A Push-Pull Manufacturing Strategy: Analytical Model in the Screw Cutting Sector

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Abstract—One of the major principles of Lean Manufacturing consists in only producing what is really needed by the customer. Despite a certain infatuation with this process, in the bar turning sector companies tend to produce more than the real demand. This production strategy can be explained by the desire of these companies to redeem their high changeover costs which are difficult to compress. In light of this context, our aim was to provide decision-making factors to find out the limits of this policy. To do this, we developed an analytical model based on calculating the total cost of producing the ordered parts and those manufactured in addition to the demand. This calculation has the advantage of taking into account both economic constraints, production means and sales opportunities. The introduction of a probabilistic model for estimating potential sales has also allowed us to consider the impact of the risk of non-sale on the total cost of production. Orienting bar turning companies towards a single strategy for all their production is complicated, this strategy has to be adapted to each case.

Keywords—Production, Lean manufacturing, Analytical model

I. INTRODUCTION

Today, the Lean system is a reference model for optimising performance. One of the major principles of Lean Manufacturing consists in only producing what is really needed by the customer. Any overproduction, as well as any excess stock, is considered to be a source of waste [1], [2], [3], [4]. This production with pull flows reduces excess stocks of finished products [1], [2], [5] and thus allows immobilisation costs related to over-storage to be decreased. Over the last ten years, this process has been applied intensively in many sectors of activity, including that of mechanical subcontracting. However, up to now, bar turning companies (mechanical subcontracting companies) have produced more than really requested by the customer. This production method can be justified due to a particularly high changeover time that is difficult to compress in the bar turning sector. Thus, producing with high flows could allow the production cost of the products manufactured to be reduced by redeeming the changeover costs. Moreover, the parts produced could be sold later, if there are sales opportunities. However, this manufacturing strategy has the disadvantage of increasing storage costs [6].

If the storage cost becomes higher than the changeover cost, this policy could lead to the opposite effect to the desired one and increase the production costs of the products. Companies could then risk heavy financial losses. What is, therefore, the best production strategy for bar turning companies? Should they produce more and stock or produce just enough? These two production approaches are rarely used exclusively [7]. Actually, several studies have shown that it is necessary to combine these two methods [8], [9]. To find the best compromise to be made between these productions approaches we have to take into consideration the changeover costs and the storage costs. Thus, a method known as the economic order quantity or the Wilson formula has been developed by F. W. Harris in 1913 [10], [11]. This model results in a batch size calculation [12]. Initially planned for calculating a quantity to order, it can be applied to identify a quantity to produce [13]. However, the costs and gains linked to sales opportunities are ignored [6]. Moreover, this method is inadequate when demand varies over time [14], [15]. Moreover, one of the most important aspects affecting the performance of a supply chain is the management of production. For these reasons, it would seem to be necessary to develop a new approach that allows us to evaluate the optimal quantity to be manufactured when a customer places an order. Calculating an optimal quantity to produce should simultaneously take into consideration the storage costs, the costs of changeover, the production means constraints, the sales demands and opportunities and the risks associated with overproduction. In this study, we develop an analytical model to respond to this problem. In order to optimise the consideration of the risk of non-sale, we suggest introducing a probabilistic model for estimating potential sales. The choice of the laws of probability used will be made depending on the sales history of a bar turning company located in the Arve valley and will lead us to identify the most adapted law model. We will put forward three probabilistic model situations for the sales history in accordance with an Exponential law and a Weibull law.

II. METHODS AND CONCEPTS USED

A. Model for calculating the optimal quantity to produce

The model for calculating the optimal quantity to produce is designed to be adapted to the bar turning sector. It is made by taking into consideration the economic constraints, the production means constraints and the sales opportunities associated with their probability.

1) Model hypotheses

The bar turning company studied has 30 employees and is located at the heart of the Arve valley (Haute-Savoie,
France). The Arve valley is considered to be one of the main local production systems in France. Companies in the Arve valley provide 60% of the French turnover in the bar turning activity [16] that means the machining of mechanical parts from mainly metal bars.

