b \le c \le d$, has the membership function by the following equation:

$$T(a,b,c,d) = \begin{cases} 0, & x < a \\ \frac{x-a}{b-a}, & a \le x < b \\ 1, & b \le x < c \\ \frac{d-x}{d-c}, & c < x < d \\ 0, & x \ge d \end{cases} \quad (2)$$

In special case $b = c$, corresponds to a fuzzy triangular number.

II. Methodology

A. Numeric Quality and Fuzzy Quality

In classical quality control, the specification limits are not dependent on the process, and may be set by management, the manufacturing engineers, the customer or product developers and designers. The upper specification limit (USL) and the lower specification limit (LSL) denote by 'U' and 'L' respectively. Products for which the quality characteristic falls within the interval $[L,U]$ are judged as 'conforming', while products out of the range are considered 'nonconforming'. If x is the quality characteristic of the produced item then the product degree of conformity and nonconformity with standard quality may be defined, by $C(x)$ and $N(x) = 1 - C(x)$, respectively, where $C(x)$ is a crisp set with following binary membership function:

$$C(x) = \begin{cases} 1, & L \le x \le U \\ 0, & otherwise \end{cases} \quad (3)$$

Yongting [10] introduced the concept of fuzzy quality as follows: Let quality be a fuzzy set with the membership function $\tilde{C}$, which has supported $[L,U]$. If the quality characteristic of a product is x, then $\tilde{C}(x)$ is called the degree of conformity with the degree of conformity. Now the degree of nonconformity is defined $\tilde{N}(x) = 1 - \tilde{C}(x)$. The graphs of $\tilde{C}$ and $\tilde{N}$ when the membership function of $\tilde{C}$ is trapezoidal, is shown in Fig. 1.

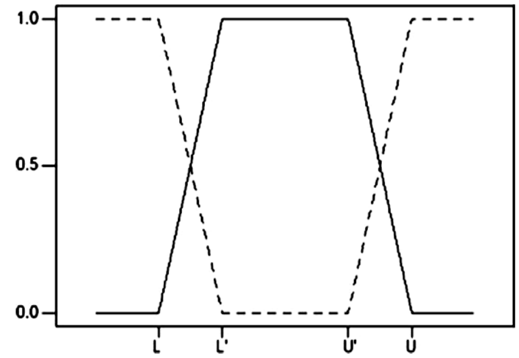

Fig. I. The graphs of $\tilde{C}$ (—) and $\tilde{N}$ (---)

The degree of nonconformity for each production is a number between 0 and 1. If $X_1,\dots,X_n$ be a real-valued random sample from the quality characteristic X . Then $\tilde{N}(X_1),\dots,\tilde{N}(X_n)$ is a random sample from $\tilde{N}(X)$. Let X be the quality characteristic of a product and suppose that $X \sim N(\mu,\sigma^2)$. Then $\tilde{N}(X)$, the degree of nonconformity of X is a bounded random variable with mean $\mu_{\tilde{N}} = E(\tilde{N}(X))$ and variance $\sigma^2_{\tilde{N}} = Var(\tilde{N}(X))$ [9].

B. Short Run Process Control

When conventional Shewhart charts are used to establish statistical control, the initial control limits are typically based on 25 to 30 subgroup samples. Often, however, this amount of data is not available in manufacturing situations where product changeover occurs frequently or production runs are limited.

A variety of methods have been introduced for analyzing data from a process that is alternating between short runs of multiple products. The methods commonly used in the United States are variations of two basic approaches:

- The *difference from nominal* approach. A product-specific nominal value is subtracted from each measured value, and the differences, together with appropriate control limits, are charted. Here it is assumed that the nominal value represents the central location of the process which, ideally estimated with historical data, and that the process variability is constant across products.
- The *standardization* approach: Each measured value is standardized with a product-specific nominal and standard deviation values. This approach is followed when the process variability is not constant across products.

C. Control Charts for the Mean Degree of Nonconformity

If X be the quality characteristic of a product and suppose that $X \sim N(\mu,\sigma^2)$. Then $\tilde{N}(X)$, the degree of nonconformity of X is a bounded random variable with mean $\mu_{\tilde{N}} = E(\tilde{N}(X))$ and variance $\sigma^2_{\tilde{N}} = Var(\tilde{N}(X))$ [9].

