Proceedings of the World Congress on Engineering 2010 Vol III
WCE 2010, June 30 - July 2, 2010, London, U.K.

Optimization of a Dynamic Supply Chain Model with Budget Constraint

Hossein Badri[a], Babak Afghahi[b]

Abstract— **This paper presents a new dynamic model in strategic and tactical planning in a multiple echelon multiple commodity production-distribution network and a solution procedure based on Lagrangian Relaxation (LR) approach. The proposed model considers different time resolutions for tactical and strategic decisions. Also expansion of supply chain in the proposed model is restricted to cumulative net profit and investments. Commercial general purpose optimization software can solve small instances of problem; however, computational times with such software become prohibitive for reasonably sized instances. For this reason, we will adopt method to solve problem based on the Lagrangian relaxation technique. In the proposed solution algorithm, feasibility of the solutions is ensured with some modifications in subproblems. Results of the computational analysis confirm efficiency of the proposed approach.**

Keywords: Strategic supply chain management, expansion planning, Lagrangian relaxation, Subgradient method.

I. INTRODUCTION

A supply chain is defined as the chain linking each entity of the manufacturing and supply process from the raw materials to the end user. A supply chain comprises many systems, including various procurement, manufacturing, storage, transportation and retail systems [1].

The term supply chain network design (SCND) is sometimes employed as synonyms of strategic supply chain planning (see [2,3,4,5]). In the current competitive world, a supply chain network is supposed to be viable for a considerable time during which many parameters can change. It may be important to consider the possibility of making future adjustments in the network configuration to allow gradual changes in the supply chain structure and/or in the capacities of the facilities. In this case, a planning horizon divided into several time periods is typically considered and strategic decisions are to be planned for each period. Such situation occurs, for instance, when the large facility investments are limited by the budget available in each period [6]. There are several models have been developed to help managers in designing and planning of their supply chain. Arntzen et al. [7] developed a global integrated model based on mixed integer linear programming for production and distribution planning with multiple products and a network of sellers. Amiri [8] proposed a mixed integer linear model to select the optimum numbers, locations and capacities of plants and warehouses to open so that all customer demand is satisfied at minimum total costs of the distribution network in a three echelons, single period and single product. In this paper an efficient heuristic solution procedure for this supply chain system problem has been provided.

Nga Thanah et.al. [9] presented a four echelon for multiple period supply chain with dynamic demands in which they suggest adding budget constraint to their model. The proposed model has been solved and analyzed using a commercial solver.

In this paper a location and production-distribution planning problem with multiple commodities during multiple periods is considered whose main objective is to make strategic and tactical decisions in a four echelon supply chain. The proposed model is a mixed integer linear programming (MILP) model for the design and expansion planning of a four echelon multiple commodity supply chain in a long term horizon. The proposed model considers different resolutions for strategic and tactical decisions. Also this model makes some decisions about supplier selection, production facility location, warehouse location, amount of raw material to be supplied from each supplier, amount of each product to be produced in each facility, amount of each manufactured product to be sent to each customer zone and expansion planning in a long term horizon.

II. FORMULATION

There are a few papers in the literature considering facility location and production-distribution problem in a dynamic model [9]. Thanha et.al. [9] in their paper suggest budget constraint to be added for the establishment of new facilities in each period. In many firms, expansion budget is supplied by cumulative net profit after tax and stakeholders' share reduction. Since costs, incomes and thus net profit is an unknown parameter before supply chain design, so mangers are not able to determine expansion budget to use in budget constraint. The proposed model in this paper uses cumulative net profit after tax and stakeholders' share reduction in budget constraint.

Some of the most important decisions in the proposed model are as follows:

- Location and establishment time of facilities (production plant, warehouse) during the planning horizon.
- Decision about establishing a new facility or adding capacity to one or more established facility.
- Supplier selection and the raw material quantity to be supplied from them.
- Products quantity to be produced in each production plant.
- Products quantity to be transported from each production plant to each warehouse.

[a] Trade Promotion Organization of Iran, Tehran, Iran.
(Phone: +98-2121912903; email: H.badri@tpo.ir).
[b] Trade Promotion Organization of Iran, Tehran, Iran.
(Phone: +98-2121912700; email: Afghahi@tpo.ir)

ISBN: 978-988-18210-8-9
ISSN: 2078-0958 (Print); ISSN: 2078-0966 (Online)

- Products quantity to be transported from each warehouse to each customer zone.

Also some of the most important features and conditions of the proposed model are as follows:

- Objective is to maximize the supply chain net profit.
- It's not necessary to satisfy all demands; we aim to meet a portion of market demands with respect to our capabilities and restrictions so that the profit is maximized.
- Customers demand is dynamic and deterministic during time periods.
- Interest rate is considered in monetary calculations.
- Each potential node has initial capacity and maximum installable capacity.
- Facilities should operate between minimum and maximum utilization rate.
- Established production plants and private warehouses cannot close.
- Closing public warehouses is permitted.