The typical functioning framework for this company leads us to put forward the following hypotheses (hypotheses validated in the company):
- The changeover time are long (> 4 hours) and difficult to compress
- Extra products to the quantity ordered are stored before being sold
- The quantities ordered are considered to be delivered immediately after manufacture.

2) Means constraints

The production means constraints allow us to take into account the “time” factor without which no manufacturing is possible. These constraints are evaluated from the available time (AT) of the resources needed for manufacturing the quantity of extra parts (X) and the part cycle time (CT). This available time is established from the resource identified as a bottleneck. Actually the bottleneck resource limits all the flows, if it is not available, the manufacturing flow of the part is stopped [17], [18]. To calculate these constraints, the following equation is suggested:

$$X \leq \frac{AT}{CT}$$

(1)

3) Economic constraints

The economic constraints are taken into account via the production cost calculation for the manufactured products. The total production cost is obtained by adding three costs [19], [13]:

Total production cost =
- Raw material purchase cost
- Production cost
- Cost not including production

Where:
- The raw material purchase costs correspond to the price invoiced by the supplier increased by the specific costs linked to purchase other than structural costs (transport costs, insurance, customs duties, etc.)
- The production costs correspond to the value added (labour, machine costs including changeover costs, plus the general costs specific to production)
- Costs not including production of the product correspond to the general costs not including production, such as distribution or administration costs.

In the bar turning sector, the changeover times are long and difficult to compress, we concentrated on the impact of the changeover cost on the cost price. This cost is independent of the quantity of parts produced; it mainly depends on the changeover time.

To calculate the cost price (CP), we can distinguish between two cases:
- For pull flow production, the cost price (CP) per part depends on the quantity ordered (QO) and the changeover cost (CC).

$$CP = \frac{CC + UC \times QO}{QO}$$

(2)

With:
- UC = the unit cost price not including the changeover cost

Thus:

$$UC = CP \times \frac{CC}{QO}$$

(3)

- In the case of push flow production, the extra parts are stored before being sold a cost of ownership has to be calculated. The total cost of ownership (CO) of the extra parts (X) for the storage period is established depending on the ownership rate (i) and the cost price not including the changeover cost (UC):

$$CO = (UC \times X) \times i$$

(4)

In this relationship, the changeover cost is not taken into consideration. Actually, in the real functioning of the company studied, the changeover cost is sold with the parts ordered. The annual ownership rate (i) is the ownership rate for one euro of stored products. It is obtained by dividing the total cost of the costs of ownership by the average stock.

These costs cover:
- The interest of the immobilised capital
- Shelving costs (renting and maintaining premises, insurance, personnel costs and handling)
- Material deteriorations
- Obsolescence risks

Depending on the type of part and the quality of stock management, the rate used in companies is between 15% and 35% ([13]). In this context, the production cost per part depends on the extra quantity produced (X) and the cost of ownership (CO) allocated:

$$CP = \frac{CC + UC \times QO + X}{QO + X}$$

(5)

Or,

$$CP = \frac{CC + UC \times (QO + X)}{QO + X} + \frac{(UC \times X) \times i}{QO + X}$$

(6)

Thus,

$$CP = \frac{UC \times (i + 1) + UC \times QO + CC}{QO + X}$$

(7)

However, this production cost calculation does not take into consideration the estimation of the sales probability (P). Actually, this sales probability is associated with two events: The case where the extra quantity (X) is sold and the case where the extra quantity (X) is not sold. The cost (C1) associated with the first event can be estimated from the production cost equation:

$$C1 = \frac{UC \times (i + 1) + UC \times QO + CC}{QO + X} \times P$$

(8)

With:
- P = the sales probability
The cost \( C_2 \) associated with the second event (corresponding to the case where the extra quantity \( X \) is not sold) is as follows:

\[
C_2 = \frac{UC(X_j + QO + X)}{QO} + CC(1 - P)
\]

(9)

In this equation, the unit cost is only redeemed depending on the quantity ordered. The parts made in excess of the demand \( (X) \) are considered to be lost.