The control lines for the chart of the mean degree of nonconformity have been introduced as follows [9]:

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$$\begin{cases} UCL = E\left(\bar{\tilde{N}}\right) + k\sqrt{Var\left(\bar{\tilde{N}}\right)} = \mu_{\bar{\tilde{N}}} + k\dfrac{\sigma_{\bar{\tilde{N}}}}{\sqrt{n}} \\ UCL = E\left(\bar{\tilde{N}}\right) = \mu_{\bar{\tilde{N}}} \\ UCL = E\left(\bar{\tilde{N}}\right) - k\sqrt{Var\left(\bar{\tilde{N}}\right)} = \mu_{\bar{\tilde{N}}} - k\dfrac{\sigma_{\bar{\tilde{N}}}}{\sqrt{n}} \end{cases} \quad (4)$$

In this equation, $\bar{\tilde{N}}$ is the mean of the degree of nonconformity. k (usually 3) is the distance of the control limits from the center line.

$$\bar{\tilde{N}} = \frac{\tilde{N}(X_1) + \ldots + \tilde{N}(X_n)}{n} \quad (5)$$

As $\bar{\tilde{N}}_1, \bar{\tilde{N}}_2, \ldots, \bar{\tilde{N}}_m$ be the mean of nonconformity degree for each subgroup, So $\bar{\bar{\tilde{N}}}$ is the best estimator for $\mu_{\bar{\tilde{N}}}$. For estimating $\sigma_{\bar{\tilde{N}}}$, the mean range of each subgroup can be used ($R = \bar{\tilde{N}}_{\max} - \bar{\tilde{N}}_{\min}$).

$$\bar{\bar{\tilde{N}}} = \frac{\bar{\tilde{N}}_1 + \bar{\tilde{N}}_2 + \ldots + \bar{\tilde{N}}_m}{m} \quad (6)$$

$$\bar{R} = \frac{R_1 + R_2 + \ldots + R_m}{m} \quad (7)$$

Therefore the control lines for the chart of the mean degree of nonconformity for a short-run process have been calculated as follows:

$$\begin{cases} UCL = \bar{\bar{\tilde{N}}} + \dfrac{3}{d_2\sqrt{n}}\bar{R} \\ UCL = \bar{\bar{\tilde{N}}} \\ UCL = \bar{\bar{\tilde{N}}} - \dfrac{3}{d_2\sqrt{n}}\bar{R} \end{cases} \quad (8)$$

where d_2 can be find in the table … of Montgomery's quality control book [1].

III. CASE STUDY

For illustrating this approach, a simple example was established. The following example is adapted from an application in aircraft component manufacturing. A metal extrusion process is used to make three different models of a component. The three product types (labeled M1, M2, and M3) are produced in small quantities because the process is expensive and time-consuming. Note that in this example the standard deviation of each components are same and they are equal to 0.46. The inside diameter measurement data of these samples are shown in Table I.

Table I
The data of metal extrusion process (mm)

Sample Number	Observations					
	M1		M2		M3	
1	15.02	15.27	16.06	15.37	14.82	14.67
2	15.46	15.45	15.27	14.95	15.01	15.24
3	15.19	15.06	16.02	15.50	15.16	15.03
4	14.52	14.65	16.02	14.87	15.02	15.08
5	15.18	15.20	15.45	15.66	15.23	15.22
6	15.29	15.00	16.12	15.73	14.38	15.21
7	15.10	15.38	15.07	15.68	14.69	14.86
8	15.41	15.01	15.08	15.53	14.74	14.97
9	15.43	14.85	15.32	15.11	15.16	14.93
10	15.02	15.01	15.40	14.90	15.05	15.00
11			15.62	16.02	14.88	15.18
12			14.95	14.89	14.86	14.37
13			15.23	15.21	14.52	14.85
14			15.12	15.08	15.21	14.39
15			14.89	15.62		
16			15.55	15.72		

IV. COMPUTATIONAL RESULT

A. Membership Functions

Now suppose that the specification limits on this metal extrusion process are 15 ± 0.55 mm, 15.5 ± 0.65 mm and 14.8 ± 0.45 mm, and the triangular membership function for conformity of each is shown in Eq. 9, 10 and 11 respectively.

$$\tilde{C}_{M1}(x) = \begin{cases} 0, & x < 14.45 \\ {x - 14.45}/{0.55}, & 14.45 \le x < 15 \\ {15.55 - x}/{0.55}, & 15 \le x < 15.55 \\ 0, & x \ge 15.55 \end{cases} \quad (9)$$

$$\tilde{C}_{M2}(x) = \begin{cases} 0, & x < 14.85 \\ {x - 14.85}/{0.65}, & 14.85 \le x < 15.5 \\ {15.15 - x}/{0.65}, & 15.5 \le x < 16.15 \\ 0, & x \ge 16.15 \end{cases} \quad (10)$$

$$\tilde{C}_{M1}(x) = \begin{cases} 0, & x < 14.35 \\ {x - 14.35}/{0.45}, & 14.35 \le x < 14.8 \\ {15.25 - x}/{0.45}, & 14.8 \le x < 15.25 \\ 0, & x \ge 15.25 \end{cases} \quad (11)$$