Notations:

$\mathscr{k}(k \in \mathscr{k}, k = 1, \dots, K)$: Set of strategic periods
$\mathcal{T}(t \in \mathcal{T}, t = 1, \dots, T)$: Set of tactical periods
INV^k: Investment in period *k*
Inc^k: Net income in period *k*
$\mathcal{F}^k$: Cumulative net profit from the first period to period *k*-1
DL^k: Cumulative net profit after tax and stakeholders' share reduction from the first period to period *k*-1
TR: Tax rate
SH: Stakeholders' share (in percent)
$\mathcal{S}(s \in \mathcal{S})$: Set of suppliers
$\mathcal{M}(i \in \mathcal{M})$: Set of production plants
$\mathcal{W}(j \in \mathcal{W})$: Set of warehouses
$\mathcal{W}\wp$: Set of private (permanent) warehouses
$\mathcal{WH}$: Set of public (hired) warehouses
$\mathcal{O}(o \in \mathcal{O})$: Set of capacity options for expansion
$\mathcal{C}(c \in \mathcal{C})$: Set of customers
$\wp(p \in \wp)$: Set of products (raw material and finished product)
$\wp_r(\wp_r \subset \wp)$: Set of raw materials
$\wp_f(\wp_f \subset \wp)$: Set of finished products
$BigM$: A large number
$\mathcal{F}$: Total profit
$\mathcal{R}$: Total return after sales
$R^{k,t}_{s,p}$: Available capacity of supplier *s* for *p* at each tactical period
MK_i : Initial capacity at *i*
NK_i: Maximal installable capacity at *i*
MU_i: Minimal utilization rate of facility *i*
NU_i: Maximal utilization rate of facility *i*
CK_o: Capacity of option *o*
$D^{k,t}_{c,p}$: Demand of customer *c* for product *p* at each tactical period
$B_{p',p}$: Quantity of p' necessary to manufacture a unit of *p*
$WL_{p,i}$: Production time of a unit *p* at plant *i*
V_p: Capacity occupied by a unit *p* at warehouse *j*
$MO_{s,p}$: Minimal allowable order of a unit *p* to supplier *s*
$A_{i,j}$: Number of deliveries from plant *i* to warehouse *j* in one period
$\mathcal{PR}_p$: Selling price of a unit *p* to customers
$\mathcal{PS}_{p,s}$: Price of raw material *p* supplied by supplier *s*
Co_i: Fixed cost for opening a facility at a potential location *i*
$CA_{i,o}$: Fixed cost for adding capacity option *o* to facility *i*
CU_i: Fixed cost for operating a facility *i*
$Cop_{i,o}$: Fixed cost for operating capacity option *o* at facility *i*
$CP_{p,i}$: Variable cost of production of a unit *p* at plant *i*
$CS_{p,j}$: Storage cost of a unit of *p* at warehouse *j*
$CT_{p,i,j}$: Transportation cost of a unit of *p* from plant *i* to warehouse *j*
$CD_{p,s,i}$: Transportation cost of a unit of *p* from supplier *s* to plant *i*
$CF_{p,j,c}$: Transportation cost of a unit of *p* from warehouse *j* to customer *c*
x^k_i: 1 if the facility *i* is active at *t*; 0 otherwise
$y^k_{i,o}$: 1 if the capacity option *o* is added to *i*; 0 otherwise
$z^{k,t}_{s,p}$: 1 if the supplier *s* is selected for the raw material *p* at *t*; 0 otherwis
$f^{k,t}_{p,i,j}$: Quantity of item *p* transferred from location *i* to *j*
$q^{k,t}_{p,i}$: Quantity of product *p* produced in plant *i*
$h^{k,t}_{p,j}$: Quantity of product *p* held in warehouse *j* at the beginning of *t*

Objective Function:

The objective function is to maximize total net income over the time periods computed by subtracting total cost from total revenue. The total cost includes the fixed costs of opening facilities, adding facility options, operating facility and variable costs of raw material, production, inventory and transportation. Equation (1) shows the objective function in which the net present value of net incomes is maximized.

$$Maximize\ \mathcal{F} = \sum_{k \in \mathscr{k}} \frac{Inc^k}{(1+Ir)^{k-1}} \tag{1}$$

Constraints:

$$\sum_{j \in \mathcal{W}} f^{k,t}_{p,j,c} \le D^{k,t}_{c,p} \quad \forall c \in \mathcal{C}, \forall p \in \wp_f \tag{2}$$

$$h^{(k-1),T}_{p,j} + \sum_{i \in \mathcal{M}} f^{k,t}_{p,i,j} = \sum_{c \in \mathcal{C}} f^{k,t}_{p,j,c} + h^{k,t}_{p,j} \ \ \forall j \in \mathcal{W}, \forall p \in \wp_f, t = 1 \tag{3}$$

$$h^{k,(t-1)}_{p,j} + \sum_{i \in \mathcal{M}} f^{k,t}_{p,i,j} = \sum_{c \in \mathcal{C}} f^{k,t}_{p,j,c} + h^{k,t}_{p,j} \ \ \forall j \in \mathcal{W}, \forall p \in \wp_f, t \ne 1 \tag{4}$$

$$\sum_{s \in \mathcal{S}} f^{k,t}_{\acute{p},s,i} = \sum_{p \in \wp_f} B_{p',p}.\, q^{k,t}_{p,i} \ \ \forall i \in \mathcal{M}, \forall p' \in \wp_r \tag{5}$$

$$q^{k,t}_{p,i} = \sum_{j \in \mathcal{W}} f^{k,t}_{p,i,j} \quad \forall i \in \mathcal{M}, \forall p \in \wp_f \tag{6}$$

$$\sum_{p \in \wp_f} WL_{p,i} .\, q^{k,t}_{p,i} \le NU_i.\,(MK_i.\, x^k_i + \sum_{o \in \mathcal{O}} CK_o y^k_{i,o}) \ \ \forall i \in \mathcal{M} \tag{7}$$

$$\sum_{p \in \wp_f} WL_{p,i} .\, q^{k,t}_{p,i} \ge MU_i.\,(MK_i.\, x^k_i + \sum_{o \in \mathcal{O}} CK_o y^k_{i,o}) \ \ \forall i \in \mathcal{M} \tag{8}$$