Taking into account these two events can be done from the following calculation of the expected value \( (E(X)) \):

\[
E(X) = \sum_{i=1}^{n} x_i P(x_i)
\]

(10)

Applied to the costs \( (C) \), this equation is expressed as follows:

\[
E(C) = \sum_{i=1}^{n} C_i P(C_i)
\]

(11)

From the calculation of these two independent costs, the expected value of the total manufacturing cost \( (E(CT)) \) is as follows:

\[
E(CT(i,X)) = \sum_{i=1}^{n} C_i P(C_i) = C1.P(C1) + C2.(1 - P(C2))
\]

(12)

Thus,

\[
E(CT(i,X)) = \frac{UC(X_j + QO + X) + CC P}{QO} + \frac{UC(X_j + QO + X) + CC}{QO}(1 - P)
\]

(13)

4) Model for sales probability

In order to model the sales probability three stages are needed.

Stage 1: looking for the development model based on the sales history

To estimate the sales probability and therefore the sales forecasts, it is necessary to take into account the sales history of the part manufactured. Sales forecast methods take into consideration the development forms of the sales history. Thus, for example, for a constant horizontal history, the simple moving average and balanced moving average methods are used. For a history with a trend (increasing or decreasing) exponential smoothing methods with a trend have the advantage of limiting the quantity of data to take into consideration. For the seasonal aspects of products, the multiplicative seasonal method is an adapted approach [20]. One of the main methods of causal forecast is linear regression [20]. Finally, the analysis of chronological series allows us to quickly create a number of short-term forecasts needed to create a production schedule.

These models allow us to estimate the development of the average future sales.

Stage 2: transforming and correcting data

To reduce and correct the possible bias linked to the trend of the sales history (increase or decrease), it is necessary to transform these data into a homogenous sample.

A sample is homogenous when its values are collected on the same date and linked to the same sales probability law. However, a sales history is made up of values collected over several months or years. In most cases, sales develop according to different models such as the trend model (increase or decrease in the average of chronological series depending on the duration), the seasonal model (variations in demand are periodical), the cyclical model (variations in demand are gradual over long periods). To obtain a homogenous sample, we suggest applying a transformation to each data to correct this bias.

Correction of the temporal bias

This bias is introduced by the development of sales over time (temporal bias) and consequently follows a known development model. The knowledge of this model allows us to emancipate this bias and to obtain a homogenous sample.

So \( Y_i^p \) is data obtained for the period \( t_i \). A sample is defined as homogenous if all the \( Y_i^p \) are collected in the same period \( t_i \) or in an equivalent period characterised in particular by the same central value \( E(Y) \) or the median.

If we know the development of these characteristics from a period \( j \) to a period \( p \), we can transform the data of period \( j \) to the estimated data of period \( p \) and thus obtain a homogenous sample. We should point out that these models correspond to the regression, to the simple exponential smoothing and to the seasonal model (with a seasonal trend).

If we apply this model to \( Y_i^p \) for a change of temporal reference from \( t_j \) to \( t_p \), then \( Y_i^p \) becomes \( Y_i^{t_p} \). With the trend model, we have:

\[
Y_i^{t_p} = Y_i^p + a(t_p - t_j)
\]

(14)

With,

\( Y_i^{t_p} = \) transformed data; \( Y_i^p = \) real data corresponding to the quantity sold in the year \( t_i \); \( a = \) coefficient calculated from the history forecast model; \( t_p = \) periodic reference; \( t_j = \) period \( j \) corresponding to the current year.

Thus the homogenous sample \( (E) \) of size \( n \) looked for is obtained by transformation:

\[
E_n^p = \{Y_1^{t_p}, Y_2^{t_p}, Y_3^{t_p}, \ldots, Y_n^{t_p}\}
\]

(15)

Stage 3: looking for the most adapted model for the data

From these data transformed into a homogenous sample, we can then look for the most appropriate law model for sales probability. Actually, extra sales possibilities are conveyed by different probability laws. Different law models are distinguished depending on the variables studied: continual law models for continual variables and discreet law models for discrete variables (for example, the number of parts manufactured). Several law models are likely to be applied for estimating sales probability. Thus, for discreet random variables, models following a geometric law, a type I discreet Weibull law, a Poisson law, a Binomial law, a hypergeometric law and an Erlang law can be used; for
continual random variables, models following an Exponential law, a Weibull law, a Log-normal law and a Gamma law can be suggested. We should note that continual law models are also used in most industrial applications (by rounding up the results found to a whole number of units).