Now for each sample $\{X_1, X_2\}$ a corresponding sample $\{N(X_1), N(X_2)\}$ of the degrees of nonconformity is calculated and the degree of nonconformity of the observations are shown in Table II ($\tilde{N}(X) = 1 - \tilde{C}(X)$).

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Table II
The degree of nonconformity of the observations

Sample	M1		M2		M3	
	$N(X_1)$	$N(X_2)$	$N(X_1)$	$N(X_2)$	$N(X_1)$	$N(X_2)$
1	0.041	0.488	0.866	0.204	0.047	0.296
2	0.828	0.819	0.358	0.851	0.458	0.983
3	0.337	0.107	0.800	0.000	0.800	0.515
4	0.872	0.634	0.804	0.975	0.499	0.632
5	0.334	0.371	0.080	0.251	0.958	0.928
6	0.531	0.001	0.954	0.349	0.938	0.912
7	0.178	0.683	0.655	0.283	0.239	0.141
8	0.754	0.020	0.644	0.048	0.128	0.383
9	0.775	0.267	0.281	0.597	0.789	0.279
10	0.045	0.020	0.147	0.931	0.550	0.439
11			0.188	0.795	0.169	0.851
12			0.842	0.938	0.138	0.960
13			0.417	0.443	0.628	0.121
14			0.580	0.640	0.908	0.902
15			0.933	0.190		
16			0.072	0.343		

B. Control Limits and Control Chart based on Nonconformity Degree

The mean and mean rang of nonconformity degree of observations were calculated and shown in Table III. For these observations, $\bar{\bar{\tilde{N}}}$ and $\bar{R}$ are equal to 0.502 and 0.357, respectively.

Table III
Estimation of the mean and standard deviation of the observations

Products	Observations		$\bar{\bar{\tilde{N}}}$	R
M1	0.041	0.488	0.264	0.447
M1	0.828	0.819	0.824	0.009
M1	0.337	0.107	0.222	0.230
…				
M2	0.866	0.204	0.535	0.662
M2	0.358	0.851	0.605	0.493
M2	0.800	0.000	0.400	0.801
…				
M3	0.138	0.960	0.549	0.821
M3	0.628	0.121	0.375	0.507
M3	0.908	0.902	0.905	0.006
			0.502	0.357

Using Eq. 8, CL, LCL and UCL for $\bar{\bar{\tilde{N}}}$ control chart are determined as follows:

$$\begin{cases} UCL = 0.502 + \dfrac{3}{1.128\sqrt{2}} 0.357 = 1.168 \\ UCL = 0.502 \\ UCL = 0.502 + \dfrac{3}{1.128\sqrt{2}} 0.357 = 0.164 \end{cases} \quad (12)$$

This chart is shown in figure 2.

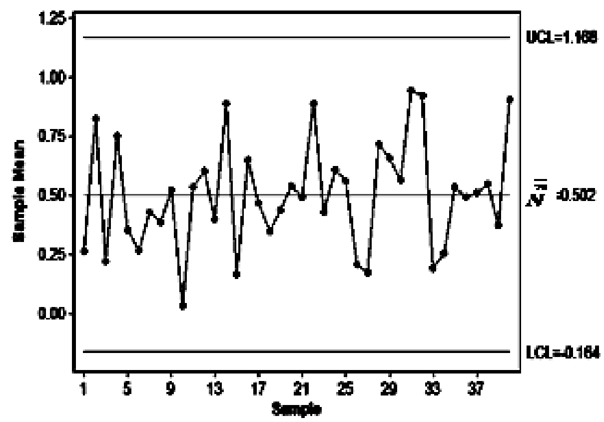

Fig. II. $\bar{\bar{\tilde{N}}}$ Control Chart

V. Conclusion

In controlling a short run process, shortage of data is the main problem. By the use of these few observations the performance of used control chart becomes low and the results cannot be applied.

Some fuzzy methods can be used for solving this problem. In this paper by the use of fuzzy membership function a method for controlling short run processes has been developed. By these membership functions, all observations of different processes convert into same scaled data and they can be monitored in the same control chart with satisfactory performance.

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