Constraint (2) states that all products transferred to costumers should not be more than their demands in any period. We should note that in this model it's not necessary to satisfy all customer demands. Constraints (3-4) are related to equilibrium of flows at warehouses. The quantity of a product stored at the end of previous tactical period plus the total quantity of that product delivered to warehouse at the current tactical period should be equal to the quantity of that product transported to customer zones plus the quantity stored at the end of the current tactical period. Constraint (5) ensures that plants receive enough raw materials in order to produce the required quantity of finished products. Constraint (6) states that the quantity of manufactured products at a plant should be equal to its delivered quantity to warehouses. Constraints (7-8) are related to capacity of production plants. These constraints prevent a plant to function under its minimum rate of utilization and to exceed the maximum rate of utilization of its installed capacity. The installed capacity is the sum of the initial capacity and the capacity of the added options.

$$\sum_{p \in \wp_f} V_p.\left(h^{k,t}_{p,j} + \sum_{i \in \mathcal{M}} \frac{1}{2\mathrm{A}_{\mathrm{i,j}}}.\, f^{k,t}_{p,i,j}\right) \le MK_j x^k_j + \sum_{0 \in \mathcal{O}} CK_o.\, y^k_{j,o} \ \ \forall j \in \mathcal{W}\wp \tag{9}$$

$$MK_i.\, x^k_i + \sum_{0 \in \mathcal{O}} CK_o.\, y^k_{i,o} \le NK_i \quad \forall i \in \mathcal{M} \cup \mathcal{W}\wp \tag{10}$$

$$\sum_{i \in \mathcal{M}} f^{k,t}_{p,s,i} \le z^{k,t}_{s,p}.\, R^{k,t}_{s,p} \quad \forall s \in \mathcal{S}, \forall p \in \wp_r \tag{11}$$

$$\sum_{i \in \mathcal{M}} f^{k,t}_{p,s,i} \ge MO_{s,p}.\, z^{k,t}_{s,p} \ \ \forall s \in \mathcal{S}, \forall p \in \wp_r \tag{12}$$

ISBN: 978-988-18210-8-9
ISSN: 2078-0958 (Print); ISSN: 2078-0966 (Online)

Warehouses must not store more than their storage capacity (9). Also the installed capacity at any plants and any warehouse must not exceed its maximal installable capacity (10). Suppliers deliver a raw material if and only if they are selected for this raw material (11) and their delivery cannot exceed their available capacity. Constraint (12) is to avoid purchasing each raw material less than predetermined minimal amount of the delivered quantity of each supplier.

$$Inc^k = \sum_{t \in \mathcal{T}} \sum_{j \in \mathcal{W}} \sum_{p \in \wp_f} \sum_{c \in \mathcal{C}} \mathcal{PR}_{p,c} . f_{p,j,c}^{k,t} \tag{13}$$

$$- \sum_{i \in \mathcal{M} \cup \mathcal{W}\wp} Co_i . (x_i^{k+1} - x_i^k) \tag{14}$$

$$- \sum_{i \in \mathcal{M} \cup \mathcal{W}\wp} \sum_{o \in \mathcal{O}} CA_{i,o} . (y_{i,o}^{k+1} - y_{i,o}^k) \tag{15}$$

$$- \sum_{i \in \mathcal{M} \cup \mathcal{W}\wp} \left(CU_i . x_i^k + \sum_{o \in \mathcal{O}} Cop_{i,o} . y_{i,o}^k \right) \tag{16}$$

$$- \sum_{t \in \mathcal{T}} \sum_{p \in \wp_f} \sum_{i \in \mathcal{M}} CP_{p,i} . q_{p,i}^{k,t} \tag{17}$$

$$- \sum_{t \in \mathcal{T}} \sum_{p \in \wp_f} \sum_{j \in \mathcal{W}} CS_{p,j} . \left(h_{p,j}^{k,t} + \sum_{i \in \mathcal{M}} \frac{f_{p,i,j}^{k,t}}{2\mathrm{A}_{\mathrm{i,j}}} \right) \tag{18}$$

$$- \sum_{t \in \mathcal{T}} \sum_{s \in \mathcal{S}} \sum_{p \in \wp_r} \sum_{i \in \mathcal{M}} CD_{p,s,i} . f_{p,s,i}^{k,t} \tag{19}$$

$$- \sum_{t \in \mathcal{T}} \sum_{p \in \wp_f} \sum_{i \in \mathcal{M}} \sum_{j \in \mathcal{W}} CT_{p,i,j} . f_{p,i,j}^{k,t} \tag{20}$$

$$- \sum_{t \in \mathcal{T}} \sum_{p \in \wp_f} \sum_{j \in \mathcal{W}} \sum_{c \in \mathcal{C}} CF_{p,j,c} . f_{p,j,c}^{k,t} \tag{21}$$

$$- \sum_{t \in \mathcal{T}} \sum_{s \in \mathcal{S}} \sum_{p \in \wp_r} \sum_{i \in \mathcal{M}} \mathcal{PS}_{p,s} . f_{p,s,i}^{k,t} \tag{22}$$

$$\mathcal{F}^k = \sum_{k=1}^{K-1} Inc^k \tag{23}$$

$$DL^k = (1 - TR) . (1 - SH) . \mathcal{F}^k \tag{24}$$

$$\sum_{i \in \mathcal{M} \cup \mathcal{W}\wp} Co_i . (x_i^k - x_i^{k-1}) + \sum_{i \in \mathcal{M} \cup \mathcal{W}\wp} \sum_{o \in \mathcal{O}} CA_{i,o} . (y_{i,o}^k - y_{i,o}^{k-1}) \le DL^k + INV^k \tag{25}$$