The choice of most adapted law model is made in two stages. The first consists in defining the scale parameter from the classic sales forecast models mentioned above. The second stage consists in defining the shape parameter of the law which characterises the random part of the sales probability. For example, for a Weibull law with two parameters, the Eta scale parameter is linked to the development of the average and the form parameter is the $\beta$ parameter.

The choice of most appropriate law model is based on the maximum likelihood method. For our application, we use Weibull ++ software.

III. PROBABILISTIC MODEL OF SALES ESTIMATION FOR INDUSTRIAL APPLICATION

A. Data collected

We studied three examples of independent mechanical parts manufactured in a bar turning company. The data collected for the manufacture of the parts named $a$, $b$ and $c$ are presented in table 1. Manufactured part $a$ has a changeover cost of €3000, part $b$ €2500 and part $c$ €2300. The unit production cost for part $a$ not including the changeover cost is equal to €0.06, for part $b$ it is €0.5 and for part $c$ it is €0.05. The allocated ownership rate for one year of immobilisation has been estimated at 15% for these three parts. The company has received an order of 10,000 parts $a$, an order for 15,000 parts $b$ and an order for 16,000 parts $c$.

<table>
<thead>
<tr>
<th>TABLE I.</th>
<th>DATA COLLECTED FOR THE MANUFACTURE OF THE PARTS NAMED $A$, $B$ AND $C$.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Changeover costs (€)</td>
<td>CS</td>
</tr>
<tr>
<td>Unit cost price (not including the changeover cost) (€)</td>
<td>CU</td>
</tr>
<tr>
<td>Quantity ordered (number of parts)</td>
<td>QC</td>
</tr>
<tr>
<td>Ownership rate for one year of storage</td>
<td>i%</td>
</tr>
</tbody>
</table>

B. Applying probabilistic models for estimating sales

1) Looking for the development model of the sales history

The sales histories for parts $a$, $b$ and $c$ is shown in table 2 and Fig. 1.

<table>
<thead>
<tr>
<th>TABLE II.</th>
<th>SALES HISTORIES FOR THE PARTS NAMED $A$, $B$ AND $C$.</th>
</tr>
</thead>
</table>

The sales history for products $a$ and $c$ can be modelled by a trend model with the following form:

$$y(t) = at + b$$  \hspace{1cm} (16)

With,

$a = \text{coefficient calculated from the history forecast model}$; $t =$ period of reference; $b =$ intercept

From the equation 16 and data collected, results are the following for parts $a$ and $c$:

$$y_a(t) = -3103t + 6245335$$  \hspace{1cm} (17)

$$y_c(t) = -5494042t + 2742$$  \hspace{1cm} (18)

The sales histories for parts can be modelled by a constant average model:

$$y'_b = c = \text{sales mean} = 10917$$  \hspace{1cm} (19)

2) Transforming and corrected data:

From equation 17, the quantity transformed from 2004 to 2009 is calculated in the following way for part $a$:

$$x'^{2009}_a = 26000 - 3103 \ast (2009 - 2004)$$

$$x'^{2009}_a = 10487$$

The transformation of the data collected in the same temporal reference provides the following results for products $a$ and $c$ (table 3). For part $b$, no transformation of data is needed since the history for this part is in the form of a constant average model.