Constraint (23) calculates the cumulative net income from the first period to period *k-1*. Constraint (24) calculates the expansion budget which is the net profit after tax and stakeholder share reduction. Constraint (25) prevents the cost of opening facility and adding option to some opened facilities be more than expansion budget in each period.

$$y_{i,o}^k \le x_i^k \quad \forall i \in \mathcal{M} \cup \mathcal{W}\wp, \forall o \in \mathcal{O} \tag{26}$$

$$x_i^{k-1} \le x_i^k \quad \forall i \in \mathcal{M} \cup \mathcal{W}\wp \tag{27}$$

$$y_{i,o}^{k-1} \le y_{i,o}^k \quad \forall i \in \mathcal{M} \cup \mathcal{W}\wp, o \in \mathcal{O} \tag{28}$$

$$\sum_{c \in \mathcal{C}} \sum_{p \in \wp_f} f_{p,j,c}^{k,t} \le x_j^k . BigM \quad \forall j \in \mathcal{W} \tag{29}$$

$$\sum_{o \in \mathcal{O}} y_{i,o}^k \le 1 \quad \forall i \in \mathcal{M} \cup \mathcal{W}\wp \tag{30}$$

$$y_{i,o}^k \le 1 - (x_i^k - x_i^{k-1}) \quad \forall i \in \mathcal{M} \cup \mathcal{W}\wp, o \in \mathcal{O} \tag{31}$$

$$x_i^k \in \{0,1\} \tag{32}$$

$$y_{i,o}^k \in \{0,1\} \tag{33}$$

$$z_{s,p}^{k,t} \in \{0,1\} \tag{34}$$

$$f_{p,i,i'}^{k,t} \ge 0 \tag{35}$$

$$q_{p,i}^{k,t} \ge 0 \tag{36}$$

$$h_{p,j}^{k,t} \ge 0 \tag{37}$$

Constraint (26) states that an opened facility can add available capacity options only. Constraint (27) prevents the opened facilities from closing. Constraint (28) states that we can add new capacity options but we cannot remove them. Constraint (29) ensures that only opened warehouses can send product to customers. Equation (30) states that we cannot add more than one capacity option to a facility in one period, and constraint (31) prevents adding any facility option at the first period of opening a facility. The constraint (32) requires that these variables are binary. The constraint (33) restricts these variables from taking non-negative values.

III. A LAGRANGIAN RELAXATION OF THE PROPOSED MODEL

The proposed model is a mixed-integer programming model which includes as a special case the classical capacitated facility location problem which is well known to be NP-hard [10]. Lagrangian relaxation, linear programming based heuristics and metaheuristics are among the most popular techniques [6]. Many algorithms have been developed based on lagrangian relaxation to solve facility location problems [11,12,13,14,15,16,17,18,19,20]. The reader is referred to references [21,22,23] for detailed discussion on the Lagrangian relaxation methodology.

We consider the Lagrangian relaxation of the problem obtained by dualizing constraints in sets (25) using multipliers γ^k for all $k \in \mathscr{k}$.

Problem L:

$$Maximize\ \mathcal{F} = \sum_{k \in \mathscr{k}} \frac{Inc^k}{(1+Ir)^{k-1}} - \sum_{k \in \mathscr{k}} \gamma^k \left(\sum_{i \in \mathcal{M} \cup \mathcal{W}\wp} Co_i . (x_i^k - x_i^{k-1}) + \sum_{i \in \mathcal{M} \cup \mathcal{W}\wp} \sum_{o \in \mathcal{O}} CA_{i,o} . (y_{i,o}^k - y_{i,o}^{k-1}) - DL^k - INV^k \right)$$

Subject to:

(2-22), (26-37)

Problem L can be further decomposed into two subproblems LR1 and LR2.

Problem LR1:

$$\begin{aligned}
Z_{LR1} = Max & \sum_{k \in \mathscr{k}} \sum_{t \in \mathcal{T}} \sum_{i \in \mathcal{M}} \sum_{p \in \wp_f} \sum_{j \in \mathcal{W}} \frac{1}{(1+Ir)^{k-1}} \mathcal{PR}_p . f_{p,i,j}^{k,t} \\
& - \sum_{k \in \mathscr{k}} \sum_{i \in \mathcal{M}} \frac{1}{(1+Ir)^{k-1}} \left(Co_i . (x_i^{k+1} - x_i^k) + \sum_{o \in \mathcal{O}} CA_{i,o} . (y_{i,o}^{k+1} - y_{i,o}^k) + CU_i . x_i^k + \sum_{o \in \mathcal{O}} Cop_{i,o} . y_{i,o}^k \right) \\
& - \sum_{k \in \mathscr{k}} \sum_{t \in \mathcal{T}} \sum_{p \in \wp_f} \sum_{i \in \mathcal{M}} \frac{1}{(1+Ir)^{k-1}} \left(CP_{p,i} . q_{p,i}^{k,t} + \sum_{j \in \mathcal{W}} CT_{p,i,j} . f_{p,i,j}^{k,t} \right) \\
& - \sum_{k \in \mathscr{k}} \sum_{t \in \mathcal{T}} \sum_{s \in \mathcal{S}} \sum_{p \in \wp_r} \sum_{i \in \mathcal{M}} \frac{1}{(1+Ir)^{k-1}} \left(CD_{p,s,i} . f_{p,s,i}^{k,t} + \mathcal{PS}_{p,s,n} . f_{p,s,i}^{k,t} \right) \\
& - \sum_{k \in \mathscr{k}} \sum_{i \in \mathcal{M}} \gamma^k \left(Co_i . (x_i^k - x_i^{k-1}) + \sum_{o \in \mathcal{O}} CA_{i,o} . (y_{i,o}^k - y_{i,o}^{k-1}) \right) \\
& - \sum_{k \in \mathscr{k}} \gamma^k . (1-TR) . (1-SH) . \sum_{\acute{k}=1}^{k-1} \sum_{i \in \mathcal{M}} \left(Co_i . (x_i^{k+1} - x_i^k) + \sum_{o \in \mathcal{O}} CA_{i,o} . (y_{i,o}^{k+1} - y_{i,o}^k) + CU_i . x_i^k + \sum_{o \in \mathcal{O}} Cop_{i,o} . y_{i,o}^k \right) \\
& - \sum_{k \in \mathscr{k}} \gamma^k . (1-TR) . (1-SH) . \sum_{\acute{k}=1}^{k-1} \sum_{t \in \mathcal{T}} \sum_{p \in \wp_f} \sum_{i \in \mathcal{M}} \left(CP_{p,i} . q_{p,i}^{k,t} + \sum_{j \in \mathcal{W}} CT_{p,i,j} . f_{p,i,j}^{k,t} \right) \\
& - \sum_{k \in \mathscr{k}} \gamma^k . (1-TR) . (1-SH) . \sum_{\acute{k}=1}^{k-1} \sum_{t \in \mathcal{T}} \sum_{s \in \mathcal{S}} \sum_{p \in \wp_r} \sum_{i \in \mathcal{M}} \left(CD_{p,s,i} . f_{p,s,i}^{k,t} + \mathcal{PS}_{p,s,n} . f_{p,s,i}^{k,t} \right) \\
& + \sum_{k \in \mathscr{k}} \gamma^k . INV^k
\end{aligned}$$

Subject to:

(5,8), (11-12), (34-36)

ISBN: 978-988-18210-8-9
ISSN: 2078-0958 (Print); ISSN: 2078-0966 (Online)

BC: (10), (26-28), (30-33)

The problem LR1 is related to supply and production echelons in supply chain network. Since objective function of the main problem is maximizing the total net profit of the supply chain and selling products is done at customer zones, here we supposed that in problem LR1 the products are purchased directly from production units. It's obvious that adopting such assumption causes no change to the nature of main model, because the main model tries to maximize production quantity as well. Some constraint are specific to the problem LR1, and others listed as BC are those in which only parameters and variables of supply and production echelons are considered.

Problem LR2:

$$
\begin{aligned}
Z_{LR2} = Max & \sum_{k \in \mathscr{k}} \sum_{t \in \mathcal{T}} \sum_{j \in \mathcal{W}} \sum_{p \in \wp_f} \sum_{c \in \mathcal{C}} \frac{1}{(1+Ir)^{k-1}} (\mathcal{PR}_p . f_{p,j,c}^{k,t}) \\
& - \sum_{k \in \mathscr{k}} \sum_{i \in \mathcal{W}_\wp} \frac{1}{(1+Ir)^{k-1}} \Big(Co_i . (x_i^{k+1} - x_i^k) + \sum_{o \in \mathcal{O}} CA_{i,o} . (y_{i,o}^{k+1} - y_{i,o}^k) + CU_i . x_i^k \\
& \qquad + \sum_{o \in \mathcal{O}} Cop_{i,o} . y_{i,o}^k \Big) \\
& - \sum_{k \in \mathscr{k}} \sum_{t \in \mathcal{T}} \sum_{p \in \wp_f} \sum_{j \in \mathcal{W}} \frac{1}{(1+Ir)^{k-1}} \left(CS_{p,j} . \left(h_{p,j}^{k,t} + \sum_{i \in \mathcal{M}} \frac{f_{p,i,j}^{k,t}}{2\mathrm{A}_{\mathrm{i,j}}} \right) + \sum_{c \in \mathcal{C}} CF_{p,j,c} . f_{p,j,c}^{k,t} \right) \\
& - \sum_{k \in \mathscr{k}} \sum_{i \in \mathcal{W}_\wp} \gamma^k \left(Co_i . (x_i^k - x_i^{k-1}) + \sum_{o \in \mathcal{O}} CA_{i,o} . (y_{i,o}^k - y_{i,o}^{k-1}) \right) \\
& + \sum_{k \in \mathscr{k}} \gamma^k . (1-TR) . (1-SH) . \sum_{\acute{k}=1}^{k-1} \sum_{t \in \mathcal{T}} \sum_{j \in \mathcal{W}} \sum_{p \in \wp_f} \sum_{c \in \mathcal{C}} (\mathcal{PR}_p . f_{p,j,c}^{k,t}) \\
& - \sum_{k \in \mathscr{k}} \gamma^k . (1-TR) . (1-SH) . \sum_{\acute{k}=1}^{k-1} \sum_{i \in \mathcal{W}_\wp} \Big(Co_i . (x_i^{k+1} - x_i^k) + \sum_{o \in \mathcal{O}} CA_{i,o} . (y_{i,o}^{k+1} - y_{i,o}^k) \\
& \qquad + CU_i . (x_i^k + \sum_{o \in \mathcal{O}} Cop_{i,o} . y_{i,o}^k \Big) \\
& - \sum_{k \in \mathscr{k}} \gamma^k . (1-TR) . (1-SH) . \sum_{\acute{k}=1}^{k-1} \sum_{t \in \mathcal{T}} \sum_{p \in \wp_f} \sum_{j \in \mathcal{W}} \Big(CS_{p,j} . \left(h_{p,j}^{k,t} + \sum_{i \in \mathcal{M}} \frac{f_{p,i,j}^{k,t}}{2\mathrm{A}_{\mathrm{i,j}}} \right) \\
& \qquad + \sum_{c \in \mathcal{C}} CF_{p,j,c} . f_{p,j,c}^{k,t} \Big)
\end{aligned}
$$