<table>
<thead>
<tr>
<th>TABLE III.</th>
<th>TRANSFORMATION OF THE DATA COLLECTED IN THE SAME TEMPORAL REFERENCE</th>
</tr>
</thead>
<tbody>
<tr>
<td>Corrected data in the year 2009 as the temporal reference</td>
<td>Year</td>
</tr>
<tr>
<td>2004</td>
<td>10487</td>
</tr>
<tr>
<td>2005</td>
<td>13090</td>
</tr>
<tr>
<td>2006</td>
<td>14452</td>
</tr>
<tr>
<td>2007</td>
<td>10285</td>
</tr>
<tr>
<td>2008</td>
<td>15287</td>
</tr>
<tr>
<td>2009</td>
<td>10000</td>
</tr>
</tbody>
</table>

3) Looking for the most adapted model for the data

The adequacy analysis between the law models previously mentioned and the sales history of the products manufactured gives the following results (Table 4):
TABLE IV. POSITION OF MODELS FOR ESTIMATION OF SALES OF PARTS A, B AND C (WITH THE MAXIMUM LIKELIHOOD METHOD)

<table>
<thead>
<tr>
<th>Model</th>
<th>Part a</th>
<th>Part b</th>
<th>Part ac</th>
</tr>
</thead>
<tbody>
<tr>
<td>Exponential 1</td>
<td>7</td>
<td>2</td>
<td>8</td>
</tr>
<tr>
<td>Exponential 2</td>
<td>3</td>
<td>2</td>
<td>1</td>
</tr>
<tr>
<td>Normal</td>
<td>2</td>
<td>5</td>
<td>3</td>
</tr>
<tr>
<td>Lognormal</td>
<td>5</td>
<td>2</td>
<td>6</td>
</tr>
<tr>
<td>Weibull 2</td>
<td>1</td>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>Gamma</td>
<td>6</td>
<td>3</td>
<td>7</td>
</tr>
<tr>
<td>Logistic</td>
<td>2</td>
<td>4</td>
<td>4</td>
</tr>
<tr>
<td>Loglogistic</td>
<td>4</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

The adequacy analysis between the law models and the sales histories shows that the Weibull model with two parameters is the most adapted one for modelling the sales estimation for parts a and b. For part c, the exponential model is the most appropriate one (Table 4).

Probabilistic model of sales for pars a and b:
The Weibull type probabilistic model of sales is expressed as follows:

\[ P_t(x) = e^{-\left(\frac{x}{\tau}\right)^\beta} \]  (20)

From equation 20 of the sales model obeying a Weibull type law, the expectation of the total manufacturing cost is as follows:

\[ E(CT(i,X)) = \frac{UC(Xi+QO+X) + CC}{QO+X} e^{-\left(\frac{x}{\tau}\right)^\beta} + \frac{UC(Xi+QO+X) + CC}{QO} \left(1 - e^{-\left(\frac{x}{\tau}\right)^\beta}\right) \]  (21)

From equation 21 and the data relating to the manufacture of part a, we obtain the following equation:

\[ E(CT) = \frac{0.06 (0.15 X + 10000 + X) + 3000}{10000 + X} e^{-\left(\frac{x}{13402}\right)^{7.69}} + \frac{0.06 (0.15 X + 10000 + X) + 3000}{10000} \left(1 - e^{-\left(\frac{x}{13402}\right)^{7.69}}\right) \]  (22)

It is possible to calculate the quantity of extra parts that minimises the total cost corresponding to the minimum of the curve from equation 22 (Fig. 2).

Probabilistic model of sales for part c:
Concerning the sales of part c, we used the exponential type law model (Table 4).

\[ P(x) = e^{-\tau X} \]  (24)

The expectation of the total cost (E(CT)) taking into account a probabilistic model of sales obeying an exponential parameter law \( \tau \) is expressed as follows:
From equation 1 the results concerning the production means constraints for parts  

\[ E(CT(i,X)) = \frac{UC(X_i + QO + X) + CC \cdot e^{-\alpha}}{QO + X} + \frac{UC(X_i + QO + X) + CC \cdot (1 - e^{-\alpha})}{QO} \]

(25)

From this last equation and the data relating to the manufacture of part c, we obtain the following equation:

\[ E(CT) = \frac{0.05(X.015 + 16000 + X) + 2300}{16000 + X} e^{5.09E-5 \cdot X} + \frac{0.05(X.015 + 16000 + X) + 2300}{16000} (1 - e^{5.09E-5 \cdot X}) \]

(26)

As we saw beforehand, it is possible to identify the total cost optimality when the company produces more than the demand from equation 26 (Fig. 4).