Subject to:

(2-4), (9), (29), (35), (37)
BC: (10), (26-28), (30-33)

The problem LR2 is related to warehouses and customer zones in which the parameters and variables of storage, distribution and selling can be seen. In this problem like the problem LR1, some constraints are specific and others (BC) are those in which only parameters and variables of warehouse and customer echelons are considered.

Solution procedure

The success of Lagrangian relaxation approach depends heavily on the ability to generate good Lagrangian multipliers [6]. Generally, the computation of a good set of multipliers is difficult [24, 25]. In this paper we use the subgradient method to drive bounds for LR. The subgradient method is an adaptation of the gradient method in which gradients are replaced by subgradients. The reader is referred to [26] which validates the use of subgradient optimization schema.

To ensure feasibility of the solutions, some constraints are added to the problem LR1 and some constraints are changed. In order to maximize the net profit, problem LR1 has tendency to maximize the production quantity, but it should be noted that it's impossible to produce products ignoring the capacity of warehouses. Constraints (38) and (39) are added to problem LR1. Constraint (38) limits the production quantity to the capacity of warehouses. To ensure feasibility of the solution a floating variable $SP(k,t)$ is defined to calculate the vacant capacity of warehouses. The objective of defining this variable will be discussed in the next section.

$$
\sum_{i \in \mathcal{M}} \sum_{p \in \wp_f} q_{p,i}^{k,t} . V_p \leq \sum_{i \in \mathcal{W}_\wp} NU_j . (MK_j . x_j^k + \sum_{o \in \mathcal{O}} CK_o y_{j,o}^k) + SP(k,t) \tag{38}
$$

$$
\sum_{i \in \mathcal{M} \cup \mathcal{W}_\wp} Co_i . (x_i^k - x_i^{k-1}) + \sum_{i \in \mathcal{M} \cup \mathcal{W}_\wp} \sum_{o \in \mathcal{O}} CA_{i,o} . (y_{i,o}^k - y_{i,o}^{k-1}) \leq DL^k + INV^k \tag{39}
$$

Constraint (39) limits establishment of the new facilities to investment and the cumulative net profit. There are some warehouse variables in these constraints and the reason is the consideration of warehouse capacities during the decision making about the location of production plants and the production quantities. Also constraints (10), (3-26) and (3- 30) whose variables and parameters had been limited to supply and production echelons, go back to the initial status. Similar to problem LR1, some modifications are done in problem LR2 to ensure feasibility of the solutions. These modifications are done by adding two constraints to problem LR2.

$$
\sum_{i \in \mathcal{M}} \sum_{j \in \mathcal{W}} f_{p,i,j}^{k,t} = \sum_{i \in \mathcal{M}} \bar{q}_{p,i}^{k,t} \tag{40}
$$

$$
\begin{aligned}
\sum_{i \in \mathcal{W}_\wp} Co_i . (x_i^k - x_i^{k-1}) + \sum_{i \in \mathcal{W}_\wp} \sum_{o \in \mathcal{O}} CA_{i,o} . (y_{i,o}^k - y_{i,o}^{k-1}) & \\
\leq DL^k + INV^k & \\
- \sum_{i \in \mathcal{M}} Co_i . (\bar{x}_i^k - \bar{x}_i^{k-1}) & \\
+ \sum_{i \in \mathcal{M}} \sum_{o \in \mathcal{O}} CA_{i,o} . (\bar{y}_{i,o}^k - \bar{y}_{i,o}^{k-1}) &
\end{aligned} \tag{41}
$$

Constraint (40) limits distributed quantity of products to the quantity of production in plants. Also constraint (41) limits establishment of warehouses to the available budget in which fixed cost of production facilities calculated in problem LR1, has been subtracted. It should be noted that $\bar{q}_{p,i}^{k,t}$, $\bar{x}_i^k$ and $\bar{y}_{i,o}^k$ are the amount of the related variables in problem LR1.

Solution algorithm

The overall solution algorithm can be summarized as follows:

Step 1:

- Set current iteration (*Iter*) to 1
- Set initial value of Lagrangian multipliers γ^k to 0, for all $k \in \mathscr{k}$.

Step 2: Set floating parameter $SP(k,t)$ to 0, for all $k \in \mathscr{k}$, $t \in \mathcal{T}$ and repeat steps 3 to 8 while the following conditions are both satisfied:

$1 - \text{Iter} \leq \max \text{Iter}$

$2 - \text{gap} \geq 0.01$

Where $gap = \frac{F_{Iter+1} - F_{Iter}}{F_{Iter+1}}$

Parameter $\mathrm{F}_{\mathrm{Iter}}$ is the objective function of the main problem at iteration $Iter$.

Step 3: Set *k* to 1 and repeat steps 4 to 6 while $\mathrm{k} \leq \mathrm{K}$.

Step 4: Solve problem LR1 only for index *k*.

Step 5: Derivate the required data from the solution of problem LR1 and feed them to Problem LR2.

Step 6: Solve problem LR2 only for index *k*.

Step 7: Derivate the final value of variables from the solutions in steps 4 and 6 according to Table I.

ISBN: 978-988-18210-8-9
ISSN: 2078-0958 (Print); ISSN: 2078-0966 (Online)

TABLE I
SOURCE OF VALUES OF DECISION VARIABLES

$LR1$		$LR2$	
x_i^k	$\forall i \in \mathcal{M}$	x_i^k	$\forall i \in \mathcal{W}$
$y_{i,o}^k$	$\forall i \in \mathcal{M}$	$y_{i,o}^k$	$\forall i \in \mathcal{W}_{\mathcal{p}}$
$z_{s,p}^{k,t}$		$f_{p,i,j}^{k,t}$	
$f_{p,s,i}^{k,t}$		$f_{p,j,c}^{k,t}$	
$q_{p,i}^{k,t}$		$h_{p,j}^{k,t}$	

Step 8: By the use of values gathered in step 7, calculate objective value of the main problem as well as the following functions, then go back to step 2.

$$\gamma_{\text{iter}+1}^k = \gamma_{\text{iter}}^k + \vartheta_{\text{iter}} \Bigg[\sum_{i \in \mathcal{M} \cup \mathcal{W}_{\mathcal{p}}} Co_i . (x_i^k - x_i^{k-1}) + \sum_{i \in \mathcal{M} \cup \mathcal{W}_{\mathcal{p}}} \sum_{o \in O} CA_{i,o} . (y_{i,o}^k - y_{i,o}^{k-1}) - DL^k - INV^k \Bigg] \quad \forall k \in \mathcal{k} \tag{42}$$

$$\vartheta_{\text{iter}} = \Delta \frac{\bar{\mathrm{F}} - \mathrm{F}_{\text{iter}}}{\sum_{k \in \mathcal{k}} \left(\sum_{i \in \mathcal{M} \cup \mathcal{W}_{\mathcal{p}}} Co_i . (x_i^k - x_i^{k-1}) + \sum_{i \in \mathcal{M} \cup \mathcal{W}_{\mathcal{p}}} \sum_{o \in O} CA_{i,o} . (y_{i,o}^k - y_{i,o}^{k-1}) - DL^k - INV^k \right)^2} \tag{43}$$

$$Iter = Iter + 1$$

Step 9: Calculate the value of parameter $SP(k,t)$ using the following equation and repeat steps 3 to 7.

$$SP(k,t) = \sum_{j \in \mathcal{W}} \left(MK_j x_j^k + \sum_{0 \in O} CK_o . y_{j,o}^k \right) - \sum_{i \in \mathcal{M}} \sum_{p \in \mathcal{p}_f} V_p . q_{p,i}^{k,t} \tag{44}$$

As mentioned before, the constraint (38) is added to problem LR1 to ensure feasibility of the solutions. This constraint limits the production quantities based on the maximum inventory. Although this constraint guarantees feasibility, but it causes low production quantity and as a result low profit for the supply chain. According to this situation, after getting an appropriate arrangement of facilities during steps 1 to 8, the vacant capacity of warehouses is calculated using parameter $SP(k,t)$, then the solution is improved significantly during the internal loop.

V. COMPUTATIONAL RESULTS

The computational analysis presented in this section is to evaluate performance of the proposed solution approach. The proposed algorithm is coded in software GAMS. The subproblems of the algorithm have been solved by the use of CPLEX MIP solver. The algorithm was run on a Dual core 2.26 GHz processor with 2GHz of RAM. Instances of were solved with random data and according to the results, increasing the size of problem, total number of variables, number of discrete variables and number of constraints increase significantly. The results illustrated in Table II confirm that the proposed algorithm effectively can reduce the solution time. We observe that in small instances because of iterative approach of the proposed algorithm, the solution time of this algorithm is more than the solution time of commercial software. In larger instances we observe that the proposed algorithm has notably better performance in solution time.

Another observation is that solution time of the commercial software increases exponentially, while the solution time of the proposed algorithm increases linearly. Since the commercial software is unable to find a feasible solution in problems of class (L), good performance of the proposed algorithm in large scale problems is proved.