IV. DISCUSSION OF THE RESULTS

Finding out the total manufacturing cost incurred by ordered production and by possible overproduction is a key item in decision-making for the strategy of a company. For the bar turning sector, calculating this cost should take into consideration both the manufacturing costs for the parts ordered and the extra parts manufactured, the high changeover costs and the storage costs.

The introduction of a probabilistic model for estimating sales in the calculation of this cost allows us to consider the impact of the risk of non-sale on the total cost of production.

In order to create a probabilistic model for sales estimation, the first-stage of our study consisted in looking for the most adapted sales forecast model. We were interested in the manufacture of three independent parts associated with three different sales histories. The sales history of parts a and c led us to use trend models whereas the one for part b led us to use a constant average model. We should point out that the sales histories used do not include a lot of data. However, for a homogenous sample, this low number of values is sufficient to find the representative model but with a high confidence interval. A larger number of data would allow this confidence interval to be reduced. However, it is difficult to have a sales history over a very long period in companies.

The following stages consisted in modelling the sales probability law by law models that are most adapted to the data transformed from the sales history. Thus, the probabilistic model for part c led to using an exponential type model whereas for parts a and b we used a Weibull type model with a \( \beta \) parameter equal to 7.69 for part a and 1.61 for part b. Parameter \( \beta \) characterises the form of the law; the higher the latter is, the less the quantities sold are dispersed.

As opposed to this, the exponential law used for the modelisation of part c characterises very dispersed random phenomena.

These results underline the importance of having the most reliable sales history possible. Actually, this history depends on the choice of sales forecast model and the choice of the most adapted law for modelling the sales probability law.

Depending on the forecast model used, the development of the total production cost will therefore be different. Using a Weibull type model first of all causes a slight reduction in the total manufacturing cost until overproduction of 6900 parts a and 532 products for part b. On the other hand, for part c, using an exponential type model causes an increase in the total manufacturing cost from the first extra part manufactured.

For a new product (that the company has no sales history for), it would then be necessary to focus on expert opinion integration methods such as Delphi and fuzzy logic methods.

The application case concerning the manufacture of part a has allowed us to identify the maximum extra quantity to produce guaranteeing a minimum total manufacturing cost (6900 parts). However, calculating the means constraints limits the company to overproduction of 3600 parts.

This last result is in contradiction with the Lean principle recommending producing the right quantity at the right time and available in the right place ([1]). This can be explained by one of the specific points of the bar turning sector: long
changeover times that are difficult to compress. Redeeming changeover costs over a larger number of parts manufactured allows us, in this case, to reduce the total manufacturing cost.

Concerning the manufacture of parts c, whatever the means constraints, the company should not produce more than really requested by the customer. Any extra overproduction would directly bring about an increase in its total production cost. This result agrees with the lean principle.

V. CONCLUSION

Thus we have developed a new approach allowing us to optimise costs by taking into account a combination of opportunities and constraints to calculate an optimal quantity to produce. A major advantage of our analytical model is the taking into consideration of the estimation of sales and the developments of them. In our study, this estimation is based on introducing a probabilistic model for estimating sales. Bar turning companies instinctively tend to produce more than the really requested by the customer in order to redeem the cost of changeover over a larger number of parts. By calculating the total cost of production, our model allows us to identify the optimal quantity of extra parts to produce without increasing this cost. The approach developed defines the limit to not be exceeded by taking into consideration the drawbacks linked to the risks of non-sale. We should point out that companies have an easily access to the data needed for using this model.

Should be produce more and stock it or produce just enough?

Orienting bar turning companies towards a single strategy for all their production, pull flow production or push flow production is complicated, and may not be adequate. Actually, as we have shown, depending on the part manufactured, it may be preferable to produce as lean as possible or to produce more than the customer's real request. To choose the best production strategy for a company, a company should consider each product independently.

REFERENCES