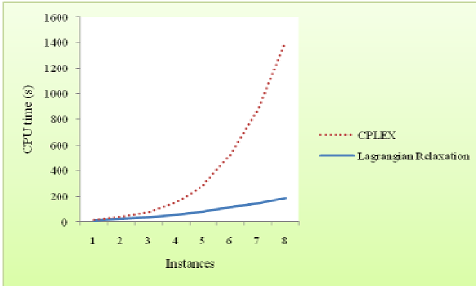

Fig.I. Solution time of CPLEX and LR approach

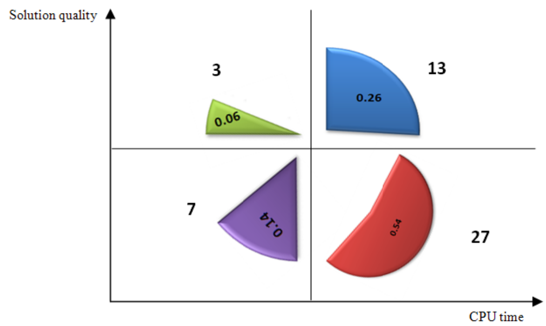

Fig. II. Comparison statistics of CPU time and solution quality

In comparison of the solution quality, %4 average of gap for all solved instances confirms that the proposed algorithm based on subgradient method has relatively good solution quality. In some instances the proposed algorithm could find a better solution than commercial software. Fig. II shows a comparison of the objective function values. The computational results

TABLE II
COMPUTATIONAL RESULTS

Class	Problem	CPLEX						Lagrangian Relaxation					
		CPU time (s)			Gap^C			CPU time (s)			$\%Gap^{LR}$		
		Min	Ave.	Max	Min	Ave.	Max	Min	Ave.	Max	Min	Ave.	Max
S	P1	2.8	3.56	4.1	0.000092	0.059562	0.089385	10.4	10.76	11.2	0.01394	0.054892	0.113981
	P2	13.8	16.52	17.7	0.027782	0.060698	0.090552	20.8	22.22	23.2	-0.07127	0.008400	0.145729
	P3	33.8	39.76	46.2	0.021684	0.060773	0.092036	33.5	33.8	34.5	-0.06401	-0.015946	0.139824
	P4	88.5	97.04	108.9	0.025007	0.042182	0.05998	53.6	55.24	56.6	-0.03286	0.018350	0.075909
	P5	191.1	208.1	228.4	0.013016	0.02249	0.040381	74.6	76.68	78.3	-0.0384	0.092581	0.168015
M	P6	320.8	408.64	469.1	0.011497	0.018737	0.024562	107.7	114.52	124.9	0.030337	0.140715	0.175535
	P7	675.6	732.52	788	0.028516	0.035685	0.043214	137.7	143.38	144.5	-0.0221	-0.00457	0.046358
	P8	1020.8	1220.48	1305.3	0.019478	0.022585	0.027465	184	186.28	187.7	-0.01674	0.04639	0.174135
L	P9	7335	7991.6	8469	0.029745	0.0474994	0.091072	623	739.8	823	0.07704	0.09494	0.122195
	P10	>3h	>3h	>3h	NFS	NFS	NFS	1399	1491	1543	NA	NA	NA

NFS: No feasible solution

shows that the proposed algorithm could make the most important strategic decisions during steps 1 to 8, but in tactical level the decision making process has not been done successfully, and it's because of solving subproblems hierarchically as well as the existence of some rational constraints added to subproblems to ensure feasibility of the solution. Table II shows the solution time and gaps in two approaches. Relations (45) and (46) are related to gaps calculation.

$$\text{Gap}^{C} = \frac{\text{Best Possible Solution} - \text{Final Solution}}{\text{Best Possible Solution}} \quad (45)$$

$$\text{Gap}^{LR} = \frac{\text{F}^{\text{C}} - \text{F}^{\text{LR}}}{\text{F}^{\text{C}}} \quad (46)$$

Totally 50 instances were designed and solved. Fig. III shows some comparison statistics about the solution time and quality. From this figure we observe that there are 7 instances in which both the solution time and quality of the proposed algorithm is worse than the solution of the commercial software. All these instances are related to problems P1 and P2. There are 3 instances in which the proposed algorithm could reach to better solution quality in more solution time. Also there are 27 instances in which the proposed algorithm could make improvement in solution time but with a worse solution quality. In 13 instances the proposed algorithm could reach to better solution quality in less CPU time.

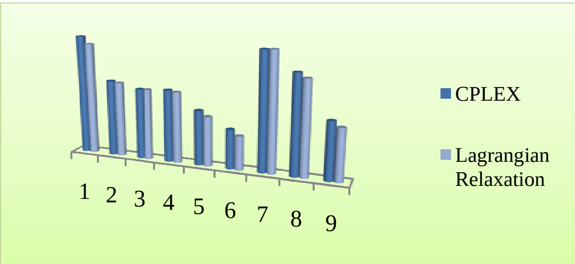

Fig.III. Objective function values in different instances

VI. CONCLUSION

In this paper a mixed integer linear programming model has been developed for the design and expansion planning of a four echelon multiple commodity supply chain in a long term horizon. Different resolutions for strategic and tactical decisions are considered in the proposed model. Also this model makes some decisions about supplier selection, production facility location, warehouse location, amount of raw material to be supplied from each supplier, amount of each product to be produced in each facility, amount of each manufactured product to be sent to each customer zone and expansion planning in a long term horizon. In the proposed model expansion of supply chain is restricted to cumulative net profit and investments. To evaluate performance of the proposed model some standard examples have been designed and solved by CPLEX solver. The results showed that the solution time of CPLEX in the real size problems is not reasonable, so a solution method was designed based on Lagrangian relaxation approach. In the proposed method, feasibility of the solutions was guaranteed by making some modifications in the subproblems. Results of the computational analysis confirmed efficiency of the proposed approach.

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ISBN: 978-988-18210-8-9
ISSN: 2078-0958 (Print); ISSN: 2078-0966 (Online)